Analyzing Jet Substructure with Energy Flow

Elementary Particle Physics Journal Club

Eric M. Metodiev

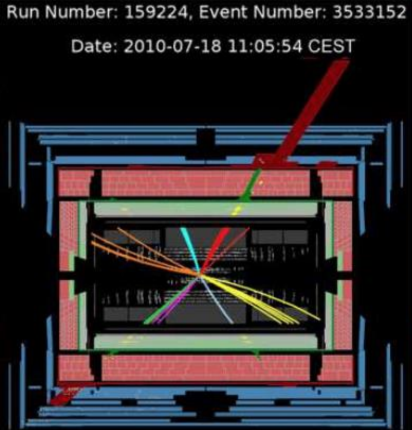
Center for Theoretical Physics

Massachusetts Institute of Technology

Joint work with Patrick Komiske and Jesse Thaler

[1712.07124] [1810.05165] [19xx.xxxxx]

April 26, 2019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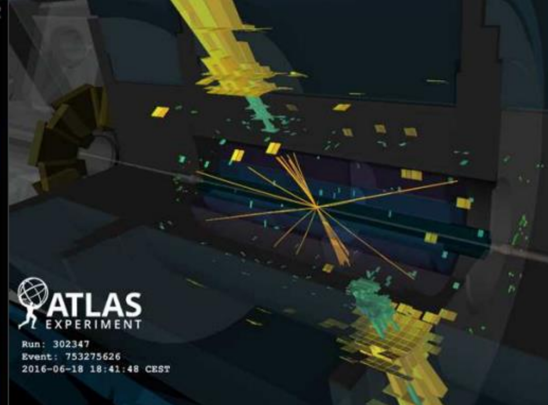
ATLAS
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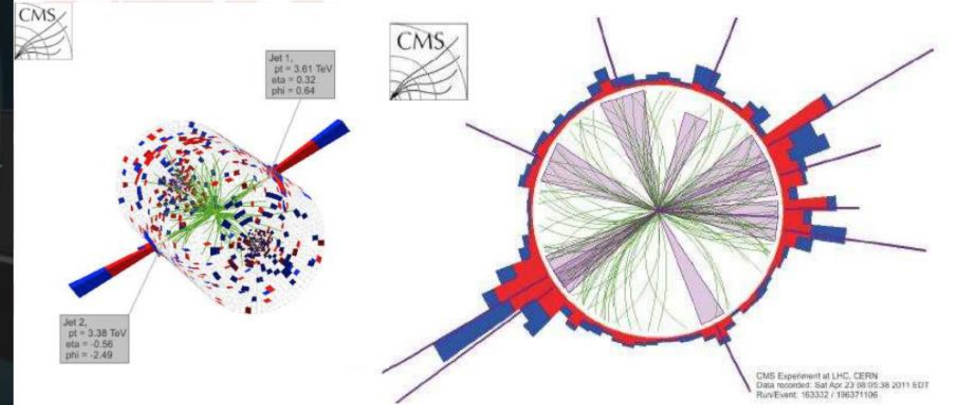
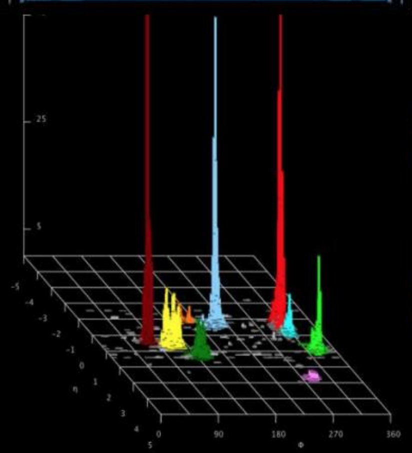
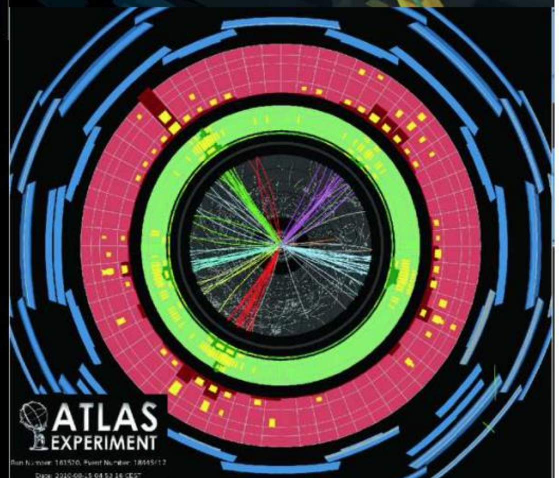

CMS
Jet 1,
pt = 3.61 TeV
eta = 0.32
phi = 0.64
Jet 2,
pt = 3.38 TeV
eta = -0.56
phi = -2.49
CMS Experiment at LHC, CERN




ATLAS
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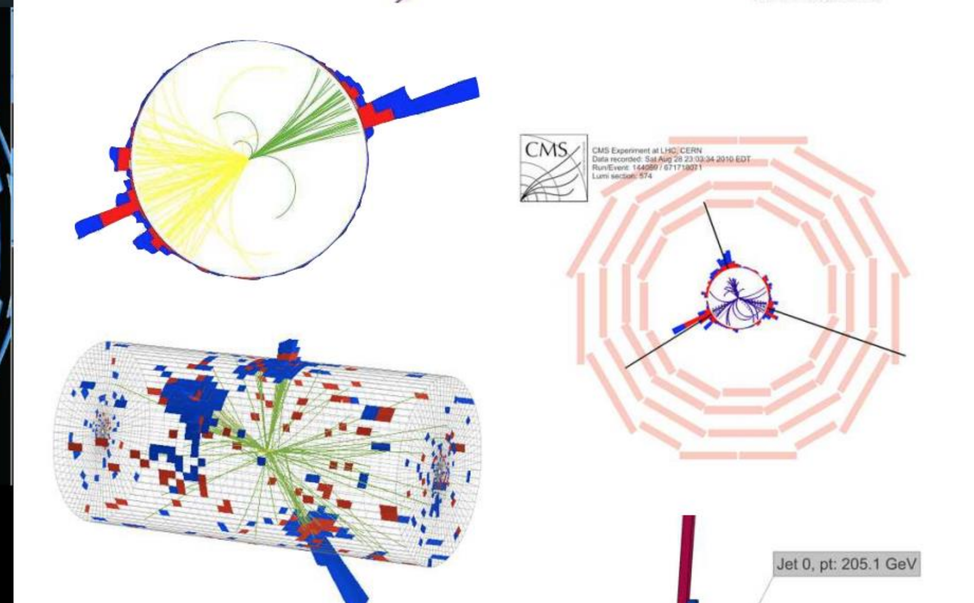
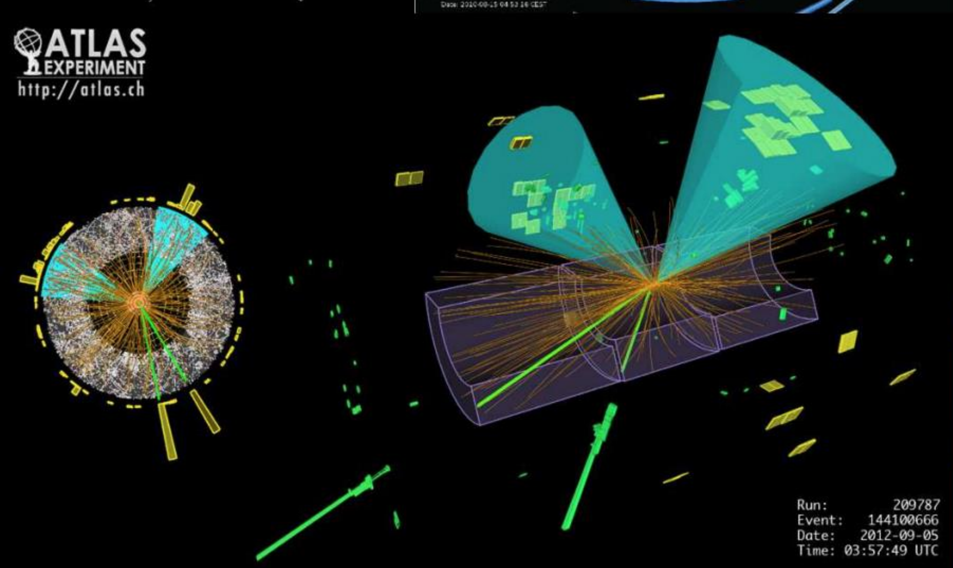

CMS
CMS Experiment at LHC, CERN


ATLAS
EXPERIMENT
http://atlas.ch
Run: 209787
Event: 144100666
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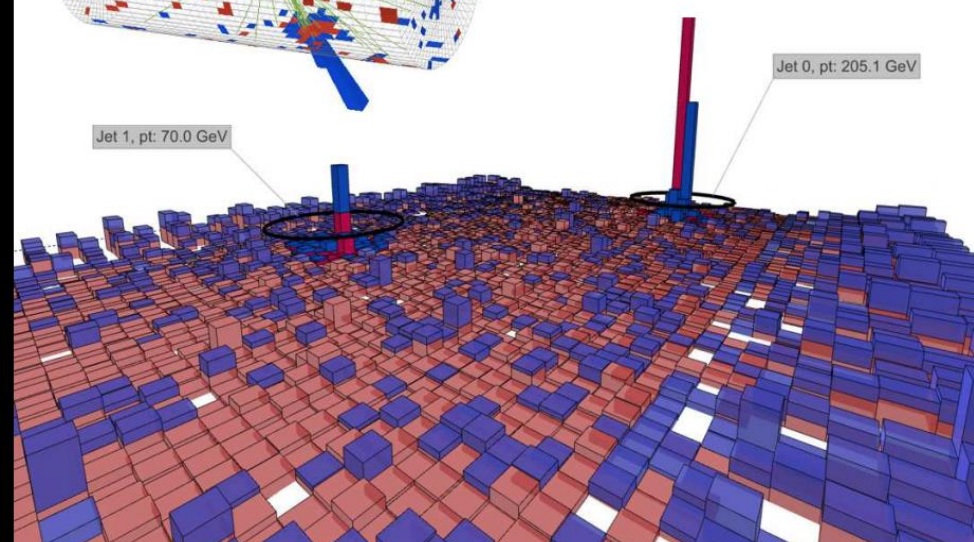
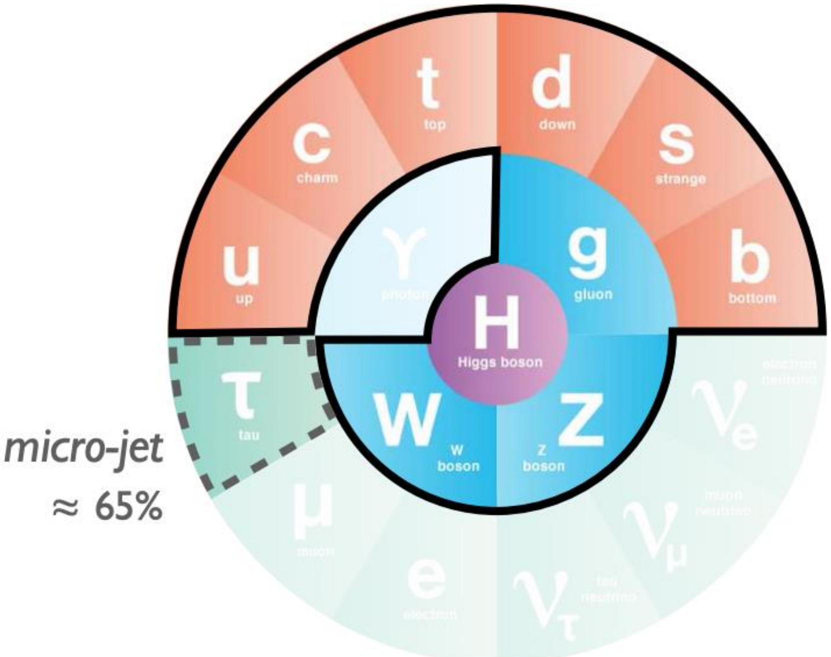
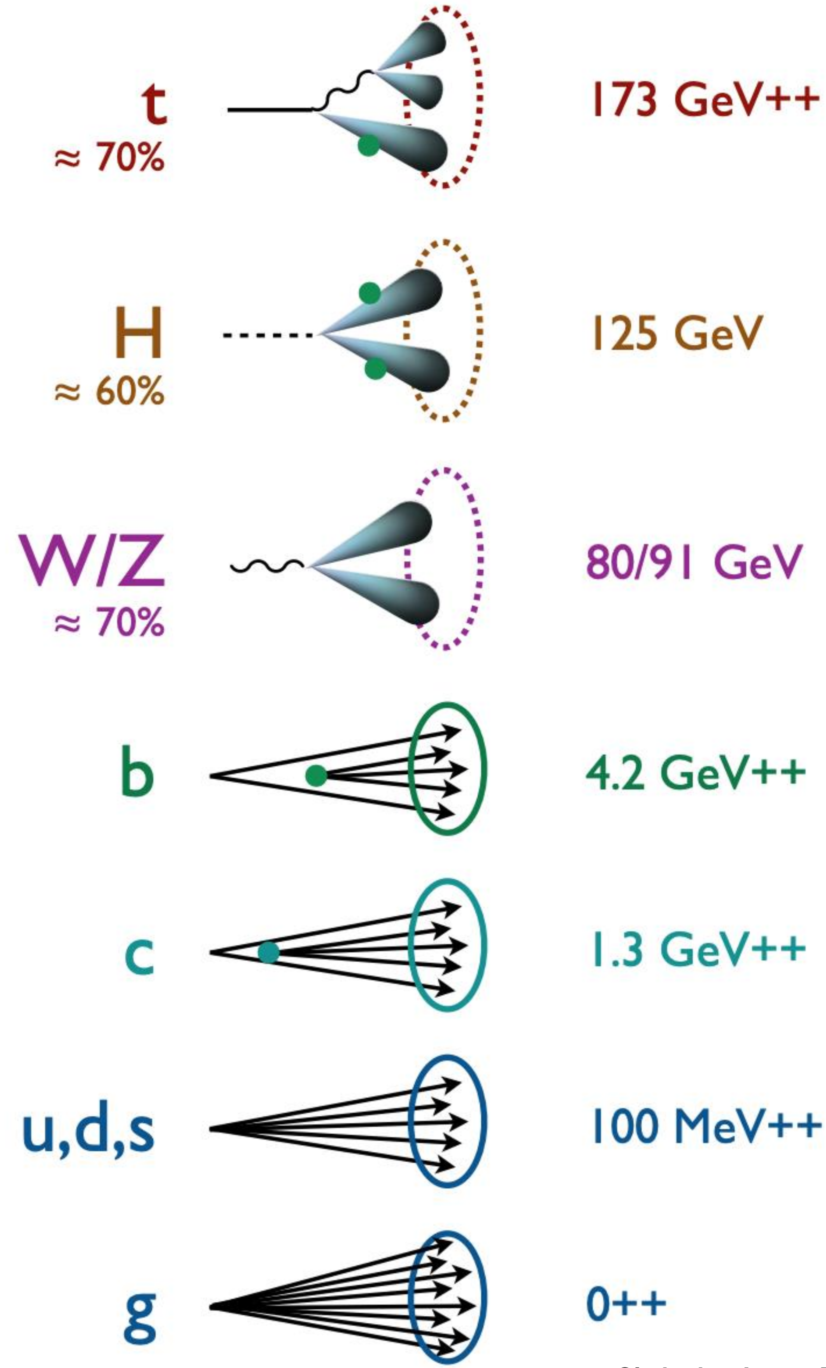
Time: 03:57:49 UTC


Jet 0, pt: 205.1 GeV
Jet 1, pt: 70.0 GeV



Jets from the Standard Model

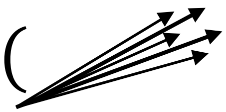
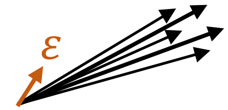


