Data ex Machina

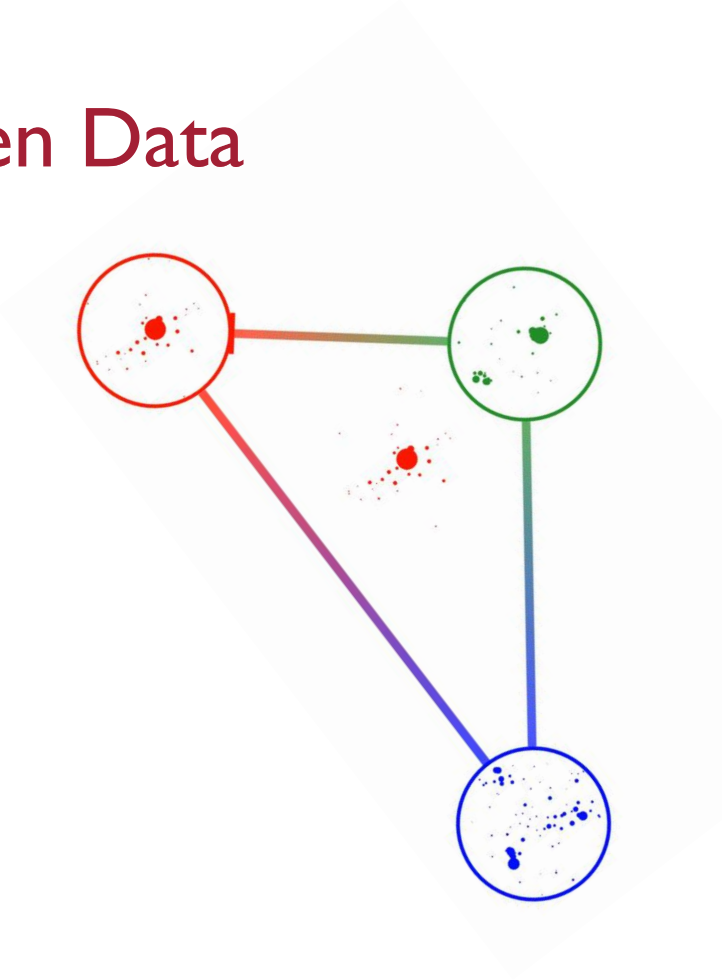
Machine Learning with Jets in CMS Open Data

Machine Learning for Jet Physics 2020

Eric M. Metodiev

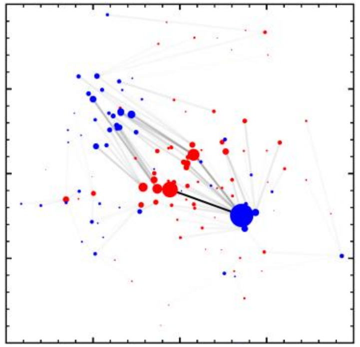
Center for Theoretical Physics

Massachusetts Institute of Technology

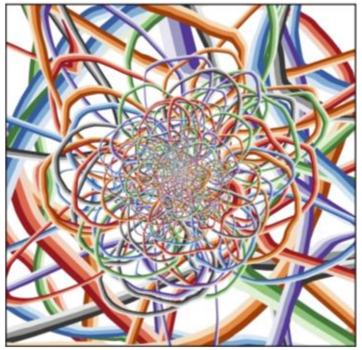


Patrick Komiske | Radha Mastandrea | Preksha Naik | Jesse Thaler

CMS Open Data
A new public dataset for jet studies



Unsupervised Learning
A metric for collider events



Supervised Learning
Training directly on collider data

CMS Open Data
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About

opendata.cern.ch

Explore more than **two petabytes** of open data from particle physics!

jet primary dataset

Search

search examples: collision datasets, keywords:education, energy:7TeV

Explore

datasets
software
environments
documentation

Focus on

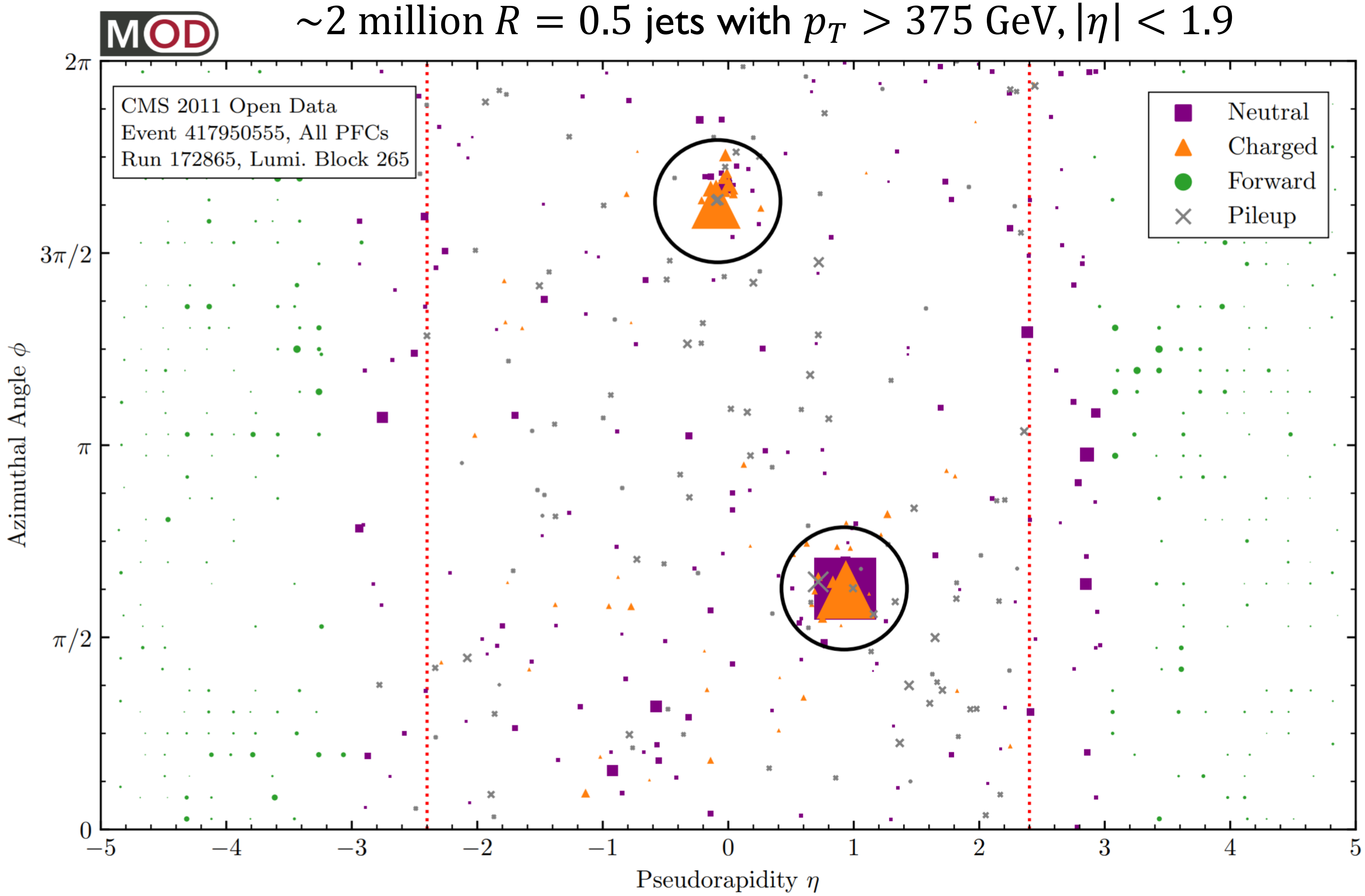
ATLAS
ALICE
CMS
LHCb
OPERA
Data Science

Get started

CMS 2011A Jet Primary Dataset (+ Simulation)

$2.3\ \mathrm{fb}^{-1}$ of 7 TeV proton-proton collision data. [link]

$\sim$2 million $R = 0.5$ jets with $p_T > 375$ GeV, $|\eta| < 1.9$



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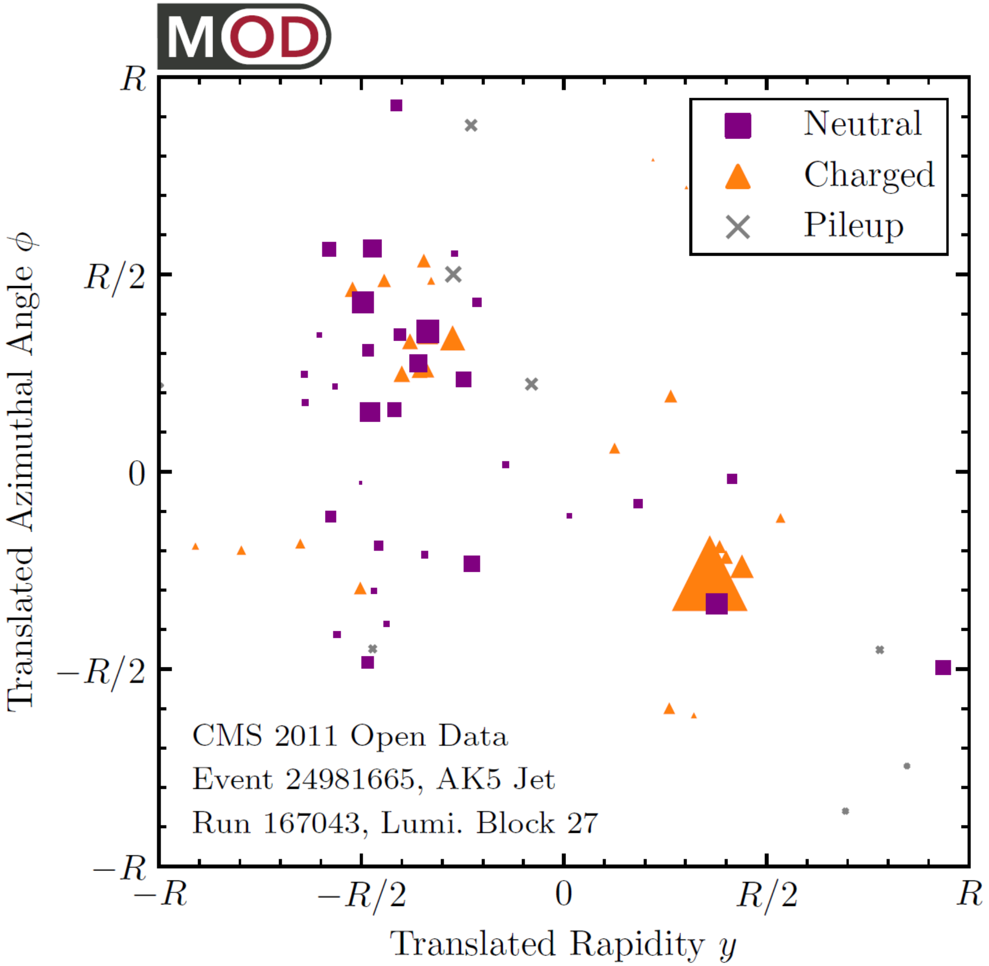

Our processed jet dataset is public!