++ = Mass from QCD Radiation

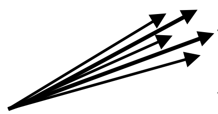
Slide by Jesse Thaler

IRC Safety

Infrared (IR) safety – observable is unchanged under addition of a soft particle:

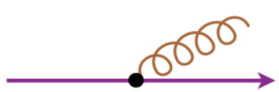
$S($  $)$ $=$ $S($  $)$

Collinear (C) safety – observable is unchanged under collinear splitting of a particle:

$S($  $)$ $=$ $S($  $)$ $\quad \forall \lambda \in [0,1]$

IRC safety guarantees that the soft and collinear divergences of a QFT cancel at each order in perturbation theory (KLN theorem)

Divergences in QCD splitting function:

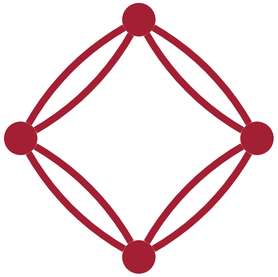


$$dP_{i\to ig} \simeq \frac{2\alpha_s}{\pi} C_i \frac{d\theta}{\theta} \frac{dz}{z}$$

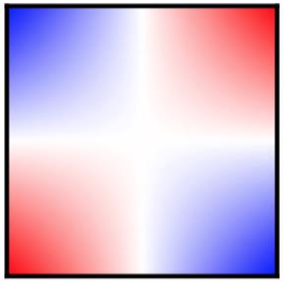
$$C_q = C_F = 4/3$$
$$C_g = C_A = 3$$

IRC-safe observables probe hard structure while being insensitive to low energy or small angle modifications

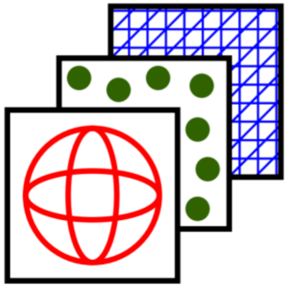
Outline



Energy Flow Polynomials
A basis of jet substructure observables

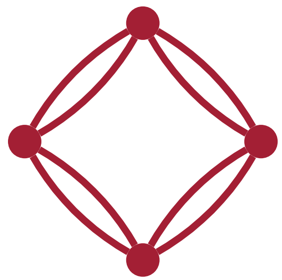


Energy Flow Moments
Tensor moments of the radiation pattern



Energy Flow Networks
ML architecture designed to learn from events

Outline



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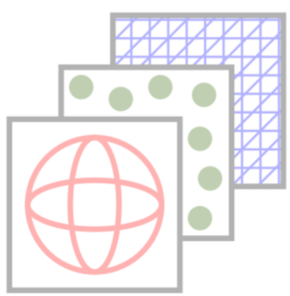
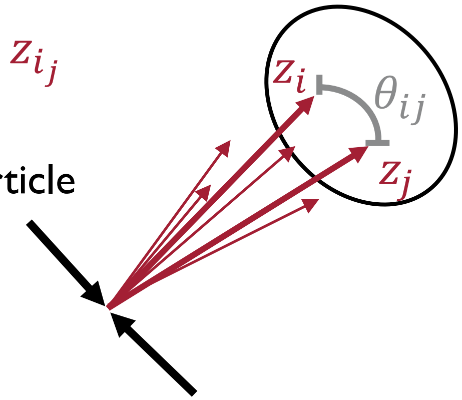


Energy Flow Networks
ML architecture designed to learn from events

Expanding an Arbitrary IRC-safe Observable

Arbitrary IRC-safe observable: $S(p_1^\mu, \dots, p_M^\mu)$ [1712.07124]

- Energy expansion: Approximate S with polynomials of z_{i_j}
  - IR safety: S is unchanged under addition of soft particle
  - C safety: S is unchanged under collinear splitting of a particle
  - Relabeling symmetry: Particle index is arbitrary



Energy correlator parametrized by angular function f

$$\sum_{i_1=1}^{M} \cdots \sum_{i_N=1}^{M} z_{i_1} \dots z_{i_N} f(\hat{p}_{i_1}, \dots, \hat{p}_{i_N})$$

[F. Tkachov, hep-ph/9601308]

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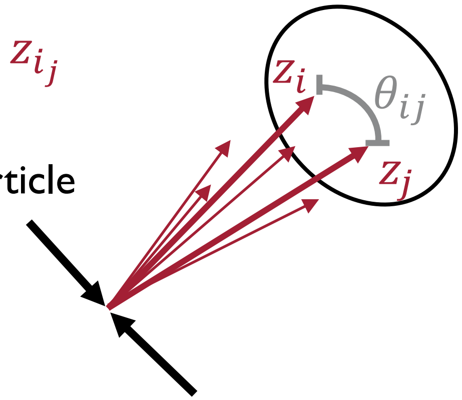

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Energy correlators linearly span IRC-safe observables

- Angular expansion: Approximate f with polynomials in θ_{ij}
- Simplify: Identify unique analytic structure that emerge

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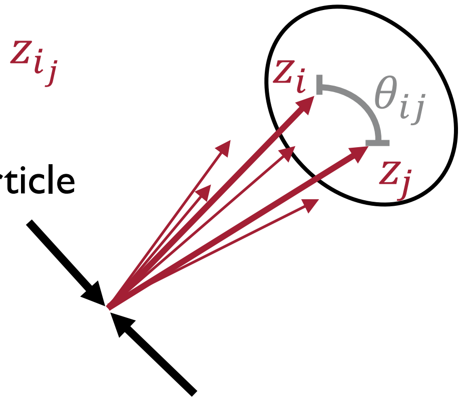

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[F. Tkachov, hep-ph/9601308]

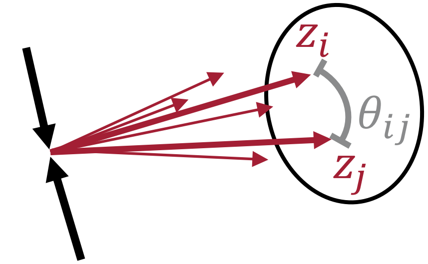
➡ Energy correlators linearly span IRC-safe observables

- Angular expansion: Approximate f with polynomials in θ_{ij}
- Simplify: Identify unique analytic structure that emerge

➡ Obtain linear spanning basis of Energy Flow Polynomials, "EFPs":

$$S \simeq \sum_{g \in G} s_G \, \mathrm{EFP}_G \,, \qquad \mathrm{EFP}_G \equiv \sum_{i_1=1}^{M} \cdots \sum_{i_N=1}^{M} z_{i_1} \cdots z_{i_N} \prod_{(k,\ell)\in G} \theta_{i_k i_\ell}$$

Anatomy of an Energy Flow Polynomial:



In equations:

$$\mathrm{EFP}_{\mathrm{G}} = \sum_{i_1=1}^{M} \sum_{i_2=1}^{M} \cdots \sum_{i_N=1}^{M} z_{i_1} z_{i_2} \cdots z_{i_N} \prod_{(k,l)\in \mathrm{G}} \theta_{i_k i_l}$$

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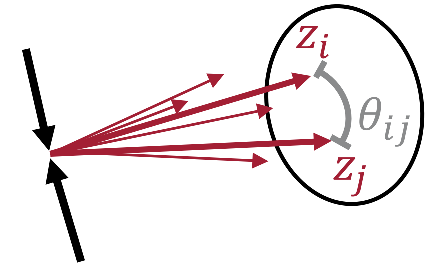


In equations:

$$\mathrm{EFP}_G = \underbrace{\sum_{i_1=1}^{M} \sum_{i_2=1}^{M} \cdots \sum_{i_N=1}^{M}} \underbrace{z_{i_1} z_{i_2} \cdots z_{i_N}} \underbrace{\prod_{(k,l)\in G} \theta_{i_k i_l}}$$

In words:

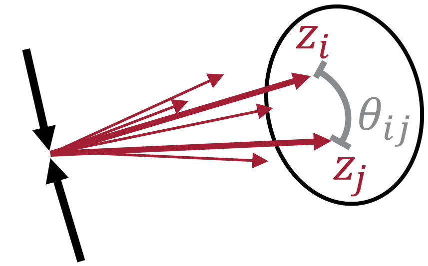
Correlator of **Energies** and **Angles**

Correlator: Sum over all N-tuples of particle in the event

Energies: Product of the N energy fractions

Angles: One $\theta_{i_k i_l}$ for each edge in $(k,l) \in G$

Anatomy of an Energy Flow Polynomial:



In equations:

$$\mathrm{EFP}_G = \sum_{i_1=1}^{M} \sum_{i_2=1}^{M} \cdots \sum_{i_N=1}^{M} z_{i_1} z_{i_2} \cdots z_{i_N} \prod_{(k,l)\in G} \theta_{i_k i_l}$$

In words:

Correlator — Sum over all *N*-tuples of particle in the event

of **Energies** — Product of the *N* energy fractions

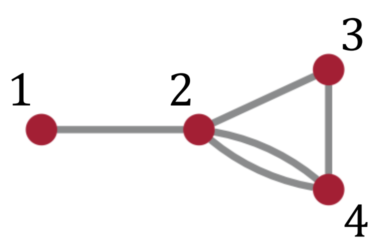
and **Angles** — One $\theta_{i_k i_l}$ for each edge in $(k,l) \in G$

In pictures:

$\longleftrightarrow$ z_{i_j}

$\longleftrightarrow$ $\theta_{i_k i_l}$

(e.g.)



(any index labelling works)

$$= \sum_{i_1=1}^{M} \sum_{i_2=1}^{M} \sum_{i_3=1}^{M} \sum_{i_4=1}^{M} z_{i_1} z_{i_2} z_{i_3} z_{i_4} \, \theta_{i_1 i_2} \theta_{i_2 i_3} \theta_{i_3 i_4} \theta_{i_2 i_4}^2$$

Organization of the basis

EFPs are truncated by angular degree *d*, the order of the angular expansion.

Finite number at each order in *d*
All prime EFPs up to *d=5* →

Exactly 1000 EFPs up to degree *d=7*

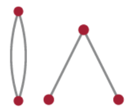
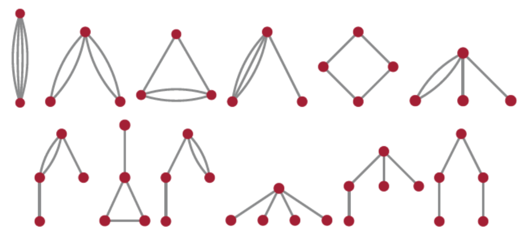
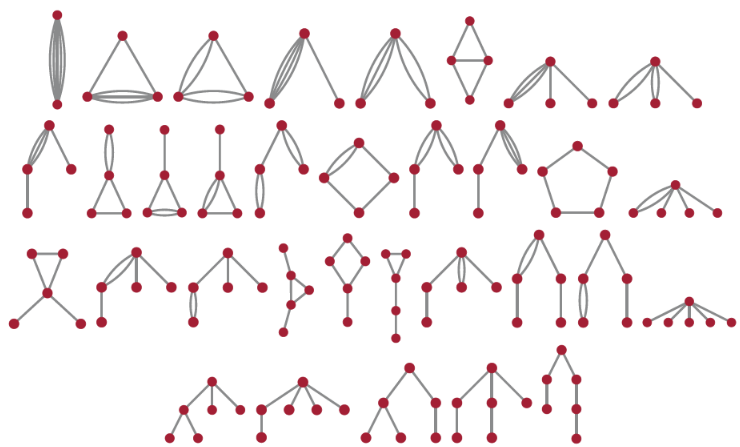
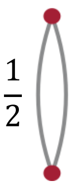
Degree	Connected Multigraphs
$d = 1$	
$d = 2$	
$d = 3$	
$d = 4$	
$d = 5$	

Image files for all of the prime EFP multigraphs up to d = 7 are available [here].

Familiar Jet Substructure Observables as EFPs

Scaled Jet Mass:

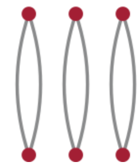
$$\frac{m_J^2}{p_{TJ}^2} = \sum_{i_1=1}^{M} \sum_{i_2=1}^{M} z_{i_1} z_{i_2} (\cosh \Delta y_{i_1 i_2} - \cos \Delta \phi_{i_1 i_2}) = \frac{1}{2}$$

$$+ \cdots$$

Jet Angularities:

$$\lambda^{(\alpha)} = \sum_{i}^{M} z_i \theta_i^{\alpha}$$

$$\lambda^{(6)} =$$

$$- \frac{3}{2}$$


$$+ \frac{5}{8}$$


$$\lambda^{(4)} =$$

$$- \frac{3}{4}$$

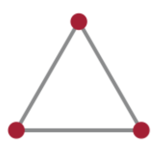

[C. Berger, T. Kucs, and G. Sterman, hep-ph/0303051]
[S. Ellis, *et al.*, 10010014]
[A. Larkoski, J. Thaler, and W. Waalewijn, 1408.3122]

Energy Correlation Functions(ECFs):

$$e_N^{(\beta)} = \sum_{i_1=1}^{M} \sum_{i_2=1}^{M} \cdots \sum_{i_N=1}^{M} z_{i_1} z_{i_2} \cdots z_{i_N} \prod_{k<l \in \{1,\cdots,N\}} \theta_{i_k i_l}^{\beta}$$

[A. Larkoski, G. Salam, and J. Thaler, 1305.0007]

$$e_2^{(\beta)} =$$

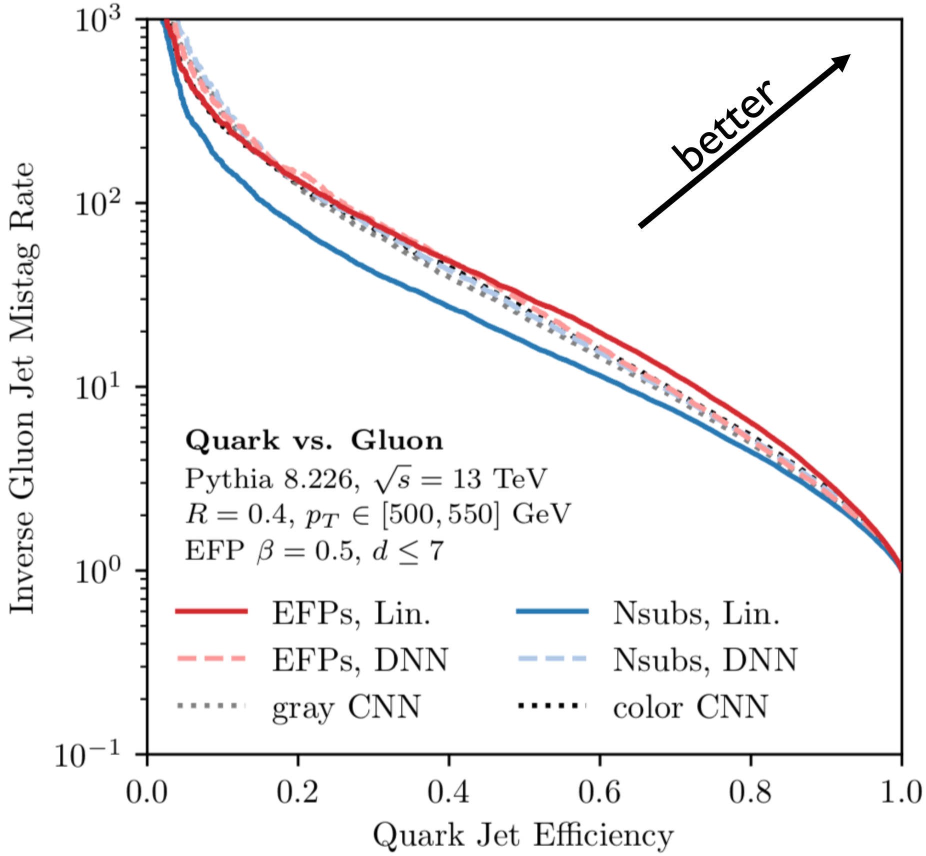

$$e_3^{(\beta)} =$$


$$e_4^{(\beta)} =$$


and many more…

Jet Tagging Performance – Quark vs. Gluon Jets

ROC curves for quark vs. gluon jet tagging



N-subjettiness:
[J. Thaler, K. Van Tilburg, 1011.2268, 1108.2701]

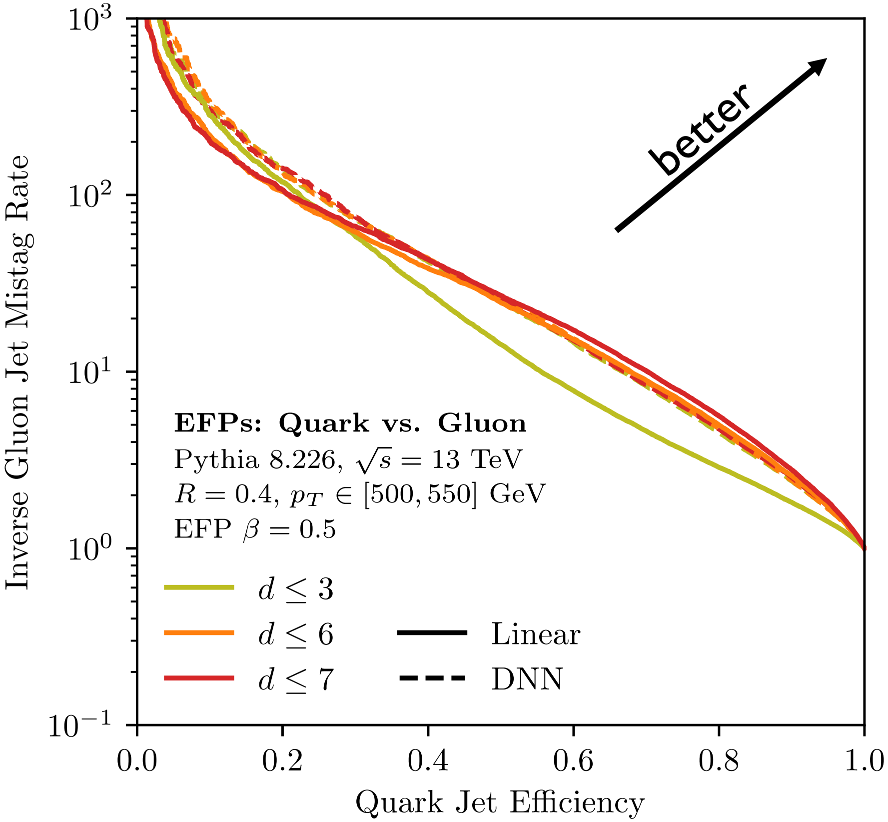
N-subjettiness basis:
[K. Datta, A. Larkoski, 1704.08249]

QG CNNs:
[P. Komiske, EMM, M. Schwartz, 1612.01551]

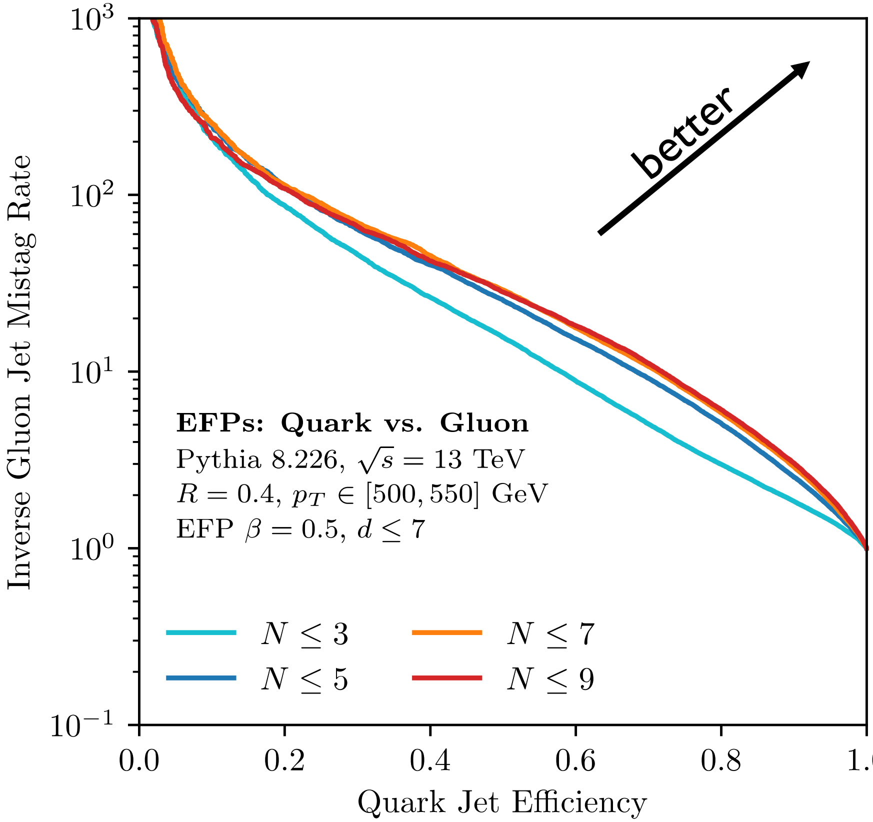
ML/NN review:
[A. Larkoski, I. Moult, B. Nachman, 1709.04464]

Linear classification with EFPs is comparable to modern machine learning techniques

Additional EFP Tagging Plots – Quark vs. Gluon Jets

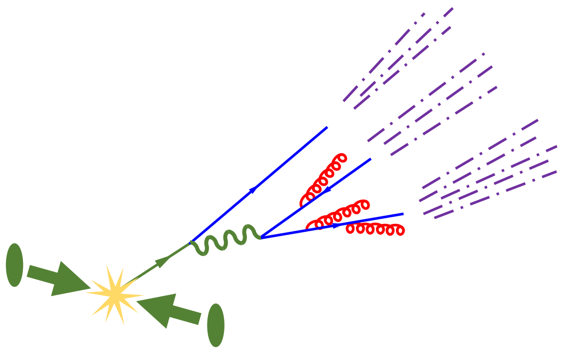


High d EFPs are important
Convergence by $d \leq 7$



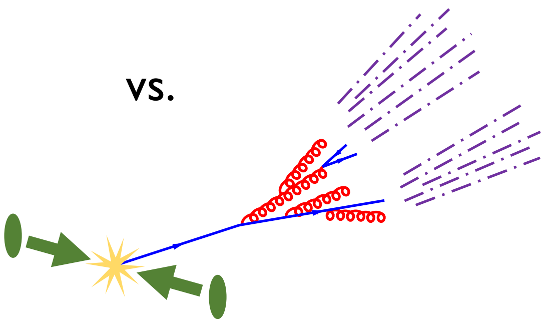
High N EFPs are important

Top Tagging Community Comparison



Top jet

vs.



QCD jet

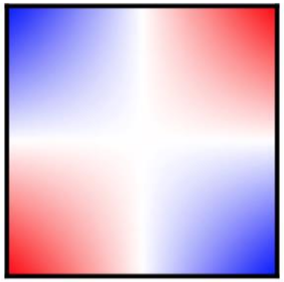
Community comparison of top tagging methods:

	AUC	Acc	#Param
CNN [16]	0.981	0.930	610k
ResNeXt [30]	0.984	0.936	1.46M
TopoDNN [18]	0.972	0.916	59k
Multi-body N-subjettiness 6 [24]	0.979	0.922	57k
Multi-body N-subjettiness 8 [24]	0.981	0.929	58k
TreeNiN [43]	0.982	0.933	34k
P-CNN	0.980	0.930	348k
ParticleNet [47]	0.985	0.938	498k
LBN [19]	0.981	0.931	705k
LoLa [22]	0.980	0.929	127k
Energy Flow Polynomials [21]	0.980	0.932	1k
Energy Flow Network [23]	0.979	0.927	82k
Particle Flow Network [23]	0.982	0.932	82k
GoaT	0.985	0.939	35k

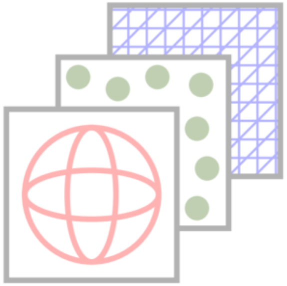
[1902.09914]

Outline

Energy Flow Polynomials
A basis of jet substructure observables



Energy Flow Moments
Tensor moments of the radiation pattern



Energy Flow Networks
ML architecture designed to learn from events

Energy Flow Moments

Take: $\theta_{ij} = n_i^{\mu} n_{j\,\mu} = (1 - \hat{n}_i \cdot \hat{n}_j)$

$$m^2 =$$

$$= \underbrace{\sum_{i=1}^{M} \sum_{j=1}^{M} E_i \, E_j \, n_i^{\mu} \, n_{j\,\mu}}_{O(M^2)} = \underbrace{\left(\sum_{i=1}^{M} E_i n_i^{\mu} \right) \left(\sum_{j=1}^{M} E_j n_{j\,\mu} \right)}_{O(M)}$$

Take: $\theta_{ij} = n_i^{\mu} n_{j\,\mu}$

Energy Flow Moments

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$$= \underbrace{\sum_{i=1}^{M}\sum_{j=1}^{M} E_i\, E_j\, n_i^{\mu}\, n_{j\,\mu}}_{O(M^2)} = \underbrace{\left(\sum_{i=1}^{M} E_i n_i^{\mu}\right)\left(\sum_{j=1}^{M} E_j n_{j\,\mu}\right)}_{O(M)}$$

$$\boxed{I^{\mu_1\mu_2\cdots\mu_v} = \sum_{i=1}^{M} E_i n_i^{\mu_1} n_i^{\mu_2} \cdots n_i^{\mu_v} =}$$

[See also J. Donogue, F. Low, S-Y. Pi, 1979]

Energy Flow Moments

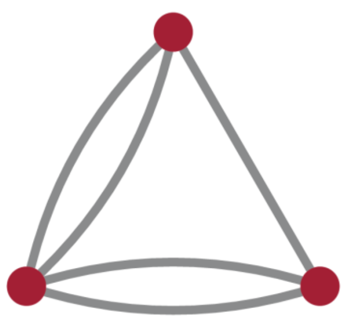
Take: $\theta_{ij} = n_i^\mu n_{j\,\mu}$

$$m^2 = $$

$$= \underbrace{\sum_{i=1}^{M}\sum_{j=1}^{M} E_i\, E_j\, n_i^\mu\, n_{j\,\mu}}_{O(M^2)} = \underbrace{\left(\sum_{i=1}^{M} E_i n_i^\mu\right)\left(\sum_{j=1}^{M} E_j n_{j\,\mu}\right)}_{O(M)}$$

$$\boxed{I^{\mu_1\mu_2\cdots\mu_v} = \sum_{i=1}^{M} E_i n_i^{\mu_1} n_i^{\mu_2} \cdots n_i^{\mu_v} =}$$

[See also J. Donogue, F. Low, S-Y. Pi, 1979]



$$= \sum_{i_1=1}^{M}\sum_{i_2=1}^{M}\sum_{i_3=1}^{M} E_{i_1} E_{i_2} E_{i_3}\, \theta_{i_1 i_2}^2\, \theta_{i_1 i_3}^2\, \theta_{i_2 i_3}$$

$$= I^{\mu\nu\rho\sigma}\, I^{\tau}{}_{\mu\nu}\, I_{\rho\sigma\tau}$$

Energy Flow Moments

$$I^{\mu_1\mu_2\cdots\mu_v} = \sum_{i=1}^{M} E_i n_i^{\mu_1} n_i^{\mu_2} \cdots n_i^{\mu_v} =$$ 

Understand EFP relations:

$$5! \times \mathcal{I}^{\mu_2}_{[\mu_1}\mathcal{I}^{\mu_3}_{\mu_2}\mathcal{I}^{\mu_4}_{\mu_3}\mathcal{I}^{\mu_5}_{\mu_4}\mathcal{I}^{\mu_1}_{\mu_5]} = 6 \times$$  $$- 5 \times$$   $$= 0$$

via Cayley-Hamilton and 3+1 dimensions

Counting symmetric kinematic polynomials:

THE ON–LINE ENCYCLOPEDIA OF INTEGER SEQUENCES®

A226919 Asymptotic number of completely symmetric polynomials of degree n up to momentum conservation in the limit as the number of particles increases.

1, 0, 1, 2, 5, 11, 34, 87, 279 (list; graph; refs; listen; history; text; internal format)

OFFSET 0,4

COMMENTS The values for n >= 6 are only conjectural.

LINKS Table of n, a(n) for n=0..8.

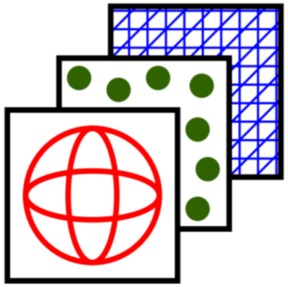
R. H. Boels, On the field theory expansion of superstring five point amplitudes, Nuclear Physics B, Vol. 876, No. 1 (2013), 215-233.

	Loopless, Leafless Multigraphs	
Edges d	Connected	Total
1	0	0
2	1	1
3	2	2
4	4	5
5	9	11
6	26	34
7	68	87
8	217	279
9	**718**	**897**
10	**2 553**	**3 129**
11	**9 574**	**11 458**
12	**38 005**	**44 576**
13	**157 306**	**181 071**
14	**679 682**	**770 237**
15	**3 047 699**	**3 407 332**
16	**14 150 278**	**15 641 159**

Outline

Energy Flow Polynomials
A basis of jet substructure observables

Energy Flow Moments
Tensor moments of the radiation pattern



Energy Flow Networks
ML architecture designed to learn from events

Towards Machine Learning

A generic IRC-safe observable O can be written via moments as:

$$O = F\left(\sum_{i=1}^{M} E_i n_i^{\mu_1} n_i^{\mu_2} \cdots n_i^{\mu_v}\right)$$

[P. Komiske, EMM, J. Thaler, 1810.05165]

Idea: Let angular structure be generic function:

$$\vec{\Phi}: R^2 \to R^\ell$$
$$F: R^\ell \to R$$

Energy Flow Network
IRC safe

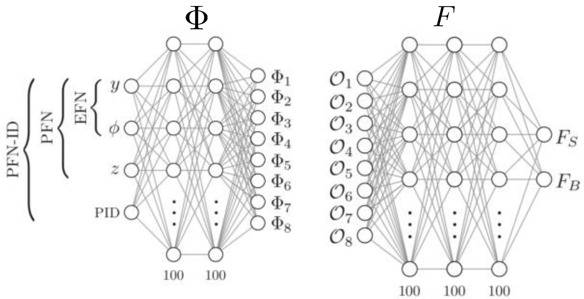
$$\mathrm{EFN} = F\left(\sum_{i=1}^{M} E_i\, \vec{\Phi}(\hat{n}_i)\right)$$

Approximate $\vec{\Phi}$ and F with neural networks to learn an observable.

Generalize beyond IRC safety?