[Komiske, Mastandrea, **EMM**, Naik, Thaler, 1908.08542]

$\sim$2 million $R = 0.5$ anti-kT jets recorded by CMS
$p_T > 375$ GeV, $|\eta| < 1.9$

Jets as lists of particle flow candidates:
[p_T, y, ϕ, ID, vertex]

Plus additional information:
Jet energy correction factors
Monte Carlo samples
Detector simulation

Zenodo record   Binder demo to download and read the dataset

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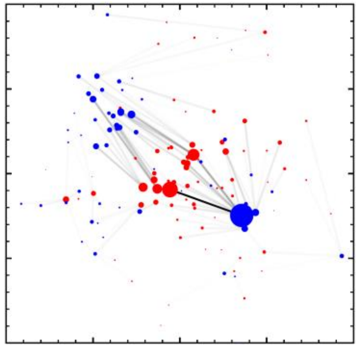

Unsupervised Learning
A metric for collider events

Supervised Learning
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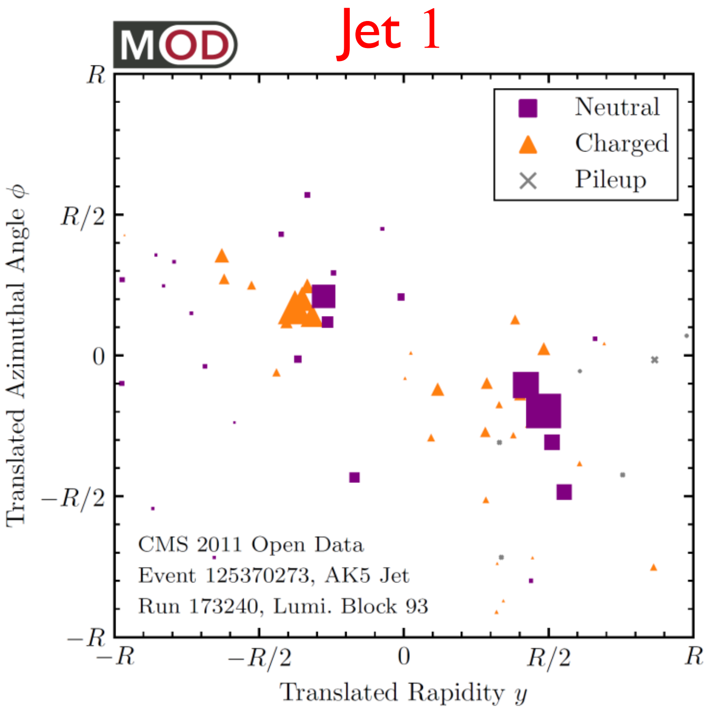
When are two jets similar?

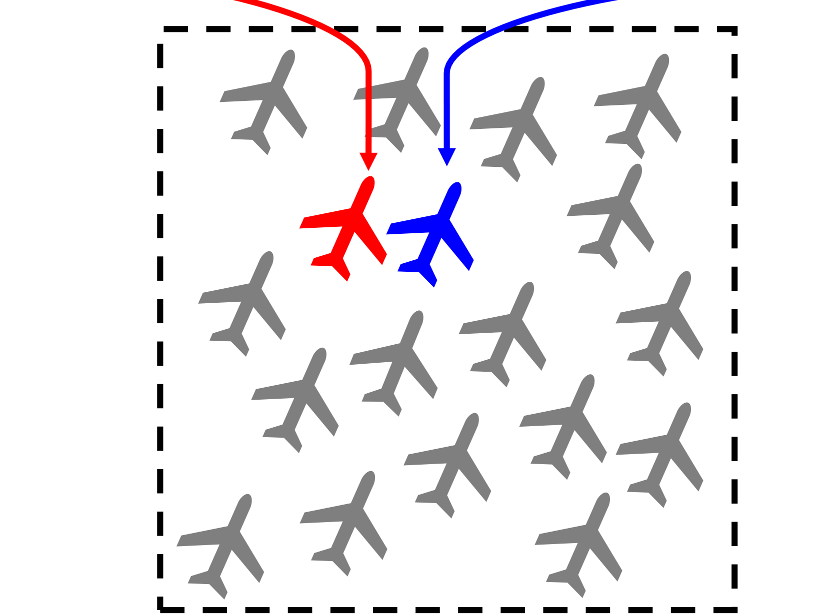
Many unsupervised methods rely on a **distance matrix.** Need a physically-sensible **metric** between jets!

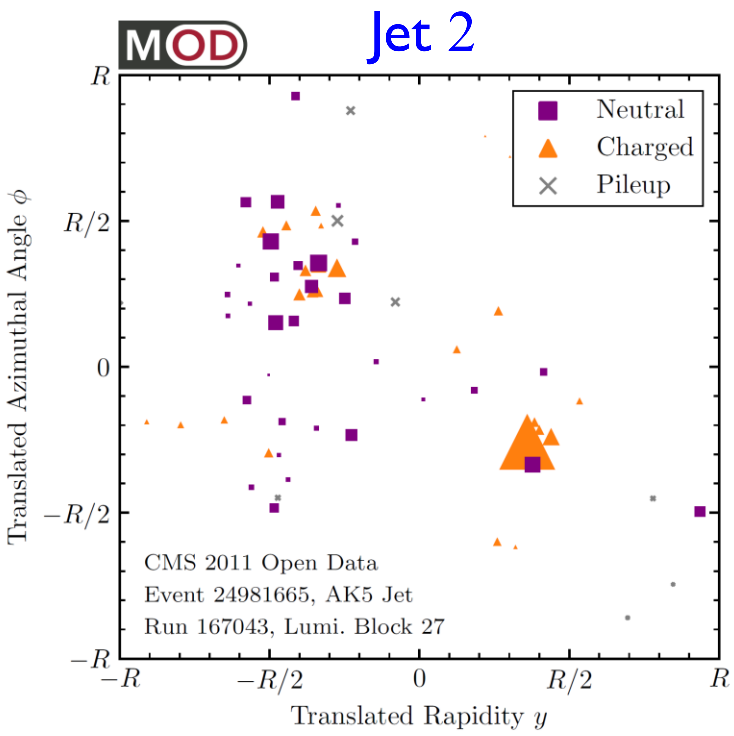
These two jets “look” similar, but have different numbers of particles, flavors, and locations.



How do we quantify this?

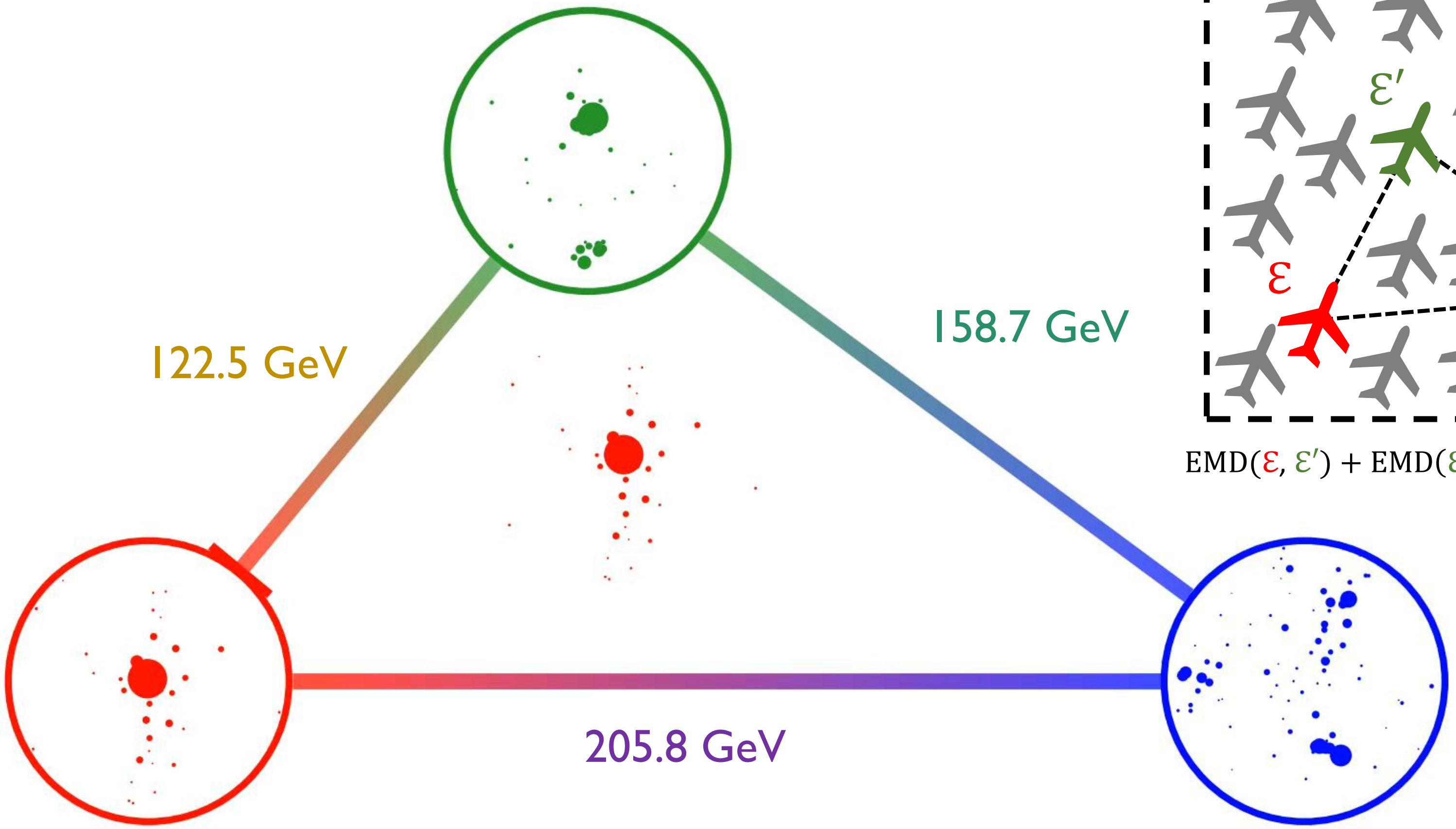
“Space of Jets”

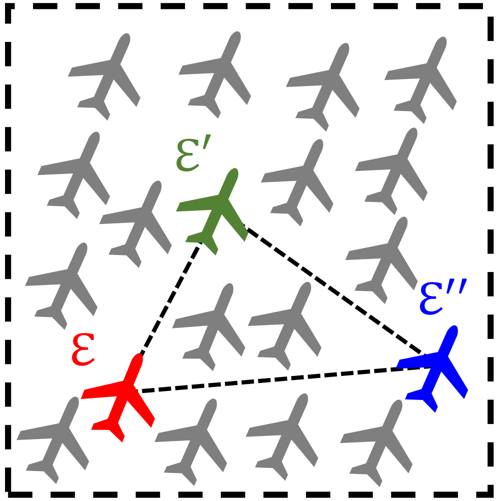




The Energy Mover's Distance

"Energy" Mover's Distance: the minimum "work" (energy x angle) to rearrange one *jet* (pile of energy) into another





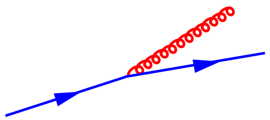
$$\mathrm{EMD}(\mathcal{E}, \mathcal{E}') + \mathrm{EMD}(\mathcal{E}', \mathcal{E}'') \geq \mathrm{EMD}(\mathcal{E}, \mathcal{E}'')$$

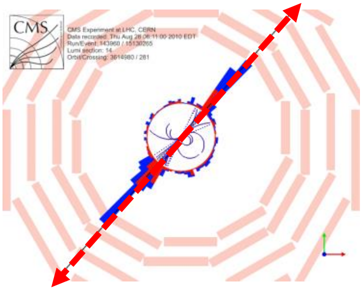
[Komiske, **EMM**, Thaler, 1902.02346]