Particle Flow Network
IRC unsafe

$$\mathrm{PFN} = F\left(\sum_{i=1}^{M} \vec{\Phi}(E_i, n_i^{\mu}, \cdots)\right)$$



Many observables are easily interpreted in EFN and PFN language

Deep Sets

All permutation symmetric functions have an additivity similar to EFMs

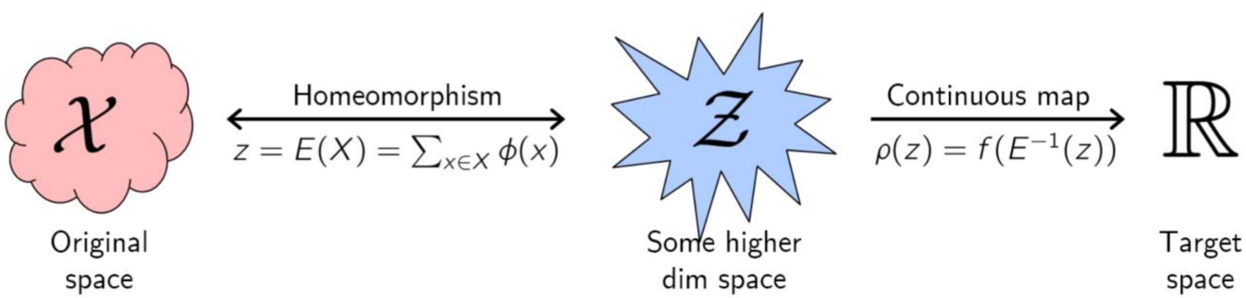
Deep Sets

[1703.06114](1703.06114)

Manzil Zaheer[1,2], **Satwik Kottur**[1], **Siamak Ravanbhakhsh**[1],
Barnabás Póczos[1], **Ruslan Salakhutdinov**[1], **Alexander J Smola**[1,2]
[1] Carnegie Mellon University   [2] Amazon Web Services

Theorem 7 *Let $f : [0,1]^M \to \mathbb{R}$ be a permutation invariant continuous function iff it has the representation*

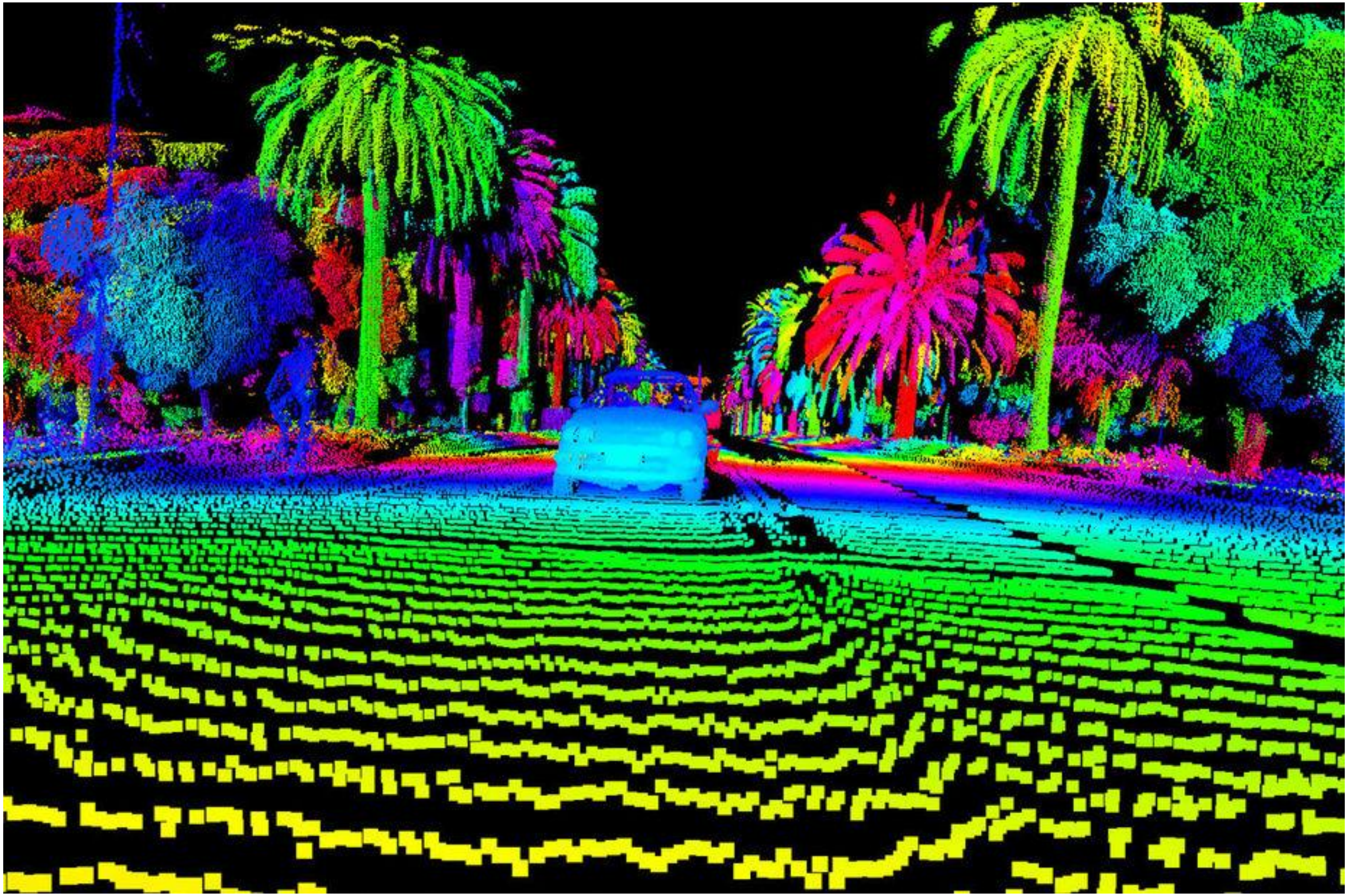
$$f(x_1, ..., x_M) = \rho\left(\sum_{m=1}^{M} \phi(x_m)\right) \tag{18}$$



Proof sketch: Stone-Weierstrass theorem with elementary symmetric polynomials

Point Cloud

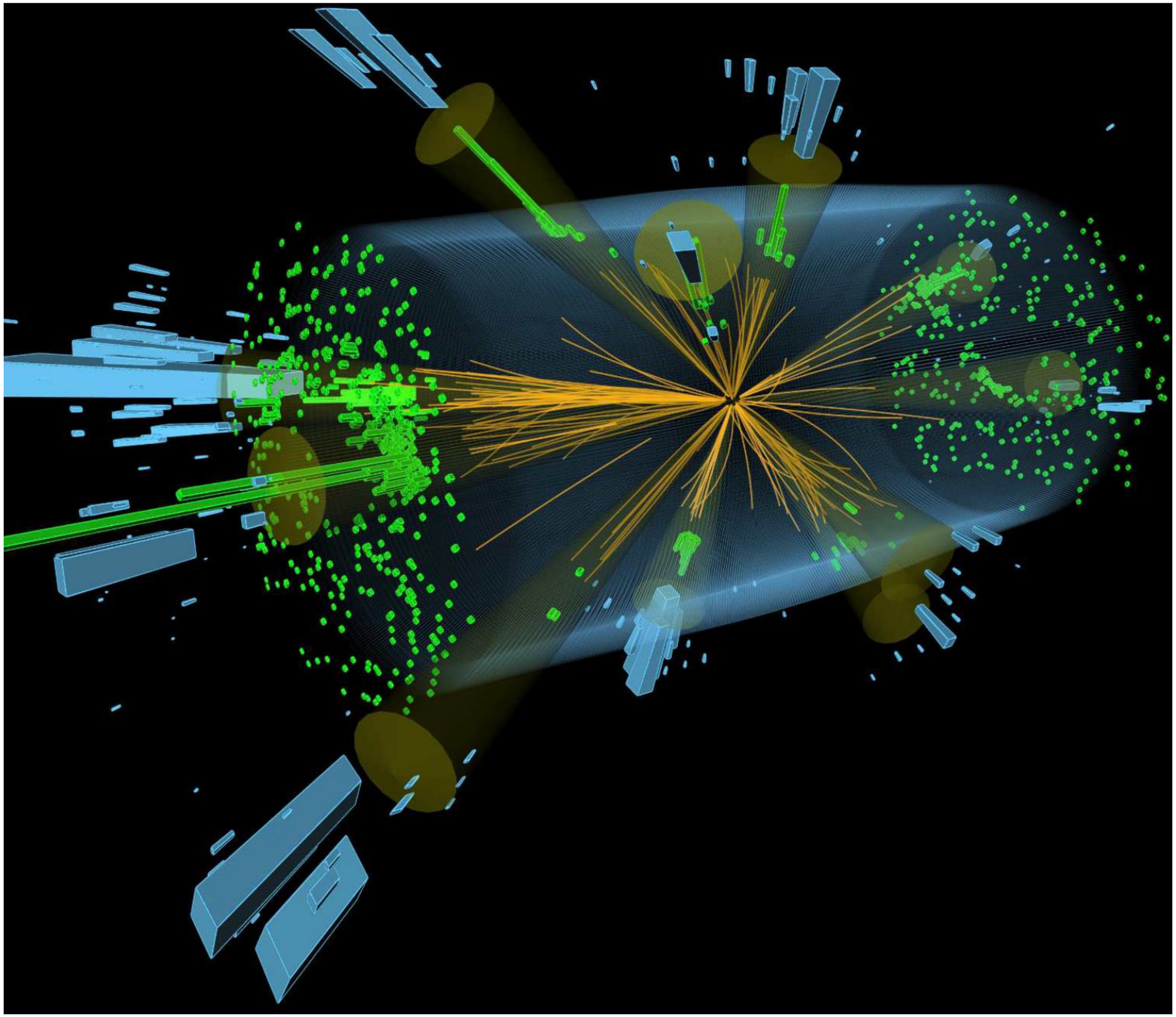
Point Cloud: “A set of points in space.” - Wikipedia



A frame from a Luminar LIDAR system

Point Cloud

Point Cloud: "A set of points in space." - Wikipedia

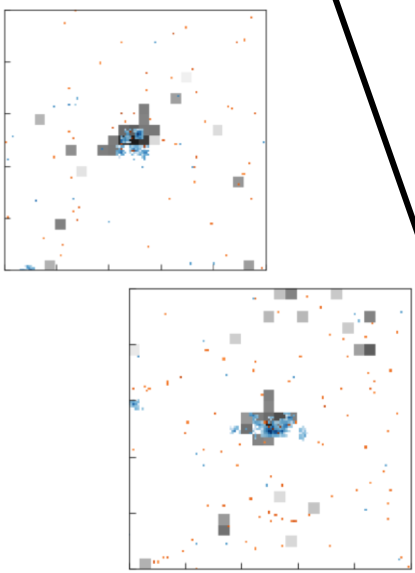


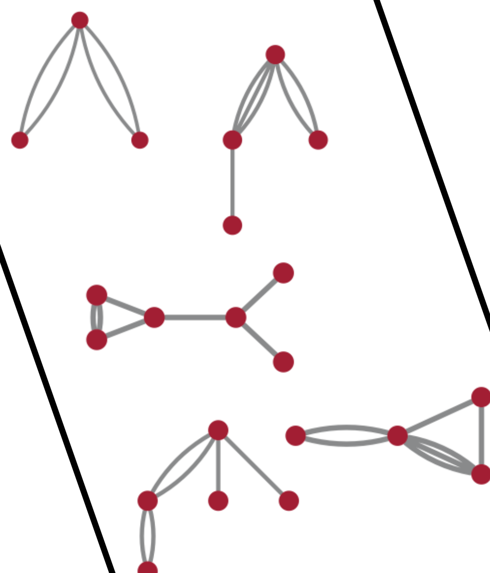
[See also H. Qu, L. Gouskos, 1902.08570]

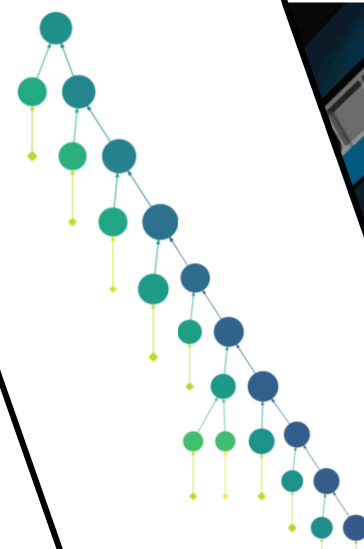
An LHC event from the CMS Detector

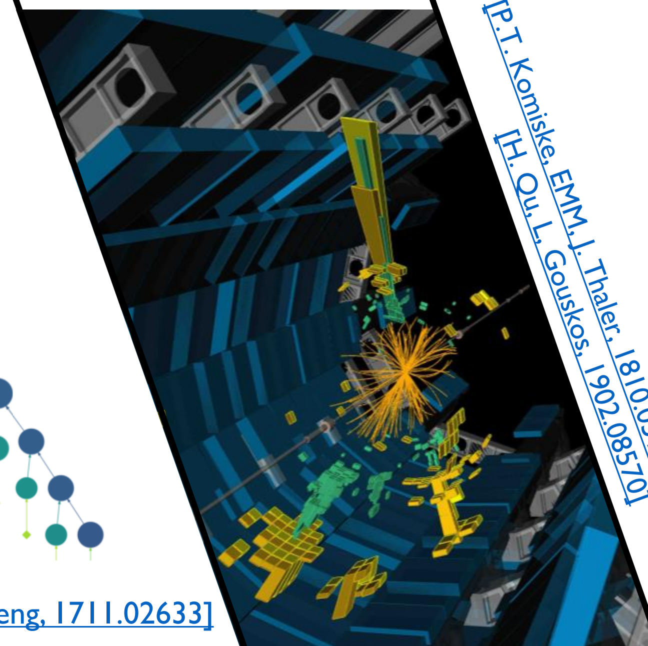
Strategies to Process Jets

Images	Observables	Sequences	Point Clouds	…









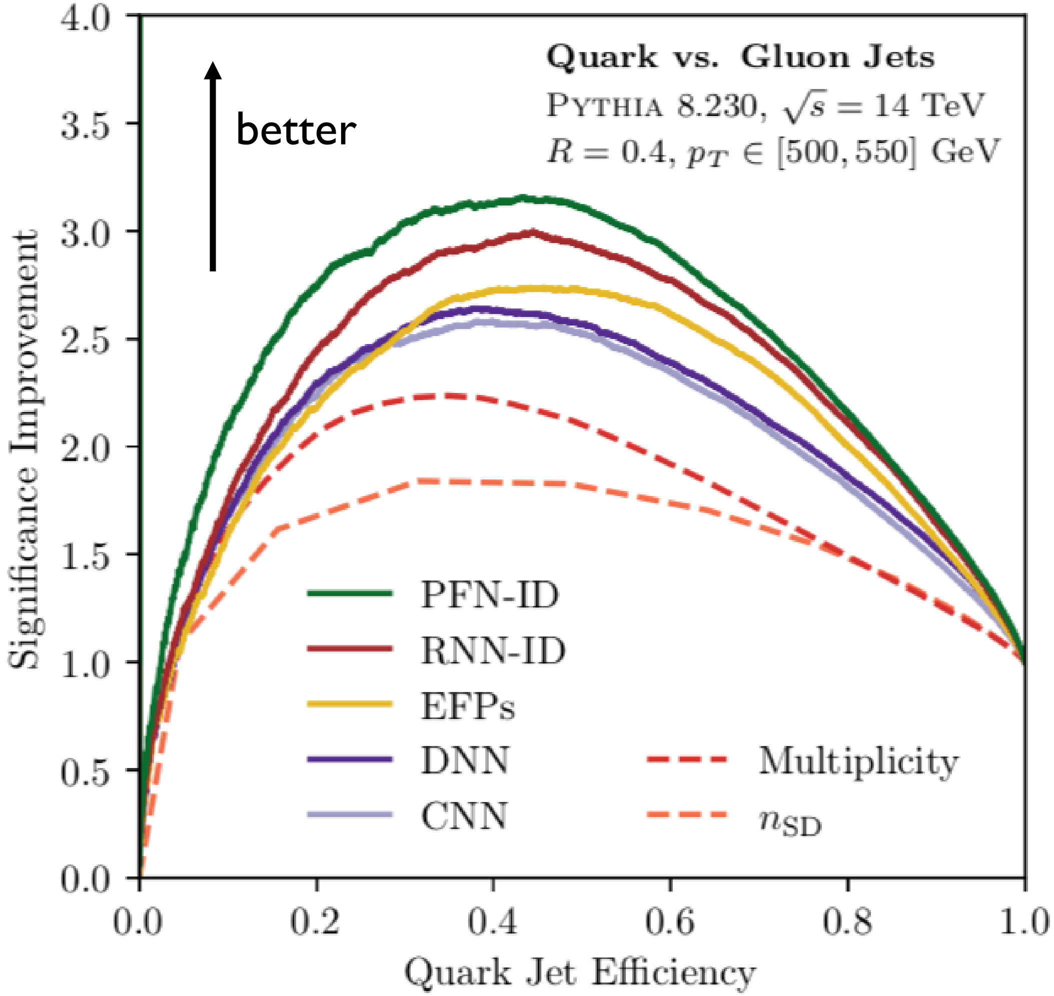
[P.T. Komiske, EMM, J. Thaler, 1810.05165]
[H. Qu, L. Gouskos, 1902.08570]

e.g. [L. de Oliveira, et al., 1511.05190]
[P.T. Komiske, EMM, M.D. Schwartz, 1612.01551]
[M. Andrews, *et al.*, 1902.08276]

e.g.
[K. Datta, A. Larkoski, 1704.08249]
[P.T. Komiske, EMM, J. Thaler, 1712.07124]

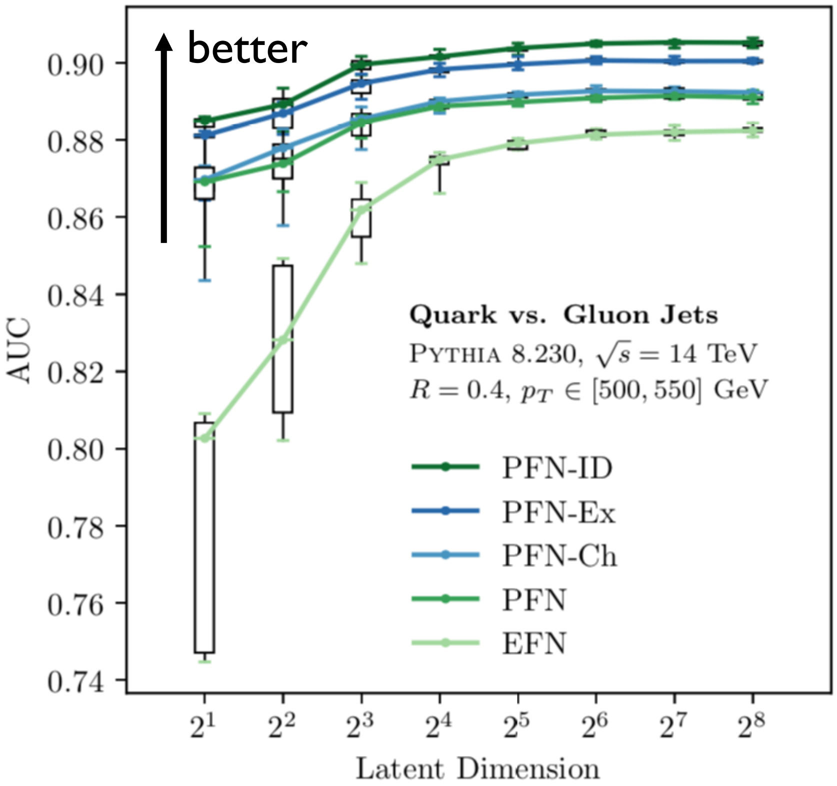
e.g. [T. Cheng, 1711.02633]
[G. Louppe, *et al.*, 1702.00748]
[G. Kasieczka, N. Kiefer, T. Plehn, J. Thompson, 1812.09223]

Classification Performance – Quark vs. Gluon Jets



PFN-ID compares favorably to other architectures and observables

EFN Latent Dimension Sweep – Quark vs. Gluon Jets



PFN-ID: Full particle flavor info
$(\pi^\pm, K^\pm, p, \bar{p}, n, \bar{n}, \gamma, K_L, e^\pm, \mu^\pm)$

PFN-Ex: Experimentally accessible info
$(h^{\pm,0}, \gamma, e^\pm, \mu^\pm)$

PFN-Ch: Particle charge info

PFN: Four momentum information

EFN: IRC-safe latent space

Performance saturates as latent dimension increases

IRC-unsafe information helpful

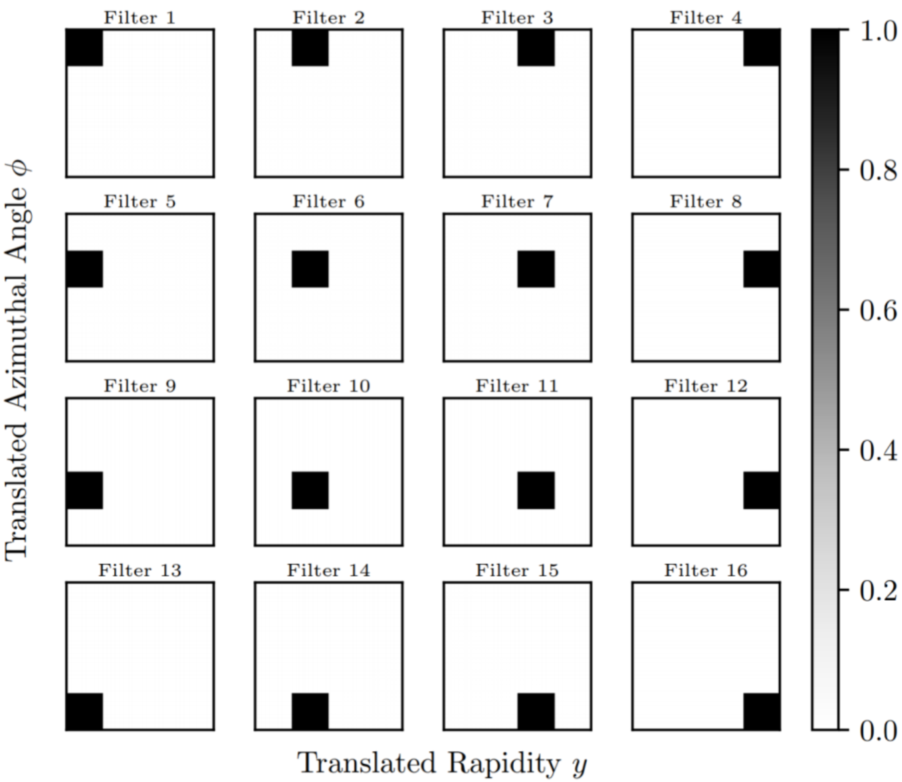
Adding particle type information helpful

What is being learned?

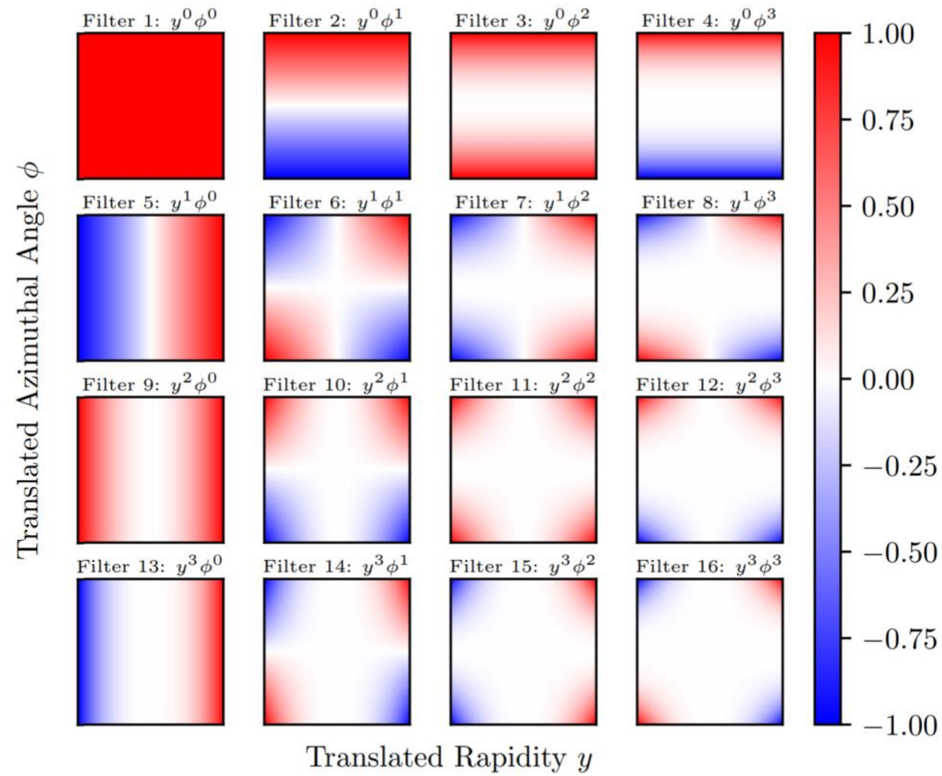
$$\mathrm{EFN}\left(\{p_1^\mu,\ldots,p_M^\mu\}\right) = F\left(\sum_{i=1}^{M} E_i\,\vec{\Phi}(\hat{n}_i)\right) \qquad \hat{n}_i = (y_i, \phi_i)$$

Manifestly IRC-safe latent space

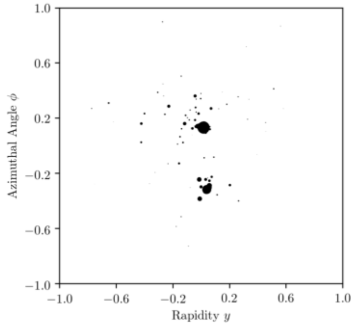
Calorimeter Images as EFN Filters



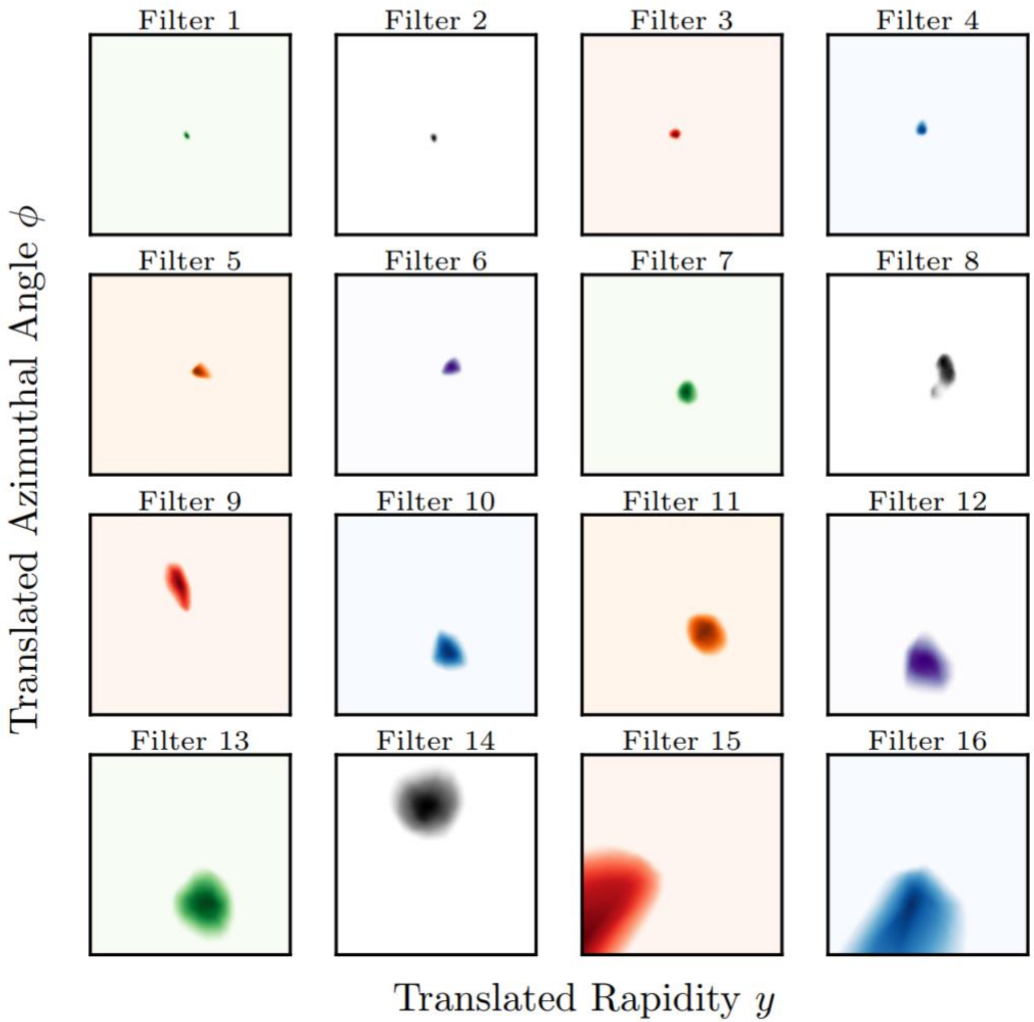
Radiation Moments as EFN Filters



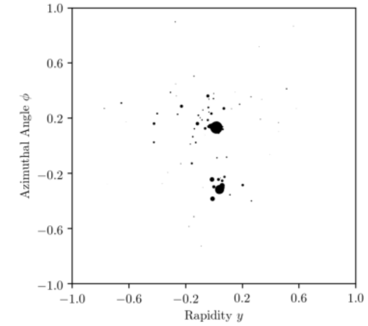
What is being learned?