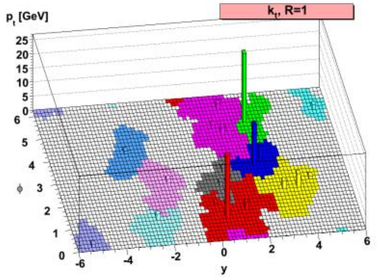
Earth Mover's Distance: [Peleg, Werman, Rom] [Rubner, Tomasi, Guibas]

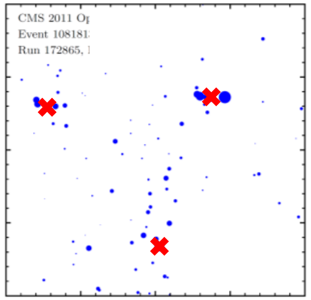
See Today's Talk on Optimal Transport

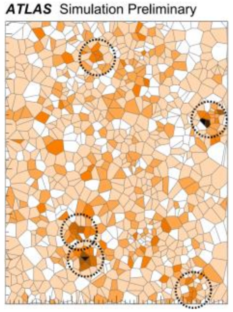
Six Decades of Collider Techniques











Taming infinities

Event Shapes

Jet Algorithms

Jet Substructure

Pileup

1960

2020

1962-1964

Infrared Safety

[Kinoshita, JMP 1962]

[Lee, Nauenberg, PR 1964]

1977

Thrust, Sphericity

[Farhi, PRL 1977]

[Georgi, Machacek, PRL 1977]

1993

k_T jet clustering

[Ellis, Soper, PRD 1993]

[Catani, Dokshitzer, Seymour, Webber, NPB 1993]

1997-1998

C/A jet clustering

[Wobisch, Wengler, 1998]

[Doskhitzer, Leder, Moretti, Webber, JHEP 1997]

2010-2015

N-(sub)jettiness, XCone

[Stewart, Tackmann, Waalewijn, PRL 2010]

[Thaler, Van Tilburg, JHEP 2011]

[Stewart, Tackmann, Thaler, Vermilion, Wilkason, JHEP 2015]

2014-2019

Constituent Subtraction

[Berta, Spousta, Miller, Leitner, JHEP 2014]

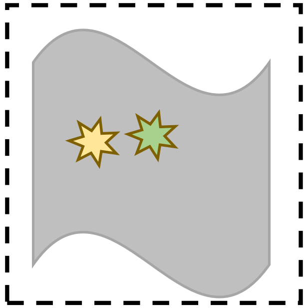
[Berta, Masetti, Miller, Spousta, JHEP 2019]

And many more!

Six Decades of Collider Techniques as Optimal Transport!

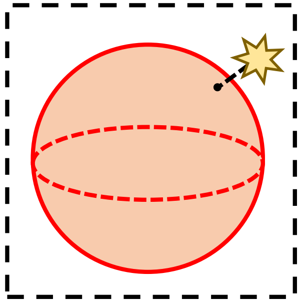
[Komiske, **EMM**, Thaler, *to appear*]

Smooth function of energy distribution are finite in QFT



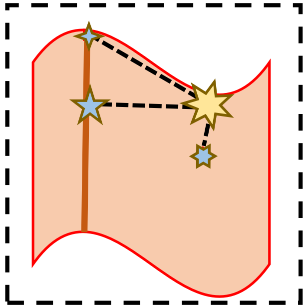
$$\mathrm{EMD}(\mathcal{E}, \mathcal{E}') < \delta$$
$$\to |\mathcal{O}(\mathcal{E}) - \mathcal{O}(\mathcal{E}')| < \epsilon$$

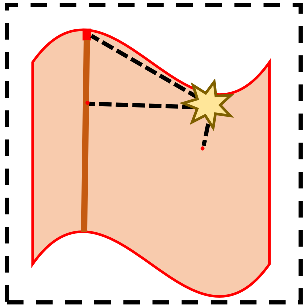
Event shapes as distances to the 2-particle manifold



$$t(\mathcal{E}) = \min_{|\mathcal{E}'|=2} \mathrm{EMD}(\mathcal{E}, \mathcal{E}')$$

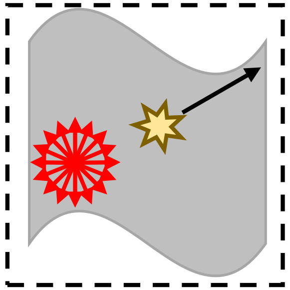
Jets are N-particle event approximations





$$\mathcal{J}(\mathcal{E}) = \operatorname*{argmin}_{|\mathcal{E}'|=N} \mathrm{EMD}(\mathcal{E}, \mathcal{E}')$$

Subtract a pileup as a uniform distribution



$$\mathcal{E} - \mathcal{U}$$

Taming infinities — Event Shapes — Jet Algorithms — Jet Substructure — Pileup

1960 — 2020

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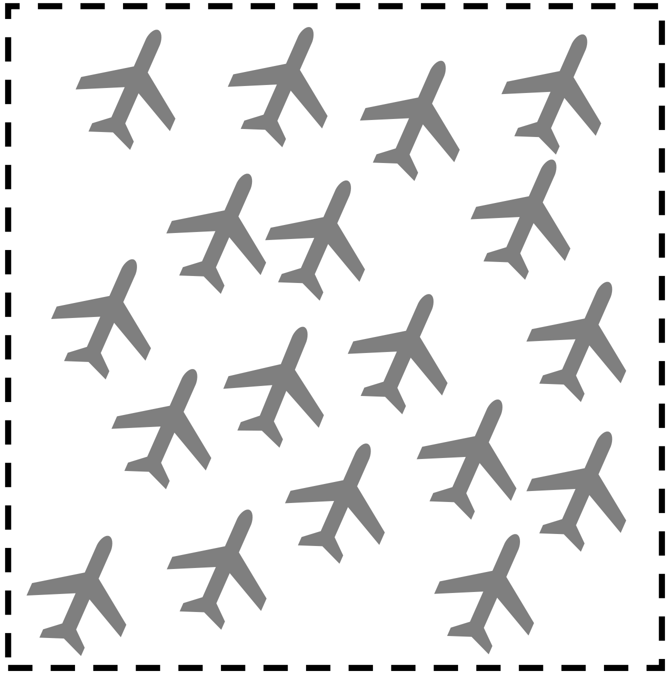
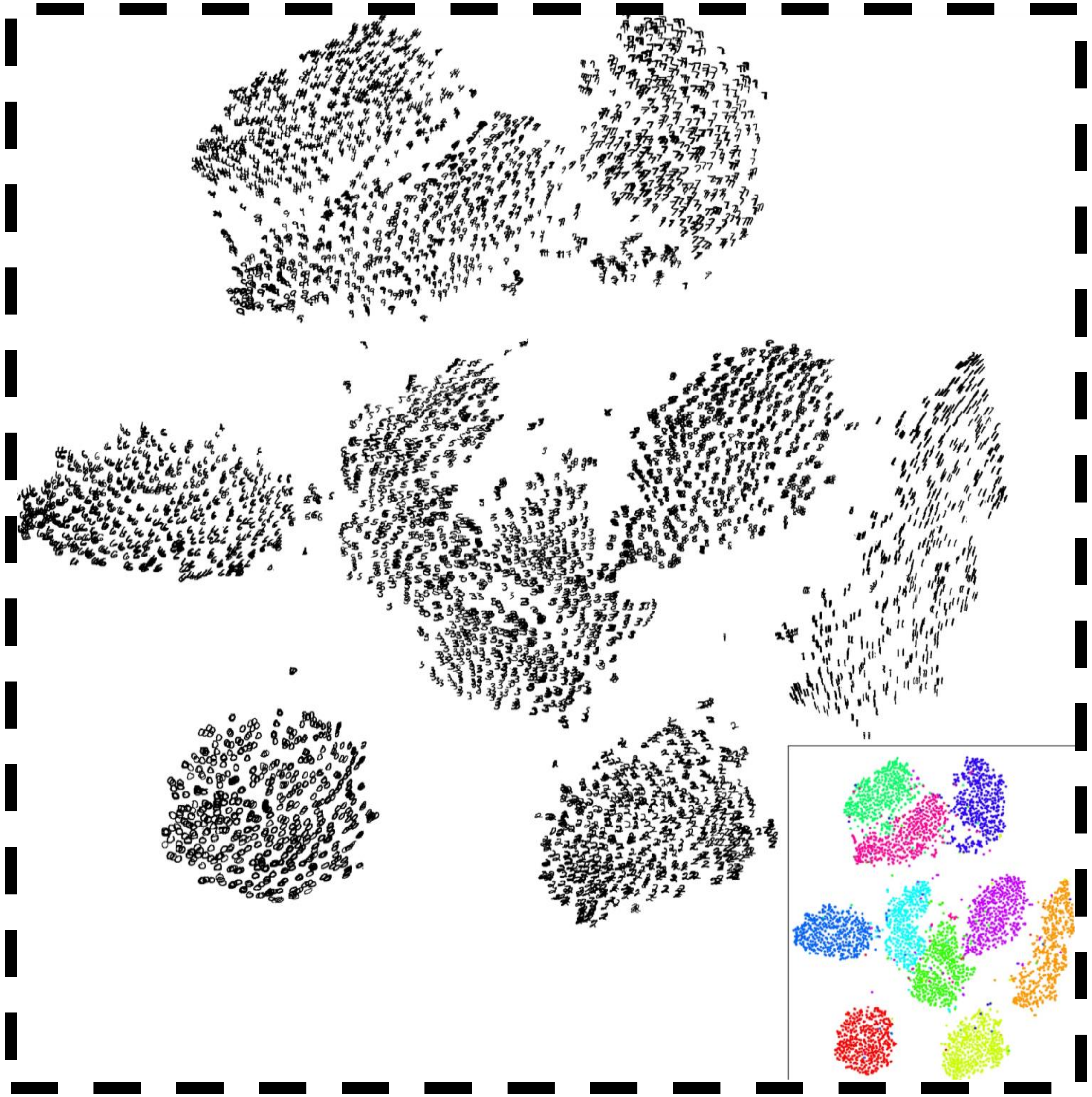
And many more!

Visualizing the Manifold

What does the space of jets look like?





t-SNE embedding

[van der Maaten, Hinton, JMLR 2008]

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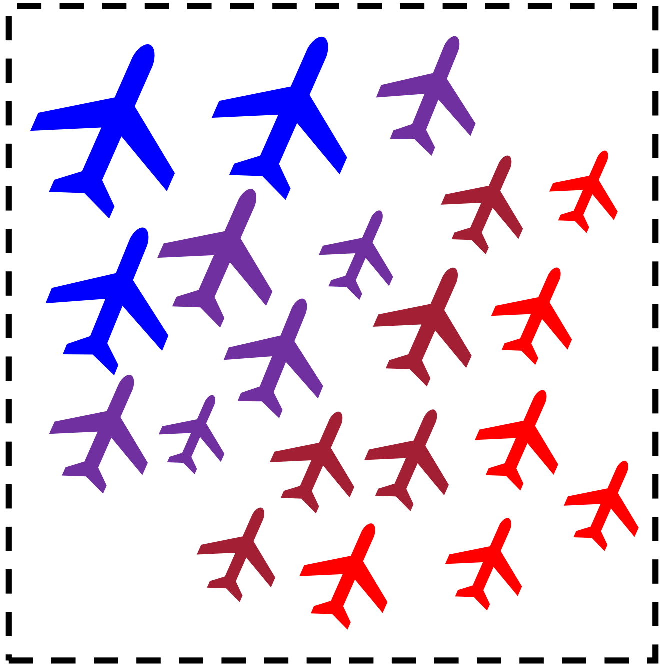

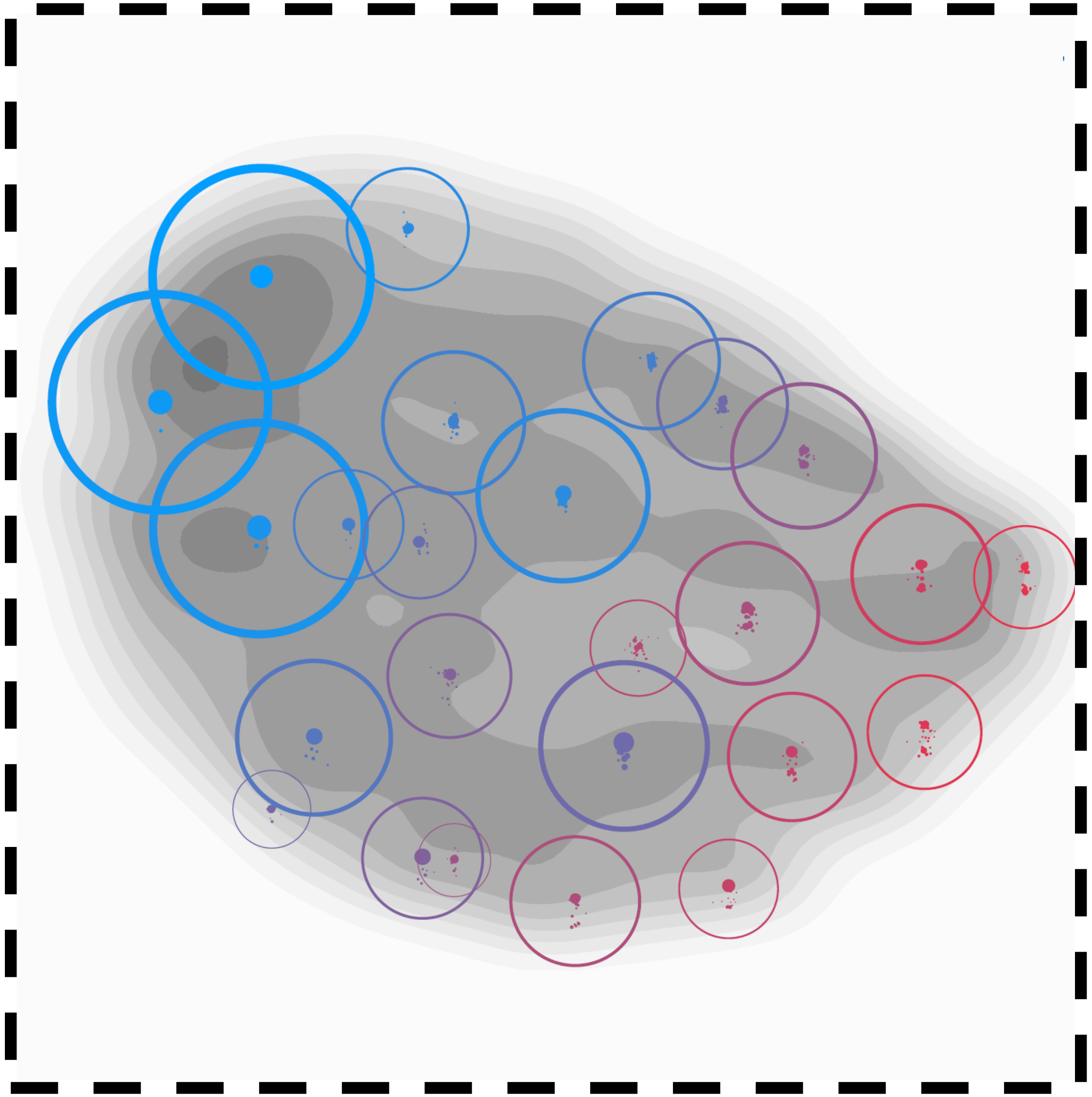

t-SNE embedding: 25-medoid jets shown

[van der Maaten, Hinton, JMLR 2008]

[Komiske, Mastandrea, **EMM**, Naik, Thaler, 1908.08542]

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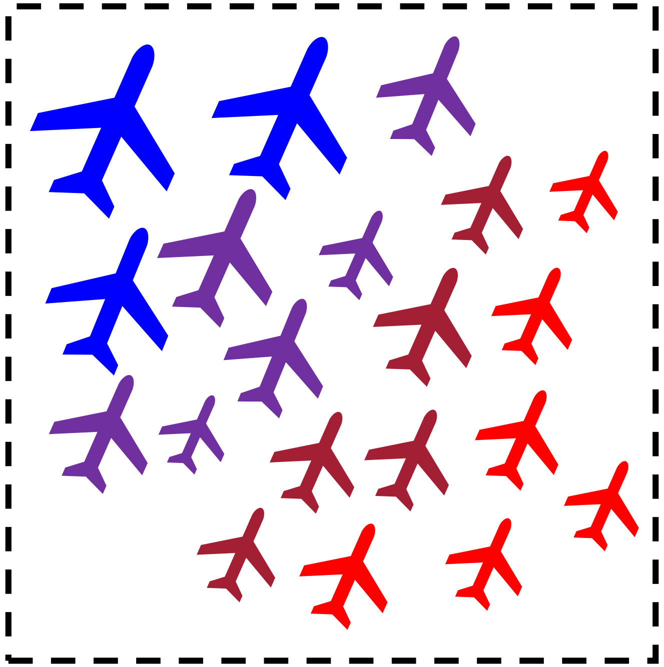

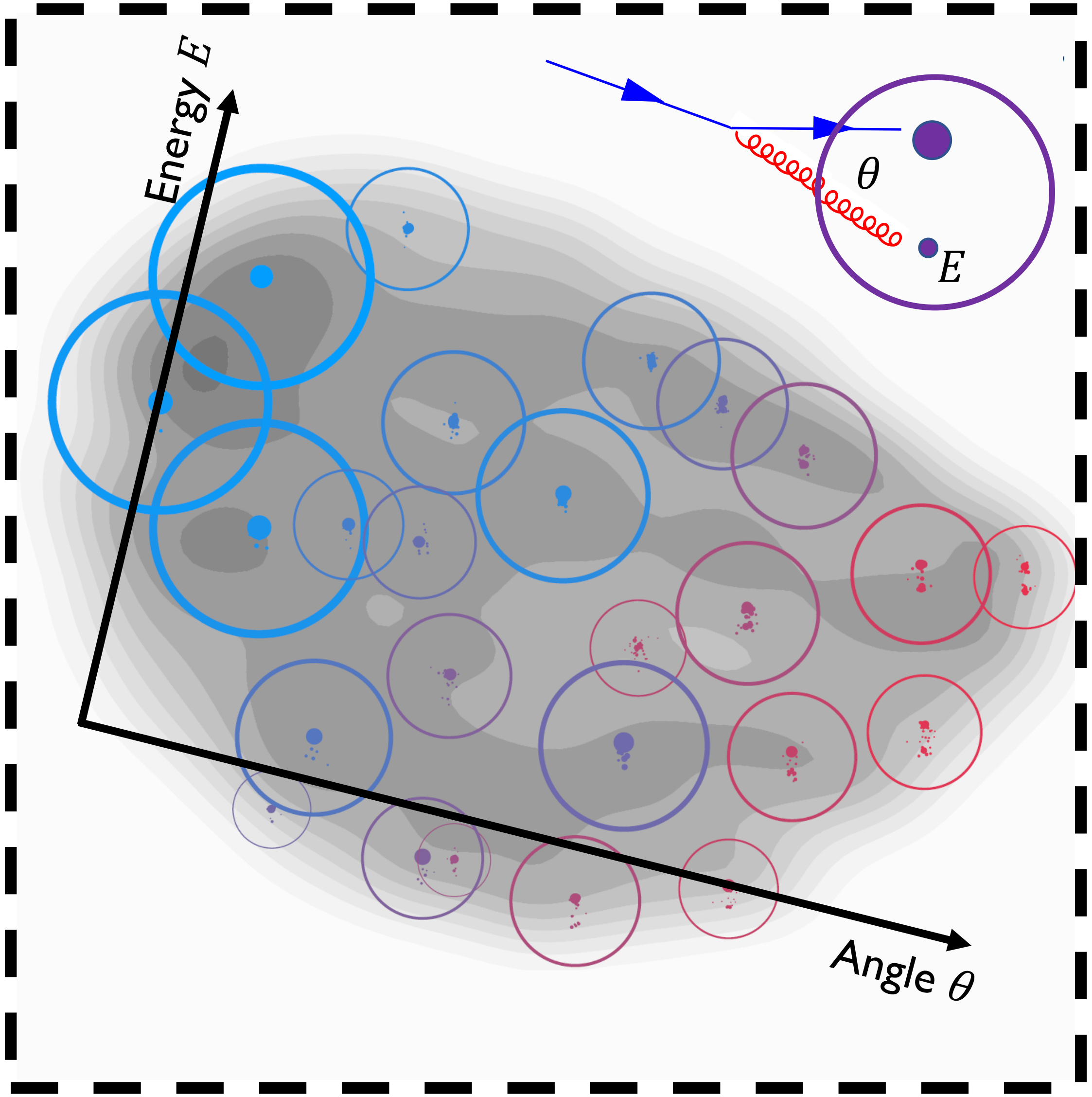

t-SNE embedding: 25-medoid jets shown

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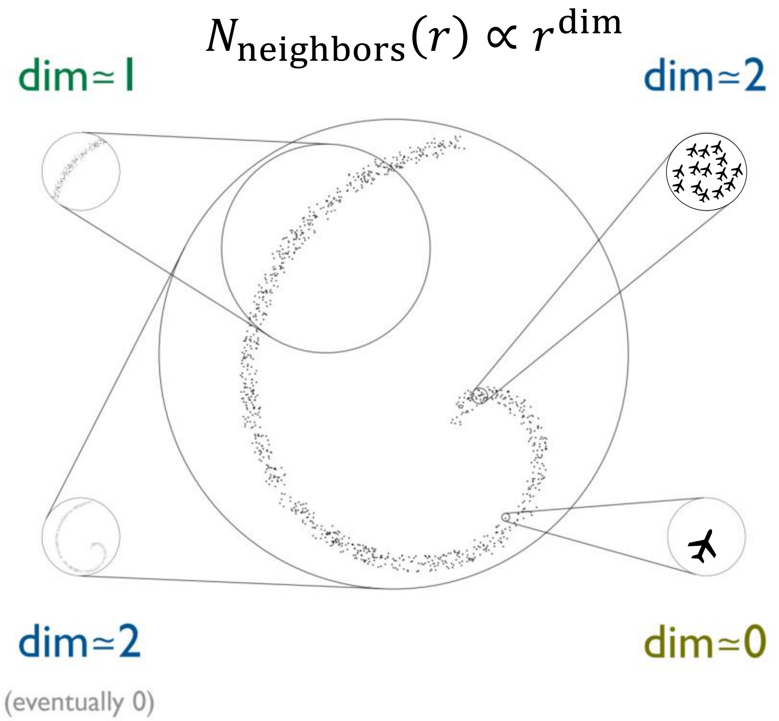
Correlation Dimension