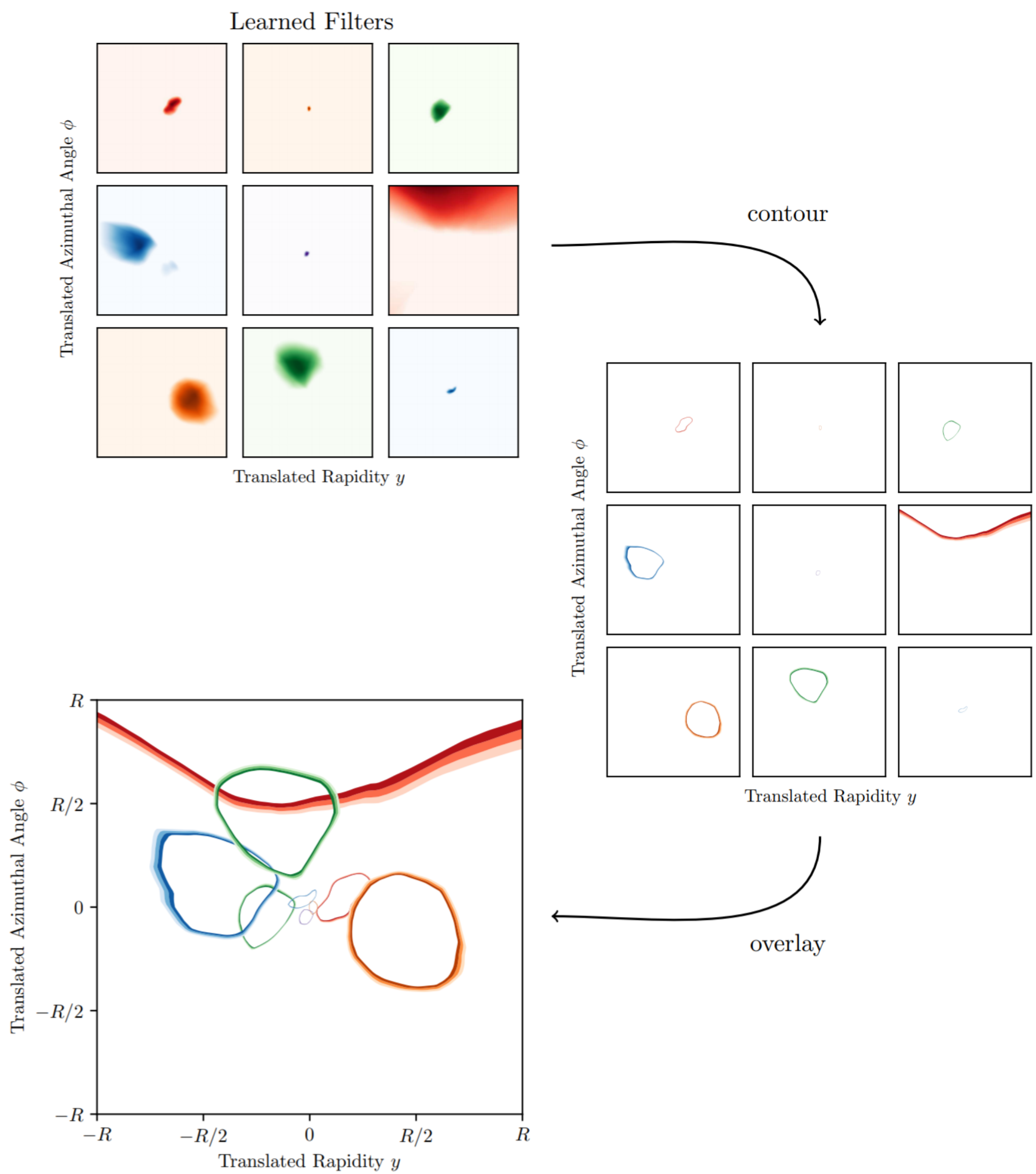


Learned EFN Filters

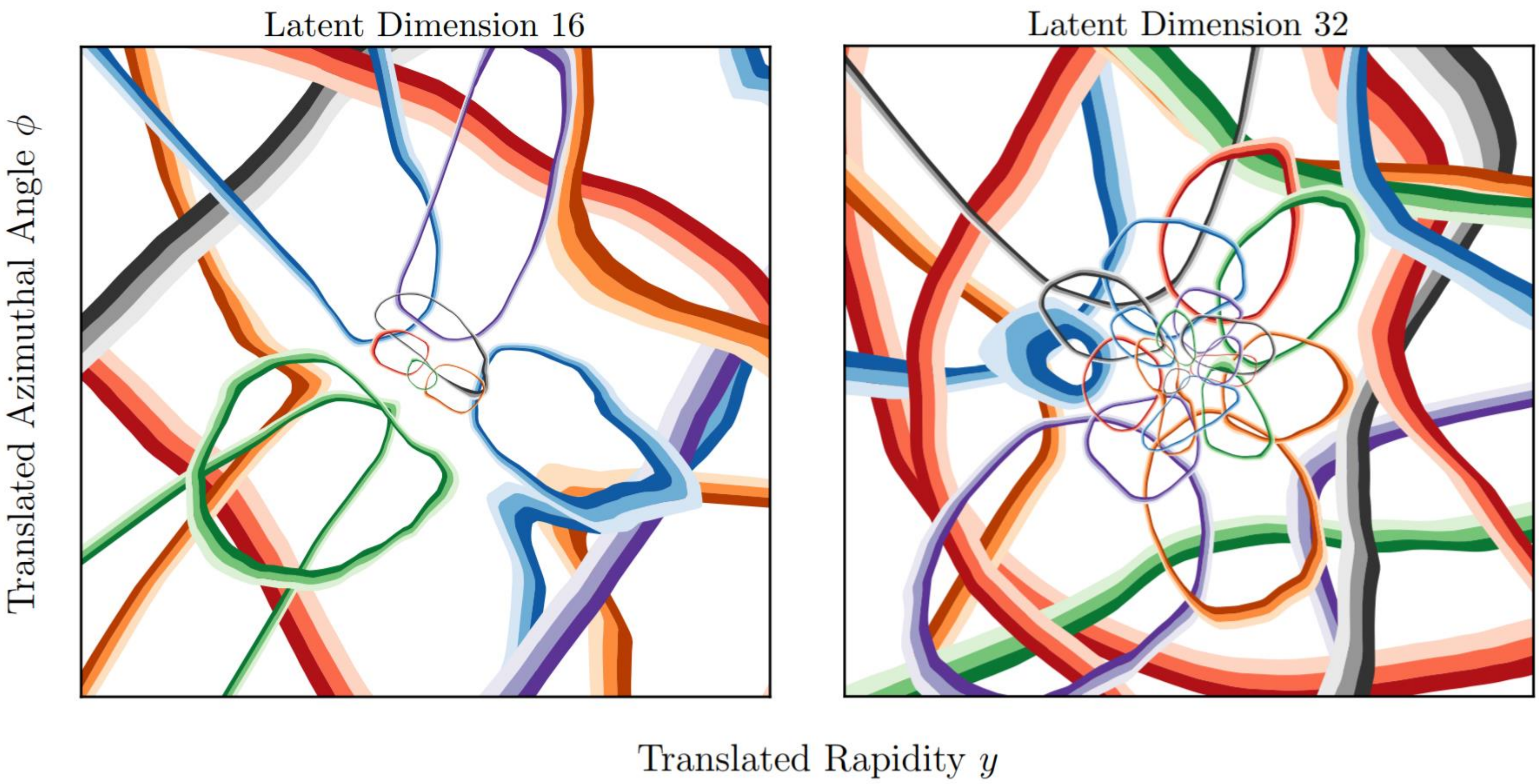


Simultaneous Visualization Strategy

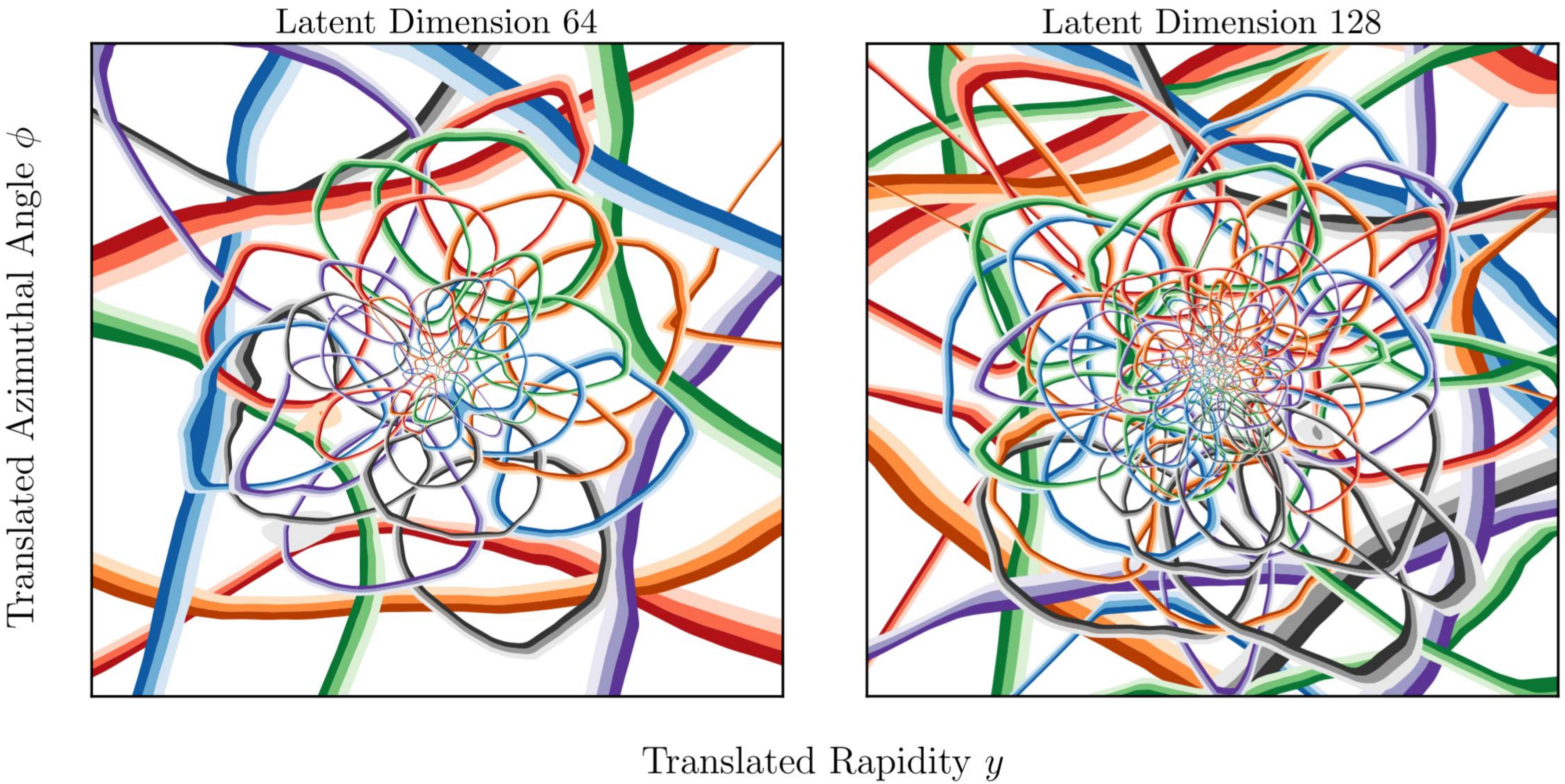




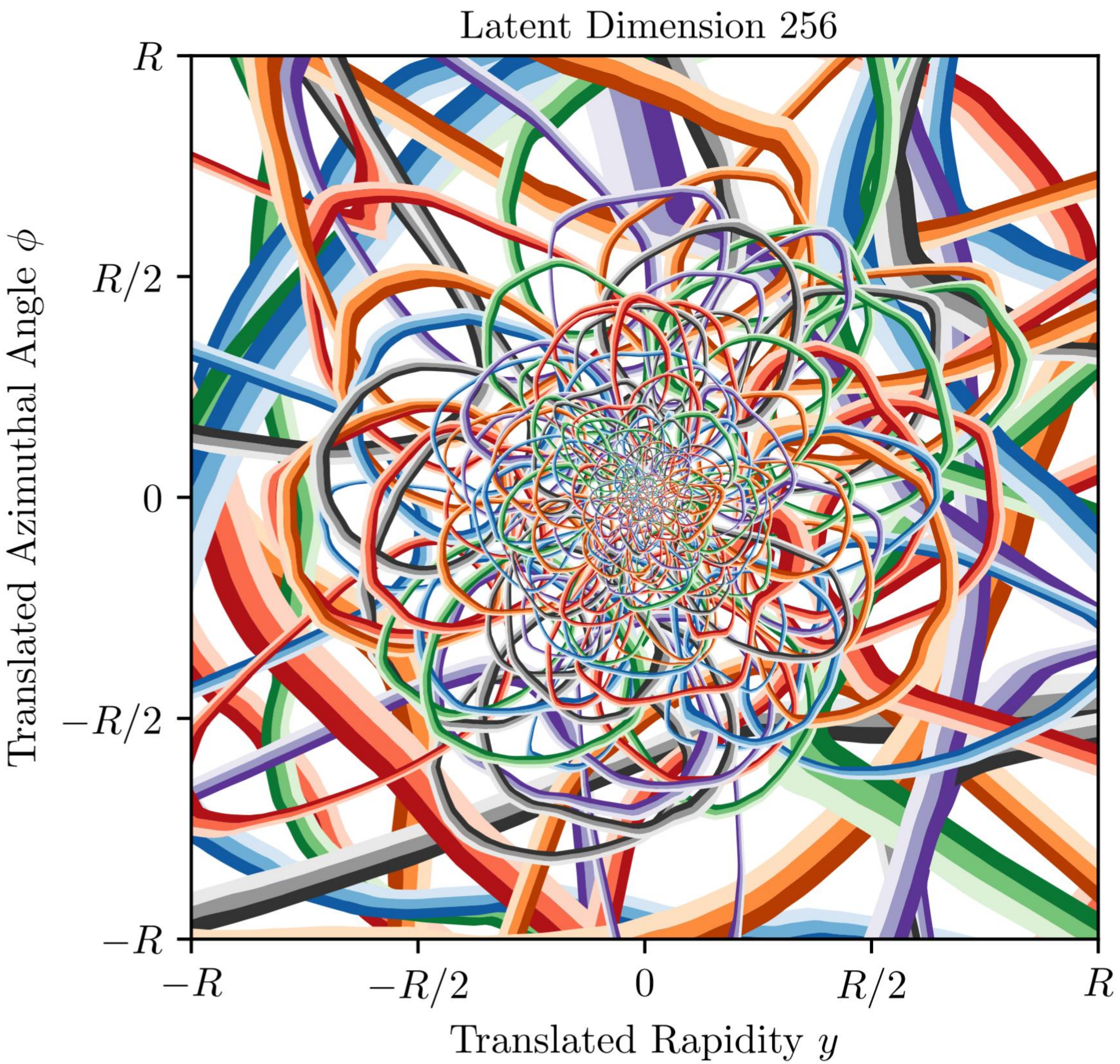
Psychedelic Visualizations



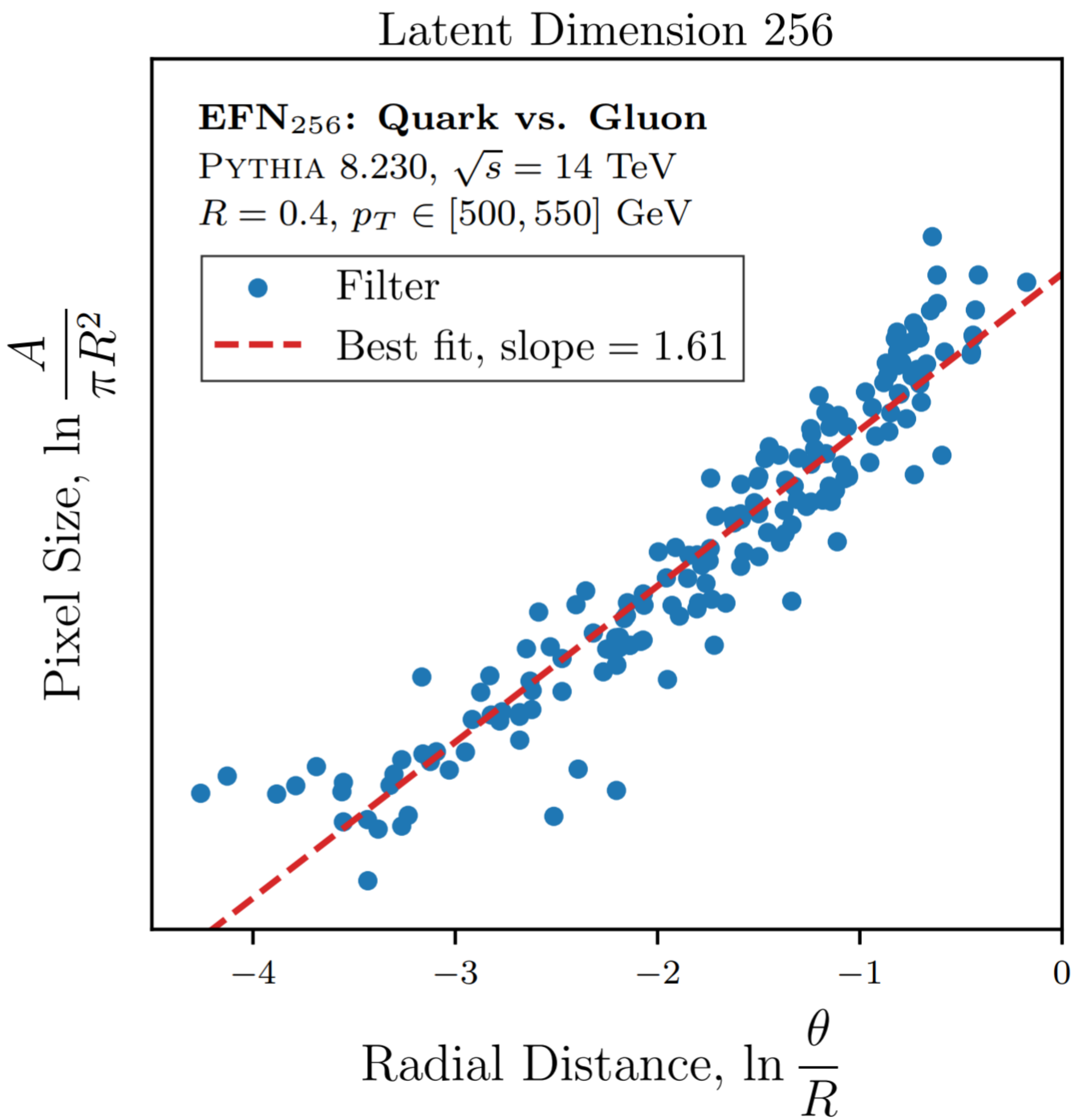
Psychedelic Visualizations

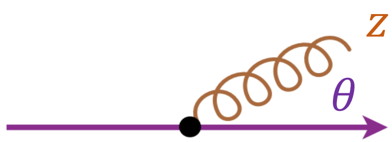


Psychedelic Visualizations



Psychedelic Visualizations





Emissions from quark or gluon are distributed according to:

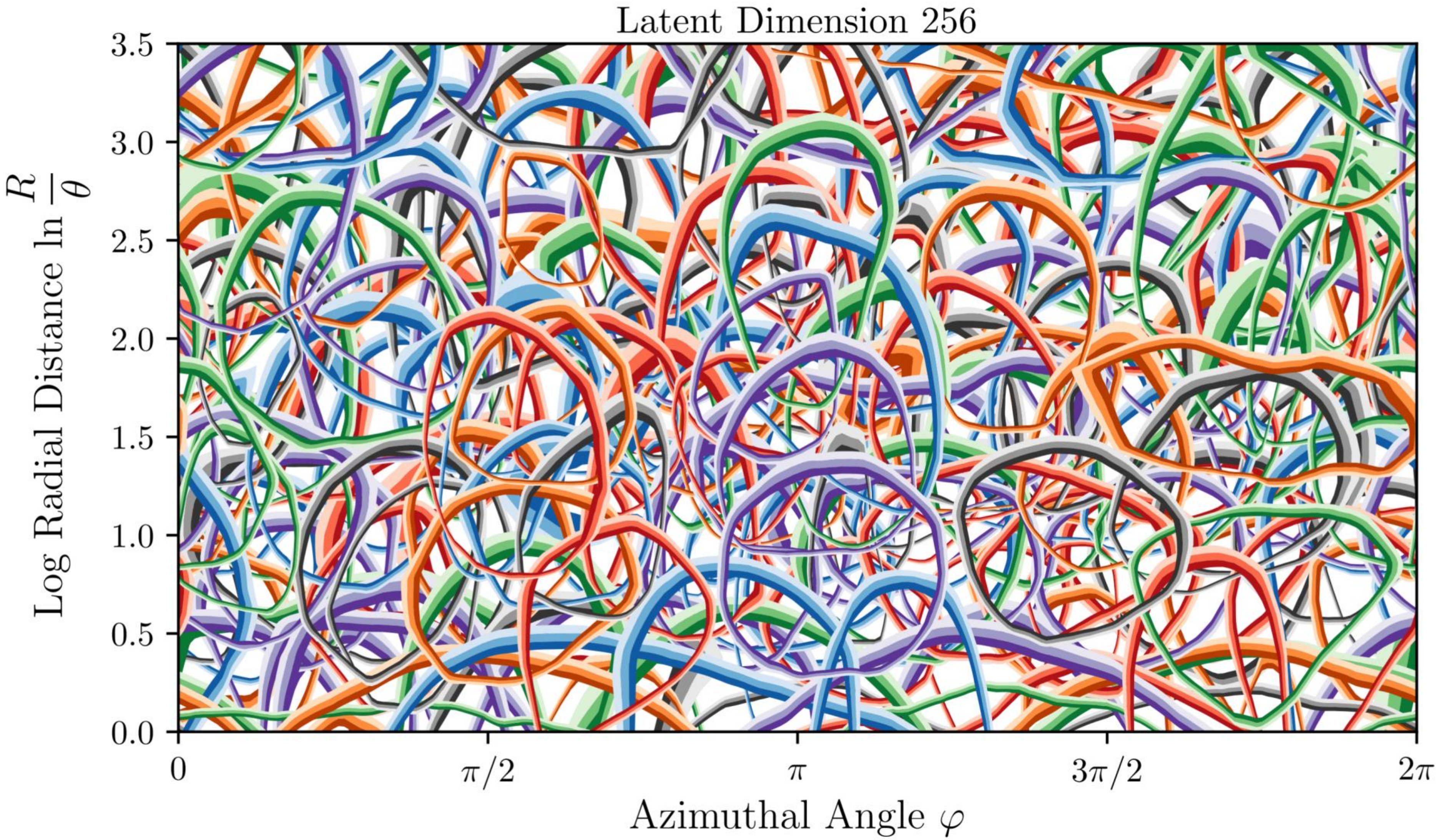
$$dP_{i\to ig} \simeq \frac{2\alpha_s C_i}{\pi} \frac{d\theta}{\theta} \frac{dz}{z}$$

Uniform pixelization in emission plane implies:

$$\frac{d\theta}{\theta} d\varphi = \theta^{-2} dy\, d\phi$$

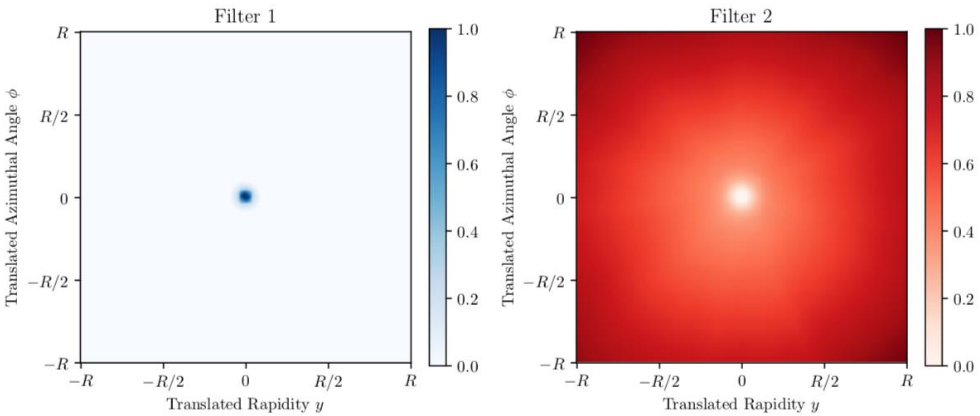
$$\ln \frac{A}{\pi R^2} = 2 \ln \frac{\theta}{R}$$

Psychedelic Visualizations



Extracting New Analytic Observables

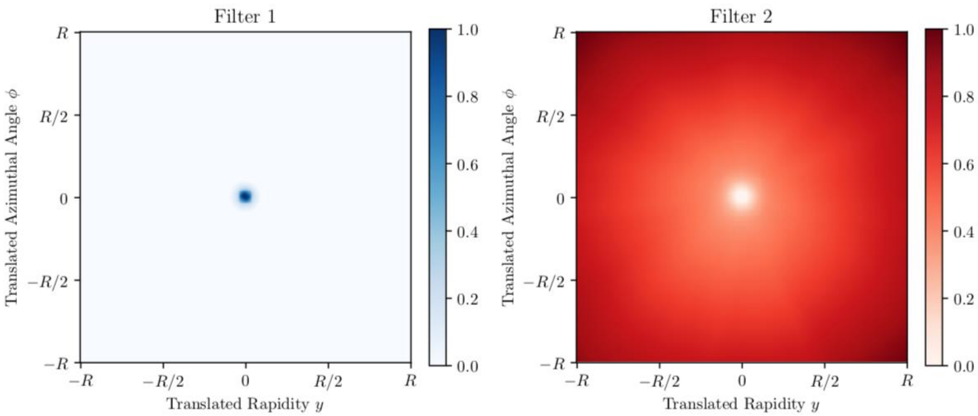
$\ell = 2$ latent space dimension has radially symmetric filters:

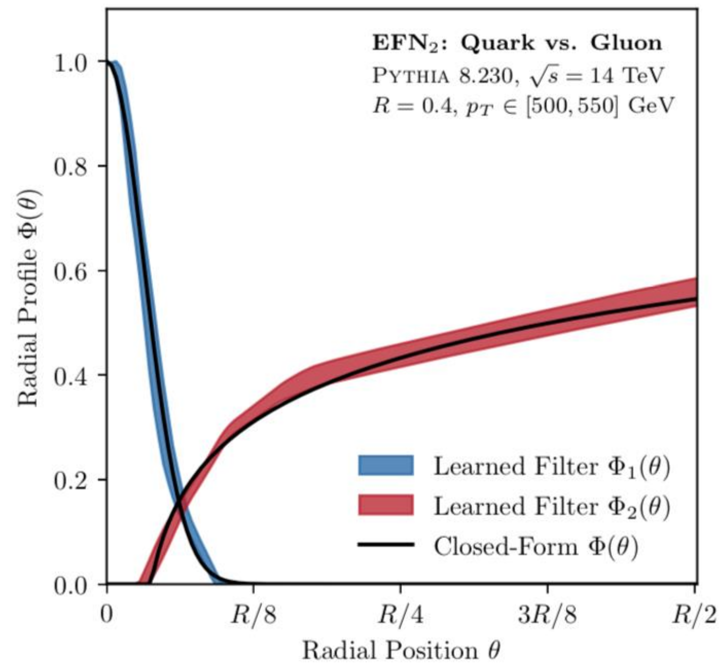


$$\mathcal{O}_1 = \sum_{i=1}^{M} z_i \, \Phi_1(\theta_i) \qquad \mathcal{O}_2 = \sum_{i=1}^{M} z_i \, \Phi_2(\theta_i)$$

Extracting New Analytic Observables

$\ell = 2$ latent space dimension has radially symmetric filters:





$$\mathcal{O}_1 = \sum_{i=1}^{M} z_i \, \Phi_1(\theta_i) \qquad \mathcal{O}_2 = \sum_{i=1}^{M} z_i \, \Phi_2(\theta_i)$$

Take radial slices to obtain envelope

Fit functions of the following forms:

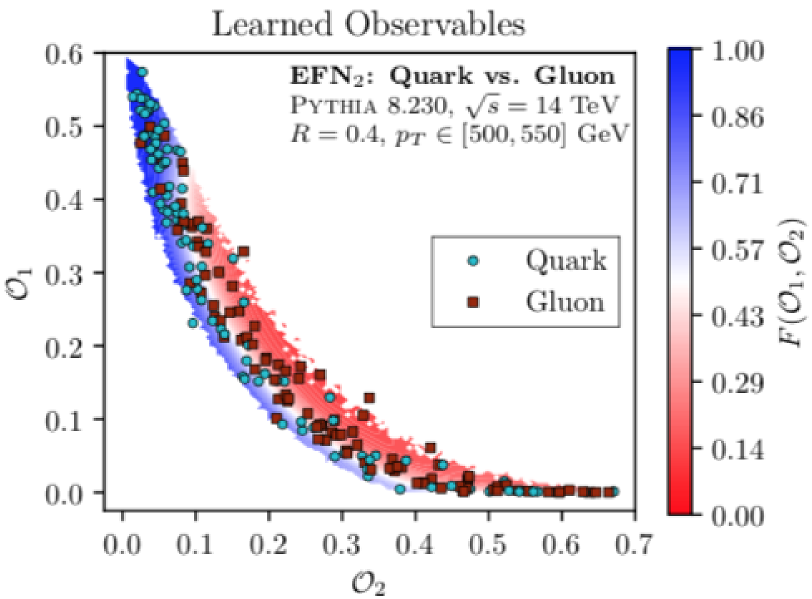
$$A_{r_0} = \sum_i z_i \, e^{-\theta_i^2/r_0^2}, \qquad B_{r_1,\beta} = \sum_i z_i \ln(1 + \beta(\theta_i - r_1)) \, \Theta(\theta_i - r_1)$$

A and B separate collinear and wide-angle regions of phase space, unlike traditional angularities which mix them

Extracting New Analytic Observables

Can also visualize F in the two-dimensional $(\mathcal{O}_1, \mathcal{O}_2)$ phase space

Learned



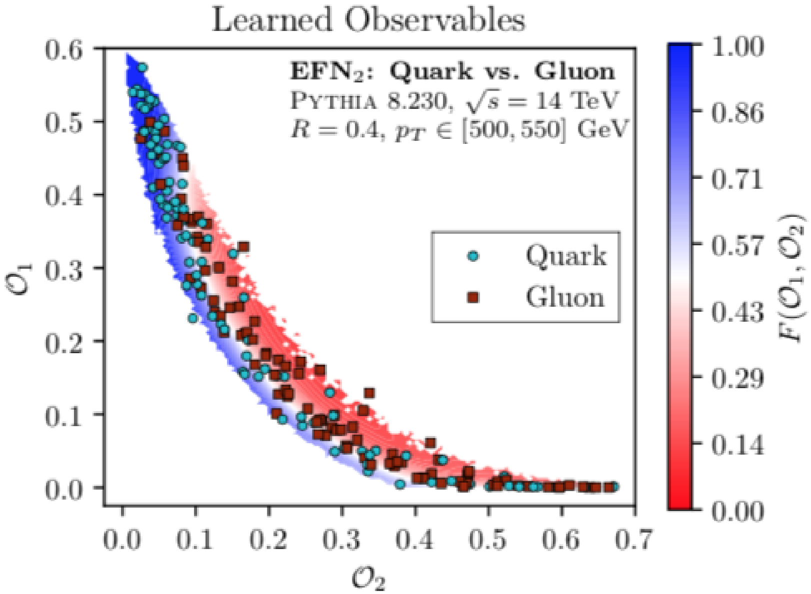
Extract analytic form for F as distance from a point:

$$C(A, B) = (A - a_0)^2 + (B - b_0)^2$$

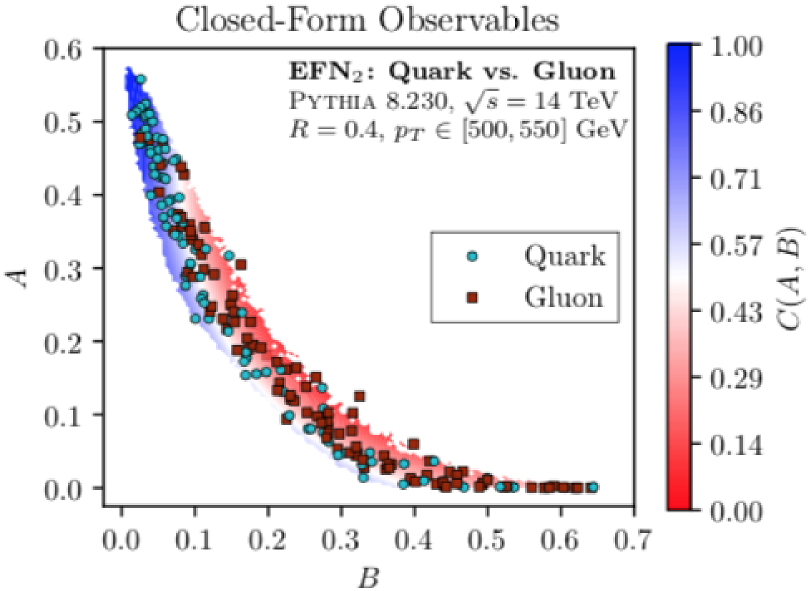
Extracting New Analytic Observables

Can also visualize F in the two-dimensional $(\mathcal{O}_1, \mathcal{O}_2)$ phase space