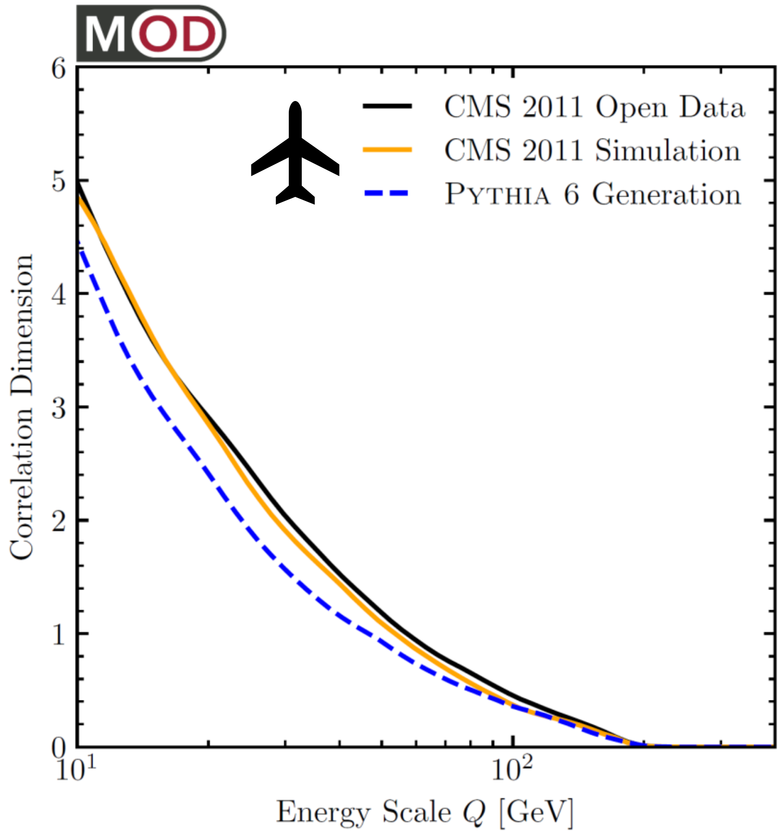
Conceptual Idea



$$\boxed{\dim(Q) = Q \frac{\partial}{\partial Q} \ln \sum_{i=1}^{N} \sum_{j=1}^{N} \Theta[\mathrm{EMD}(\mathcal{E}_i, \mathcal{E}_j) < Q]}$$

[Grassberger, Procaccia, PRL 1983] [Kegl, NeurIPS 2002]

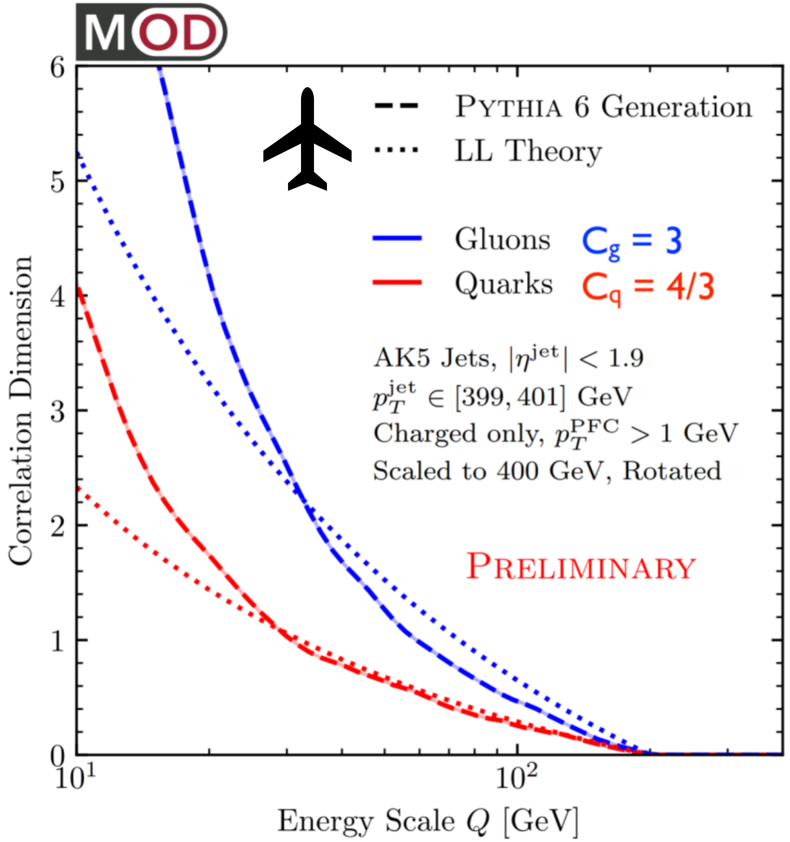
Experimental Data



Dimension blows up at low energies.

[Komiske, Mastandrea, **EMM**, Naik, Thaler, 1908.08542]

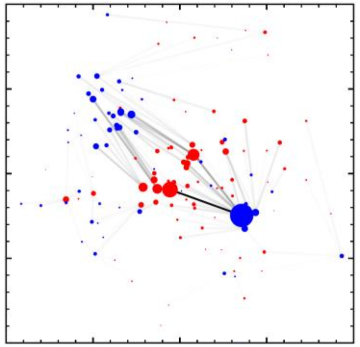
Theoretical Calculation



See extra slides for calculation sketch.

See Jack's Talk for more on dimensionality

CMS Open Data
A new public dataset for jet studies



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A metric for collider events

Supervised Learning
Training directly on collider data

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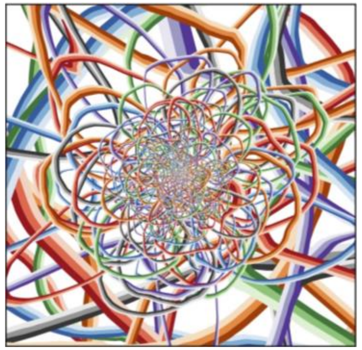
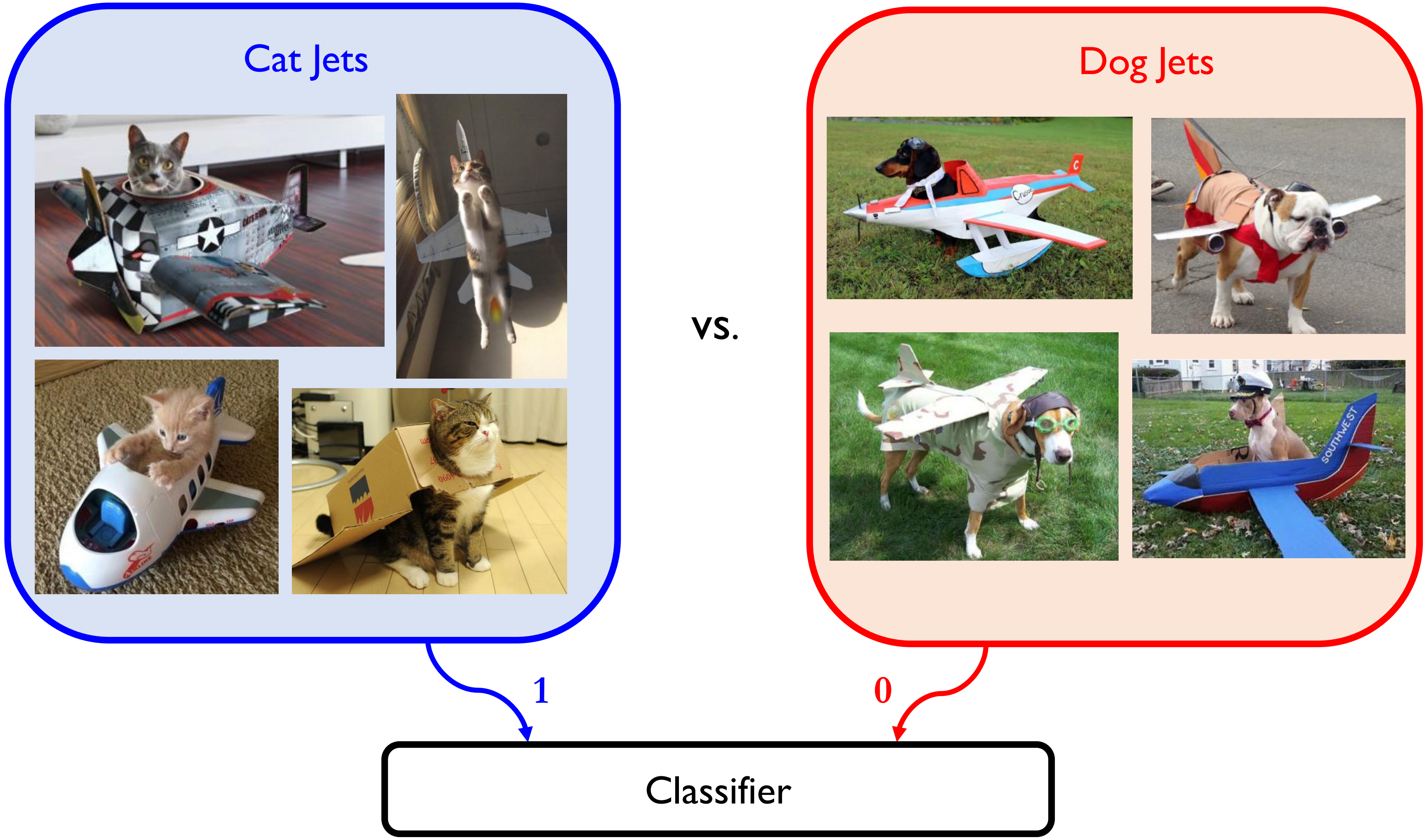


Supervised Learning
Training directly on collider data

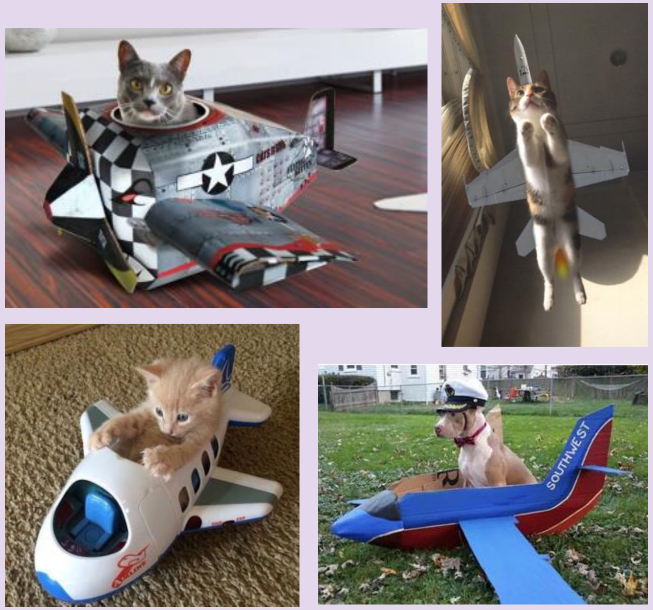
Training on pure samples: Cat jets vs. Dog jets



Training on mixed samples: Cat jets vs. Dog jets

Classification Without Labels (CWoLa)

Cat-enriched Jets



vs.

Dog-enriched Jets

1

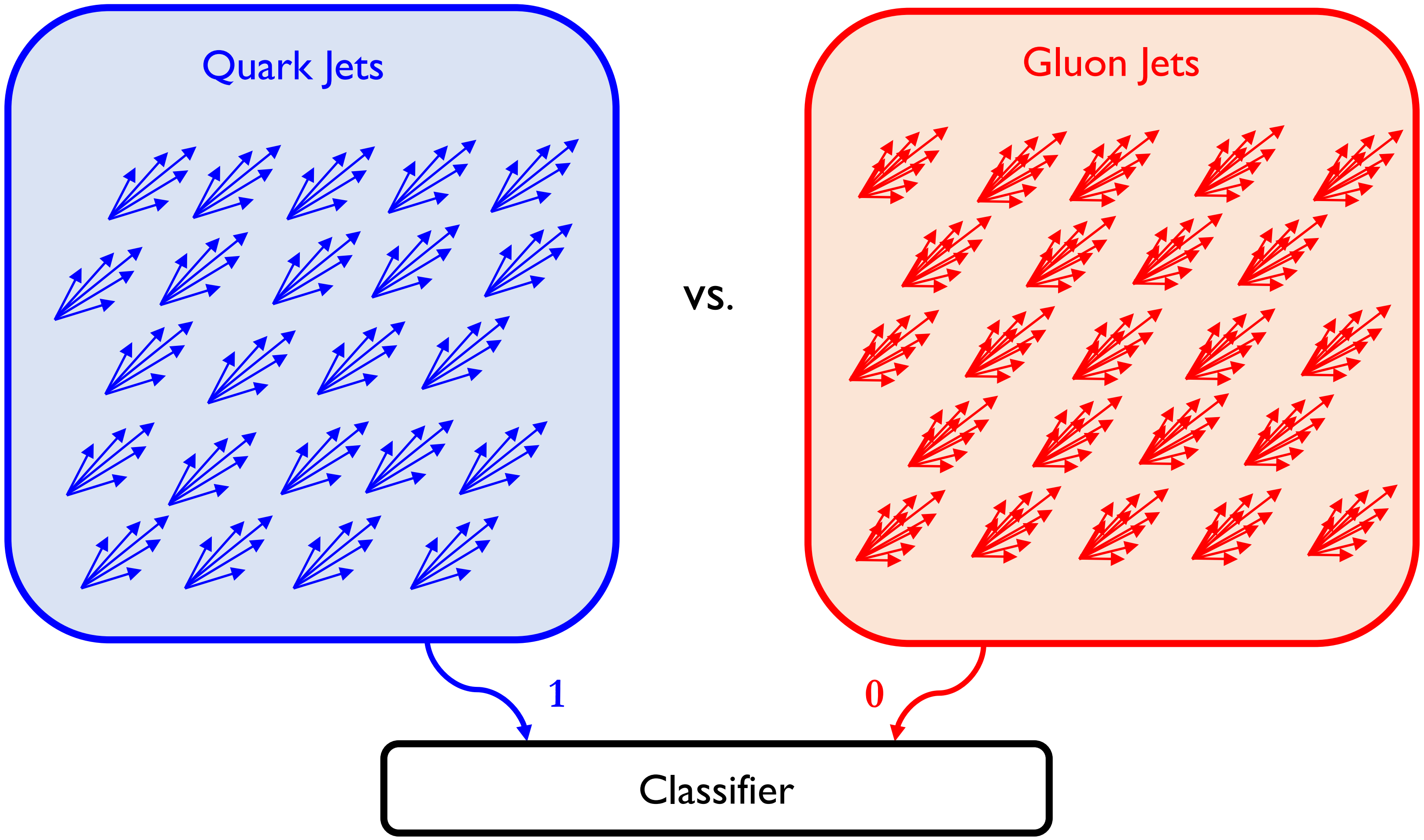
0

Classifier

Used by CMS for ttbb! [CMS 1909.05306]

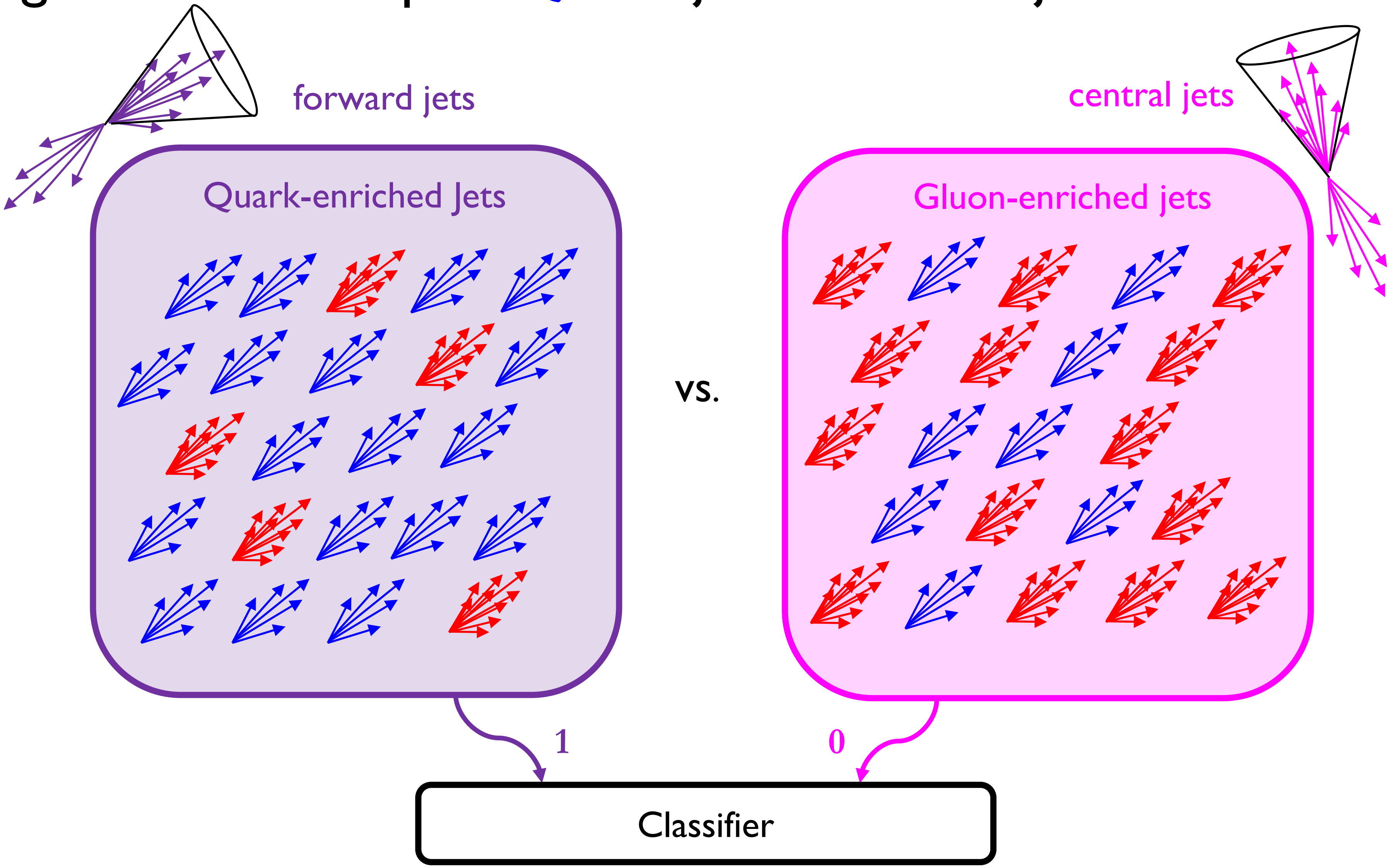
This defines an equivalent classifier to the pure case!

Training on pure samples: Quark jets vs. Gluon jets



Training on mixed samples: Quark jets vs. Gluon jets

Classification Without Labels (CWoLa)



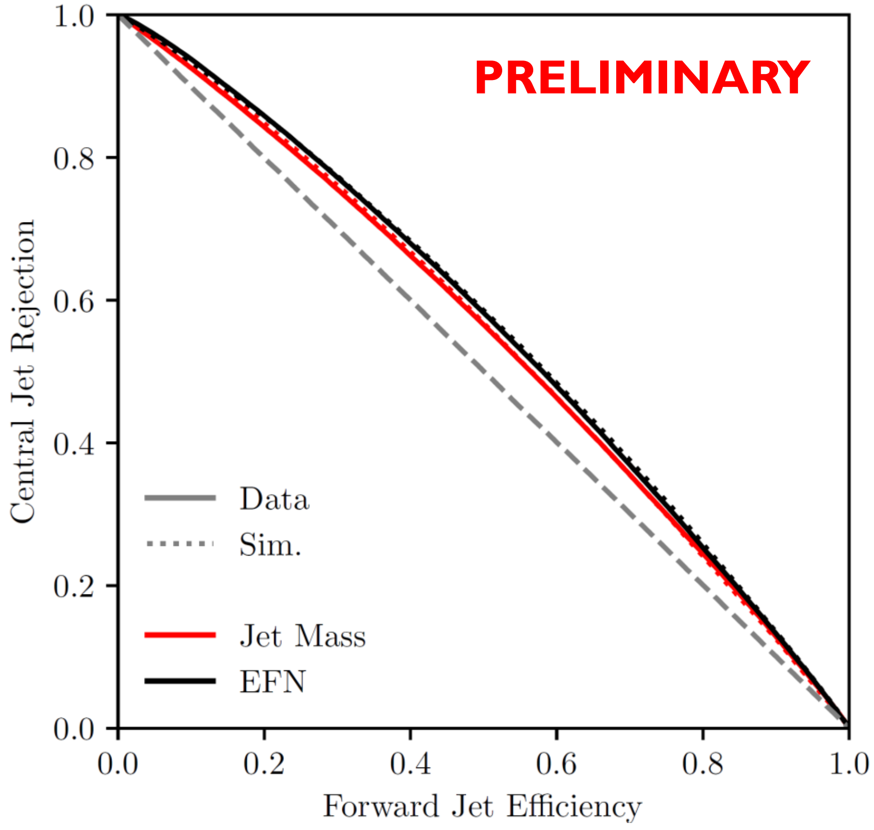
[**EMM**, B. Nachman, J. Thaler, 1708.02949]
[P.T. Komiske, **EMM**, B. Nachman, M.D. Schwartz, 1801.10158]
[L. Dery, B. Nachman, F. Rubbo, A. Schwartzman, 1702.00414] [T. Cohen, M. Freytsis, B. Ostdiek, 1706.09451]

Training on Data!

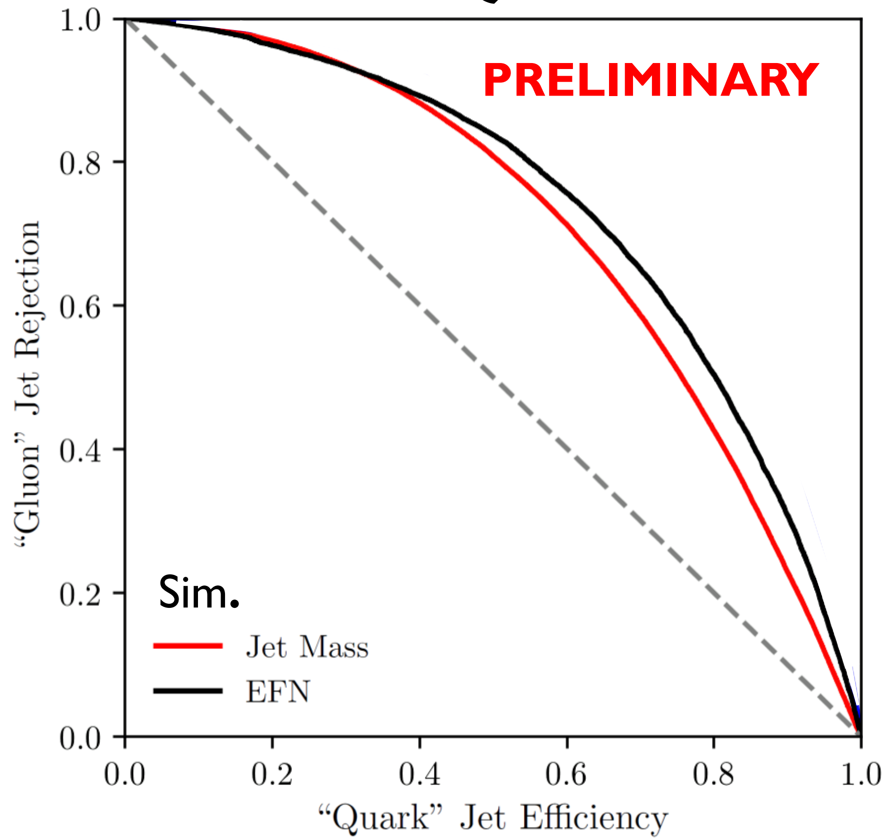
Central Jets $(|\eta^{\text{jet}}| < 0.7)$: ~45% quark jets

Forward Jets $(|\eta^{\text{jet}}| > 0.7)$: ~65% quark jets

Weak QG Labels



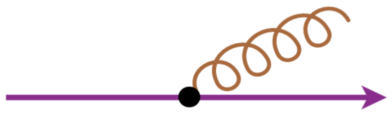
Sim. MC QG Labels



To reduce sample dependence, we train an EFN on tracks with $p_T^{\text{PFC}} > 1$ GeV and remove pileup.