Learned



Closed-Form

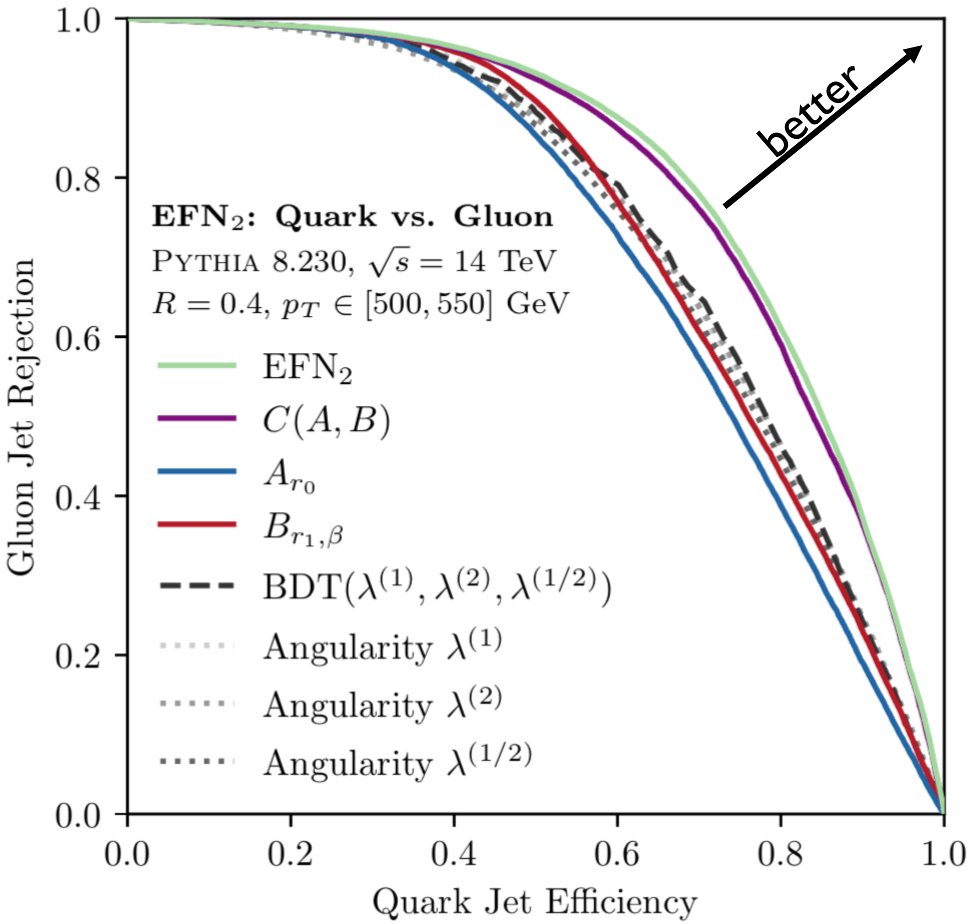


Extract analytic form for F as distance from a point:

$$C(A, B) = (A - a_0)^2 + (B - b_0)^2$$

Extracted C, A, B do a good job of reproducing the learned $\mathcal{O}_1, \mathcal{O}_2, F$

Benchmarking New Analytic Observables

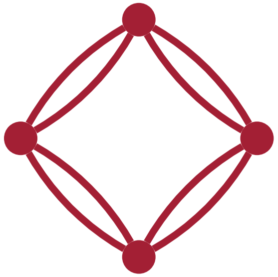


Extracted $C(A, B)$ performs nearly as well as EFN_2

Multivariate combination (BDT) of three other angularities does not do as well

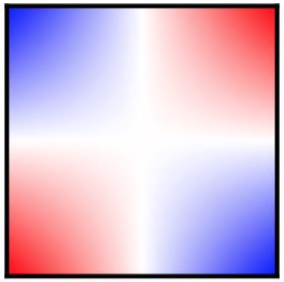
Successfully reverse engineered what the machine learned

Outline



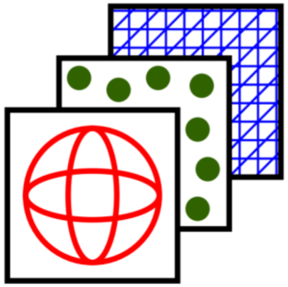
Energy Flow Polynomials

A basis of jet substructure observables



Energy Flow Moments

Tensor moments of the radiation pattern

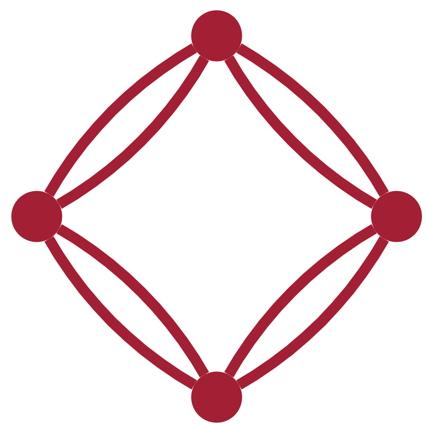


Energy Flow Networks

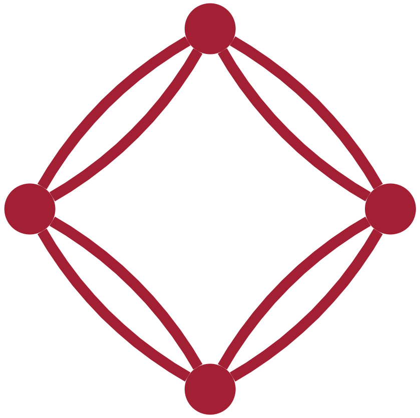
ML architecture designed to learn from events

The End

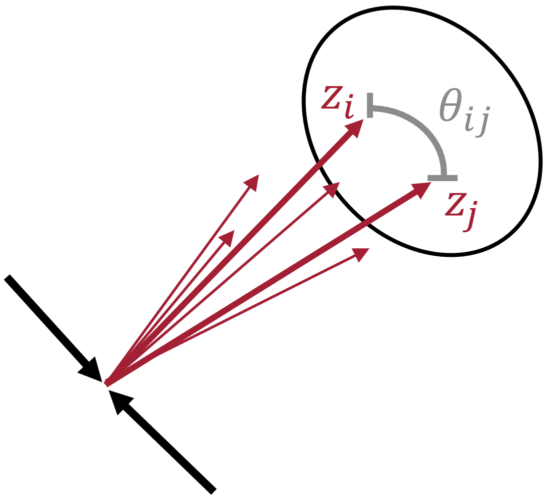
Thank you!



Extra Slides



Connection with the Stress-Energy Operator



At the heart is the Energy Flow Operator:

Energy Flow to infinity

$$\hat{\mathcal{E}}(\hat{n}, v) = \lim_{t\to\infty} \hat{n}_i T^{0i}(t, vt\hat{n})$$

in the $\hat{n}$ direction
at velocity v

[N. Sveshnikov and F. Tkachov, hep-ph/9512370]
[V. Mateu, I.W. Stewart, and J. Thaler, arXiv:1209.3781]

Progress has been made in computing correlations of $\hat{\mathcal{E}}(\hat{n}, v)$ in conformal field theory

[D. Hofman and J. Maldecena, 0803.1467]

IRC-safe observables are built out of energy correlators:

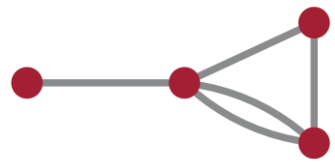
[F. Tkachov, hep-ph/9601308]

Rigid energy structure

$$C_f = \sum_{i_1=1}^{M} \cdots \sum_{i_N=1}^{M} z_{i_1} \cdots z_{i_N} f(\hat{p}_{i_1}, \cdots, \hat{p}_{i_N})$$

Arbitrary angular function f

Multigraph/EFP Correspondence

Multigraph	$\longleftrightarrow$	EFP
		$= \sum_{i_1=1}^{M} \sum_{i_2=1}^{M} \sum_{i_3=1}^{M} \sum_{i_4=1}^{M} z_{i_1} z_{i_2} z_{i_3} z_{i_4}\, \theta_{i_1 i_2} \theta_{i_2 i_3} \theta_{i_3 i_4} \theta_{i_2 i_4}^2$
	$\longleftrightarrow$	z_{i_j}
	$\longleftrightarrow$	$\theta_{i_k i_l}$

N	Number of vertices	$\longleftrightarrow$	N-particle correlator
d	Number of edges	$\longleftrightarrow$	Degree of angular monomial
χ	Treewidth + 1	$\longleftrightarrow$	Optimal VE Complexity

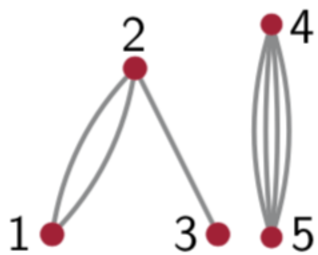
e.g. Tree graph EFPs are $O(M^2)$!
Surprisingly efficient to compute.
Stay tuned… See P. Komiske's talk.

Connected	$\longleftrightarrow$	Prime
Disconnected	$\longleftrightarrow$	Composite

⋮

Computational Complexity of EFPs

- Like other energy correlators, EFPs are naively $\mathcal{O}(M^N)$
- *Factorability* of summand in EFP formula can speed up computation



$$= \left(\sum_{i_1=1}^{M}\sum_{i_1=1}^{M}\sum_{i_3=1}^{M} z_{i_1} z_{i_2} z_{i_3} \theta_{i_1 i_2}^2 \theta_{i_2 i_3}\right)\left(\sum_{i_4=1}^{M}\sum_{i_5=1}^{M} z_{i_4} z_{i_5} \theta_{i_4 i_5}^4\right)$$

Composite EFPs are products of prime EFPs

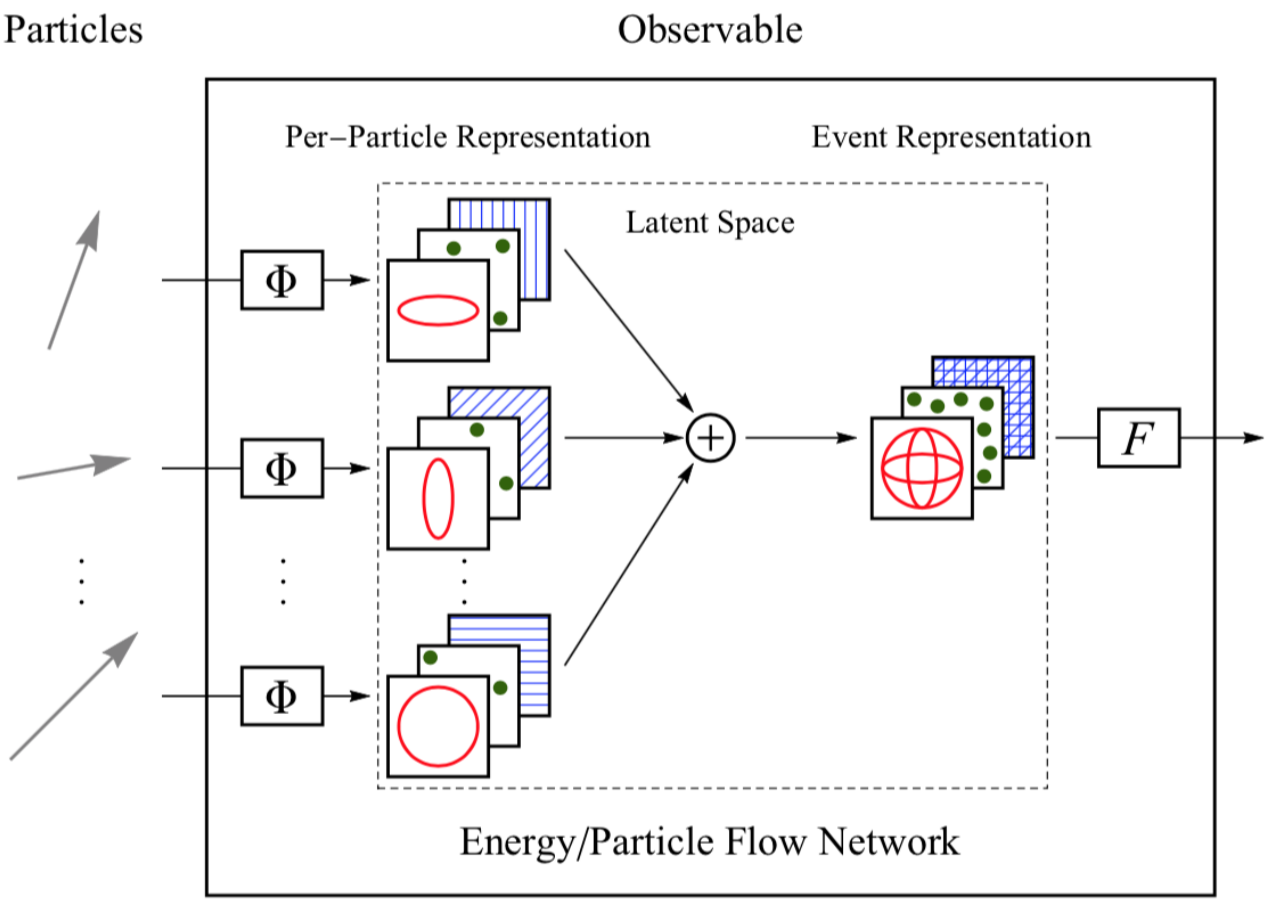
$$= \underbrace{\sum_{i_1=1}^{M}\sum_{i_2=1}^{M}\sum_{i_3=1}^{M}\sum_{i_4=1}^{M}\sum_{i_5=1}^{M}\sum_{i_6=1}^{M}\sum_{i_7=1}^{M}\sum_{i_8=1}^{M} z_{i_1} z_{i_2} z_{i_3} z_{i_4} z_{i_5} z_{i_6} z_{i_7} z_{i_8} \theta_{i_1 i_2}\theta_{i_1 i_3}\theta_{i_1 i_4}\theta_{i_1 i_5}\theta_{i_1 i_6}\theta_{i_1 i_7}\theta_{i_1 i_8}}_{\mathcal{O}(M^8)}$$

$$= \underbrace{\sum_{i_1=1}^{M} z_{i_1}\left(\sum_{i_2=1}^{M} z_{i_2}\theta_{i_1 i_2}\right)^7}_{\mathcal{O}(M^2)}$$

Other algebraic simplifications are also possible by choosing parentheses wisely

Energy Flow Networks

[P. Komiske, EMM, J. Thaler, 1810.05165]



$$\mathrm{EFN}\left(\{p_1^\mu,\ldots,p_M^\mu\}\right) = F\left(\sum_{i=1}^{M} E_i\,\vec{\Phi}(n_i^\mu)\right)$$

Manifestly IRC-safe latent space

$$\mathrm{PFN}\left(\{p_1^\mu,\ldots,p_M^\mu\}\right) = F\left(\sum_{i=1}^{M} \Phi(p_i^\mu)\right)$$

Fully general latent space

Many observables are easily interpreted in EFN language

Familiar Jet Substructure Observables as EFNs or PFNs

$$\mathrm{EFN}\left(\{p_1^\mu,\ldots,p_M^\mu\}\right) = F\left(\sum_{i=1}^{M} z_i \Phi(\hat{p}_i)\right)$$

Manifestly IRC-safe latent space

$$\mathrm{PFN}\left(\{p_1^\mu,\ldots,p_M^\mu\}\right) = F\left(\sum_{i=1}^{M} \Phi(p_i^\mu)\right)$$

Fully general latent space

Observable $\mathcal{O}$		**Map** Φ	**Function** F
Mass	m	p^μ	$F(x^\mu) = \sqrt{x^\mu x_\mu}$
Multiplicity	M	1	$F(x) = x$
Track Mass	m_{track}	$p^\mu \mathbb{I}_{\text{track}}$	$F(x^\mu) = \sqrt{x^\mu x_\mu}$
Track Multiplicity	M_{track}	$\mathbb{I}_{\text{track}}$	$F(x) = x$
Jet Charge [69]	$\mathcal{Q}_\kappa$	$(p_T, Q\, p_T^\kappa)$	$F(x, y) = y/x^\kappa$
Eventropy [71]	$z \ln z$	$(p_T, p_T \ln p_T)$	$F(x, y) = y/x - \ln x$
Momentum Dispersion [90]	p_T^D	(p_T, p_T^2)	$F(x, y) = \sqrt{y/x^2}$
C parameter [91]	C	$(\lvert\vec{p}\,\rvert, \vec{p}\otimes\vec{p}/\lvert\vec{p}\,\rvert)$	$F(x, Y) = \frac{3}{2x^2}[(\mathrm{Tr}\, Y)^2 - \mathrm{Tr}\, Y^2]$

Many observables are easily interpreted in EFN language