Or high-dimensional unfolding? [See Patrick's Talk]

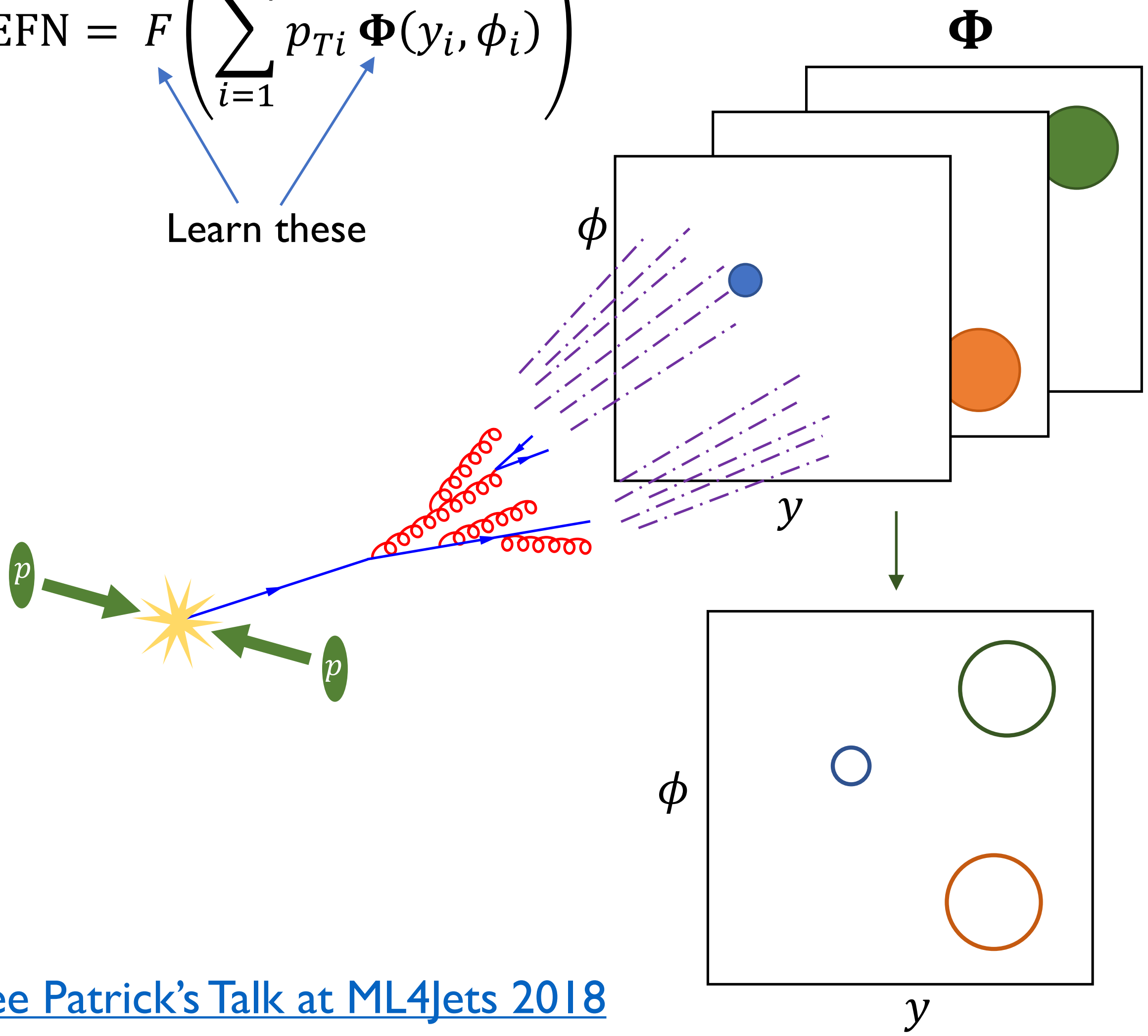
What is the model learning?

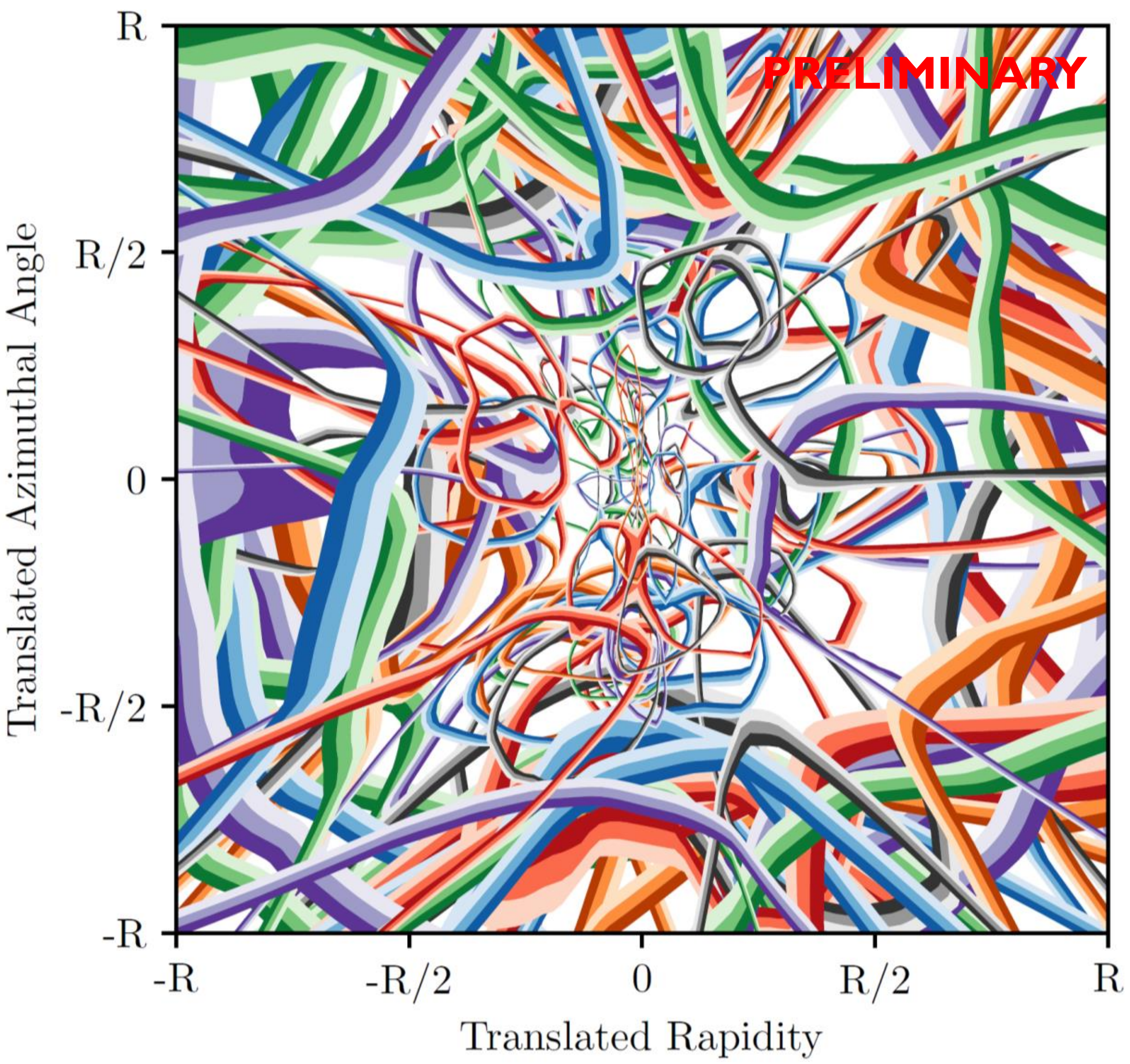


$$dP_{i\to ig} \simeq \frac{2\alpha_s C_i}{\pi}\frac{d\theta}{\theta}\frac{dz}{z}$$

$$\text{EFN} = F\left(\sum_{i=1}^{M} p_{Ti}\,\boldsymbol{\Phi}(y_i,\phi_i)\right)$$

Learn these





Visualizing 256 filters for EFN (weakly) trained on data

See Patrick's Talk at ML4Jets 2018

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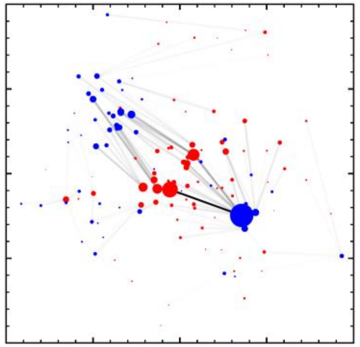

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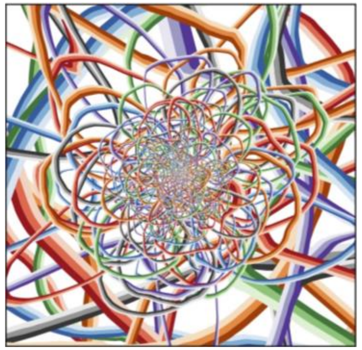


Supervised Learning
Training directly on collider data

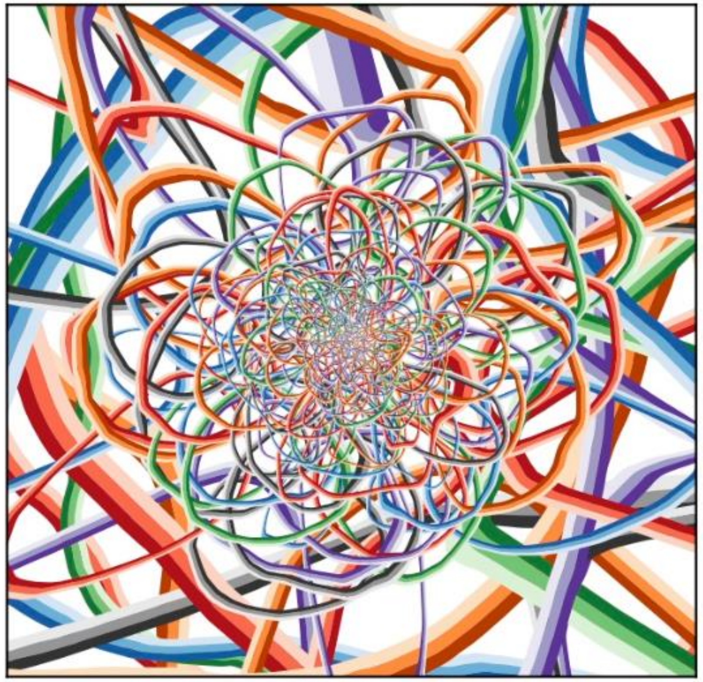
EnergyFlow

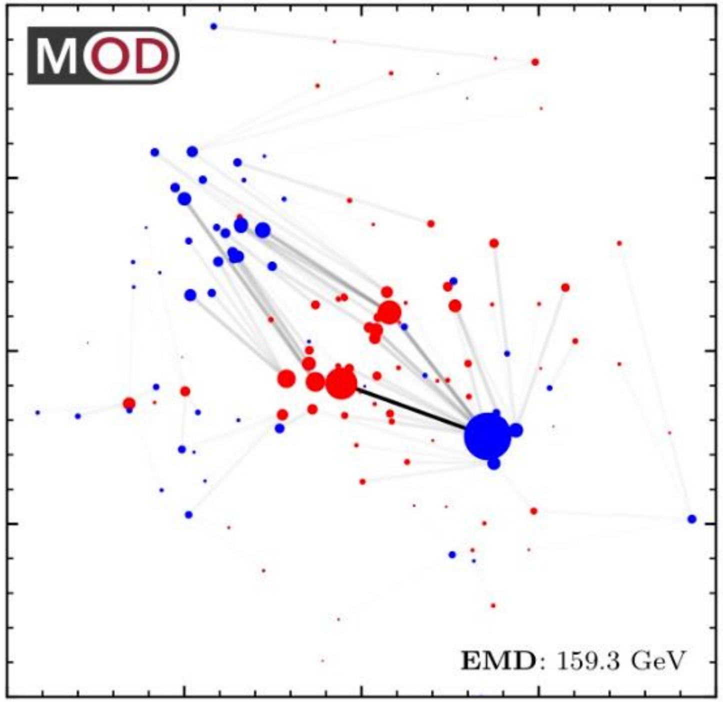
Search docs

https://energyflow.network

```
pip install energyflow
```

Welcome to EnergyFlow

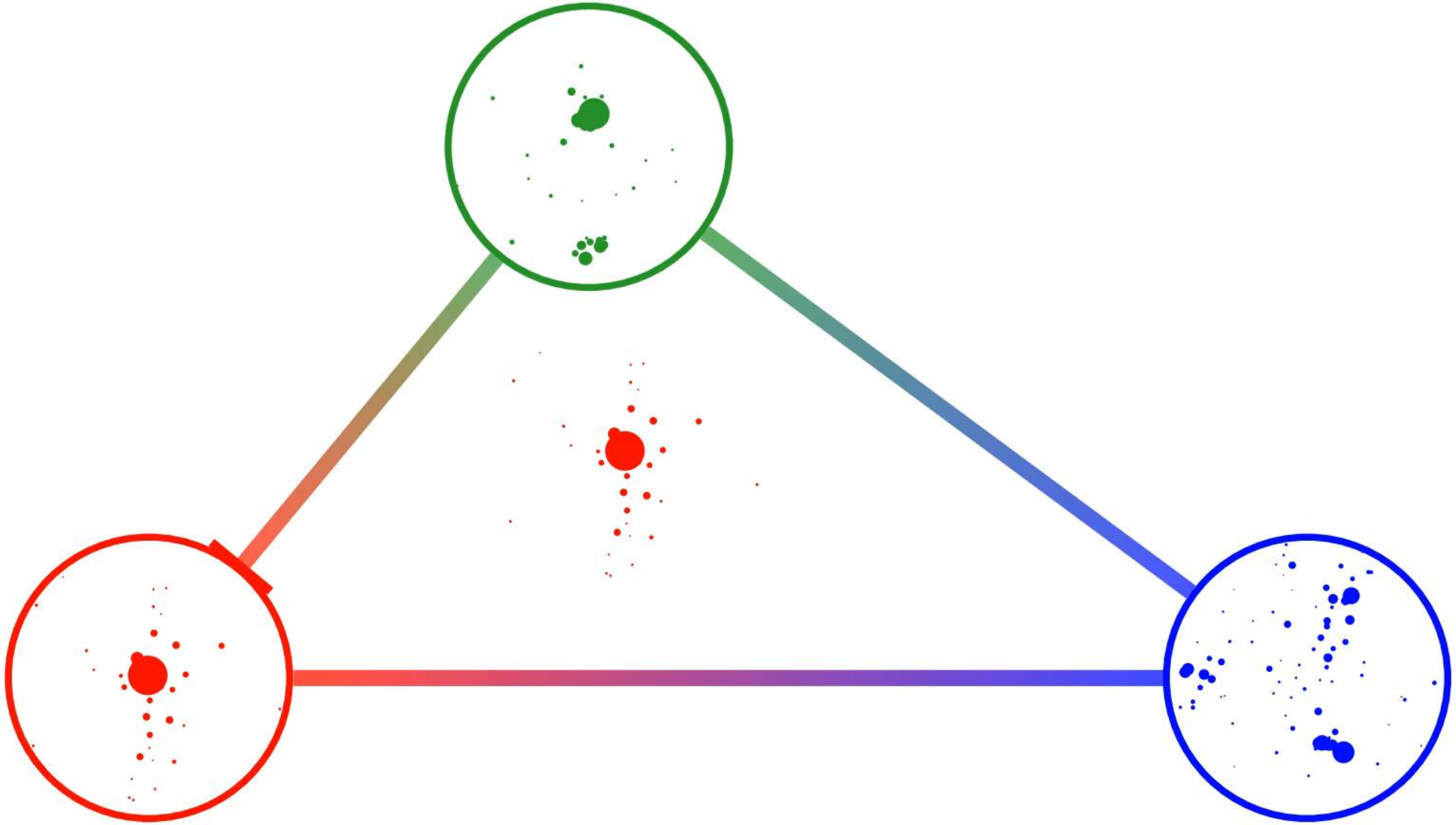




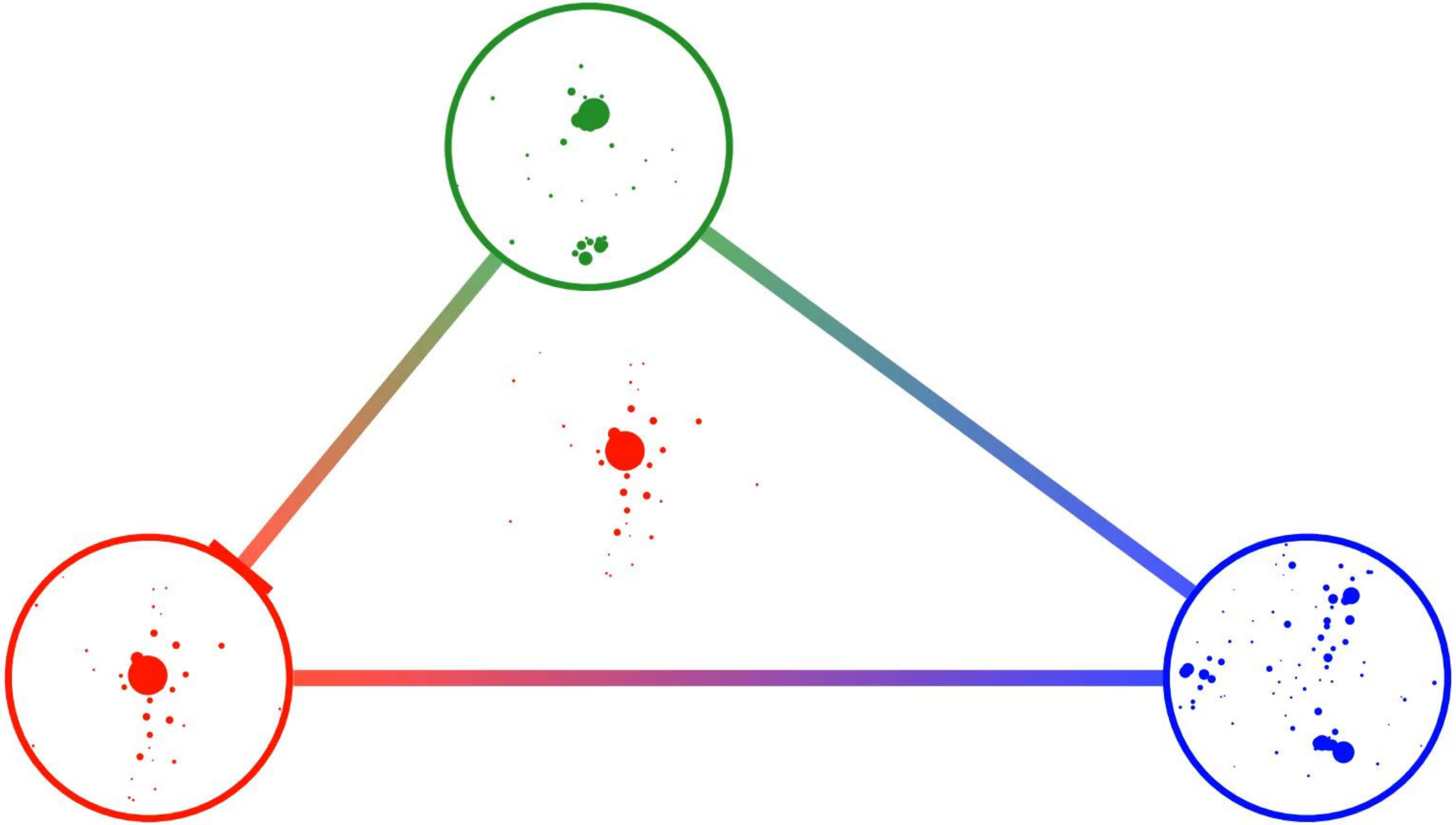
EnergyFlow is a Python package containing a suite of particle physics tools:

- Energy Flow Polynomials: EFPs are a collection of jet substructure observables which form a complete linear basis of IRC-safe observables. EnergyFlow provides tools to compute EFPs on

The End
Thank you!



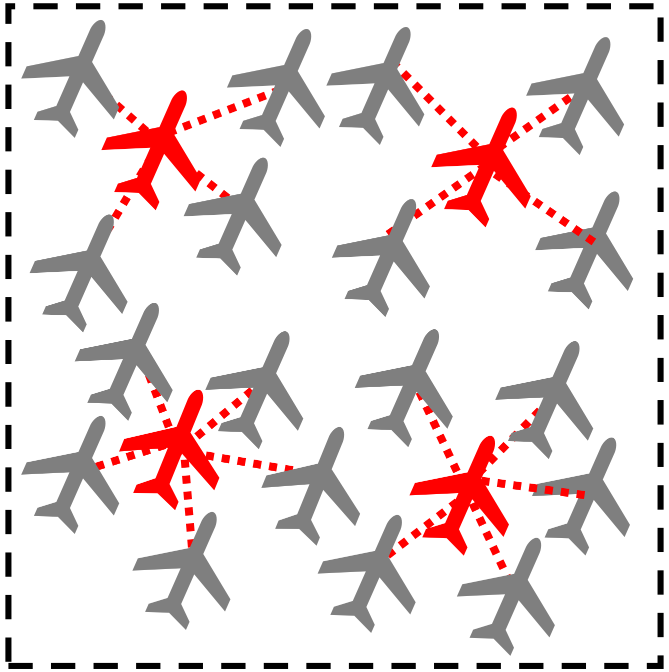
Extra Slides



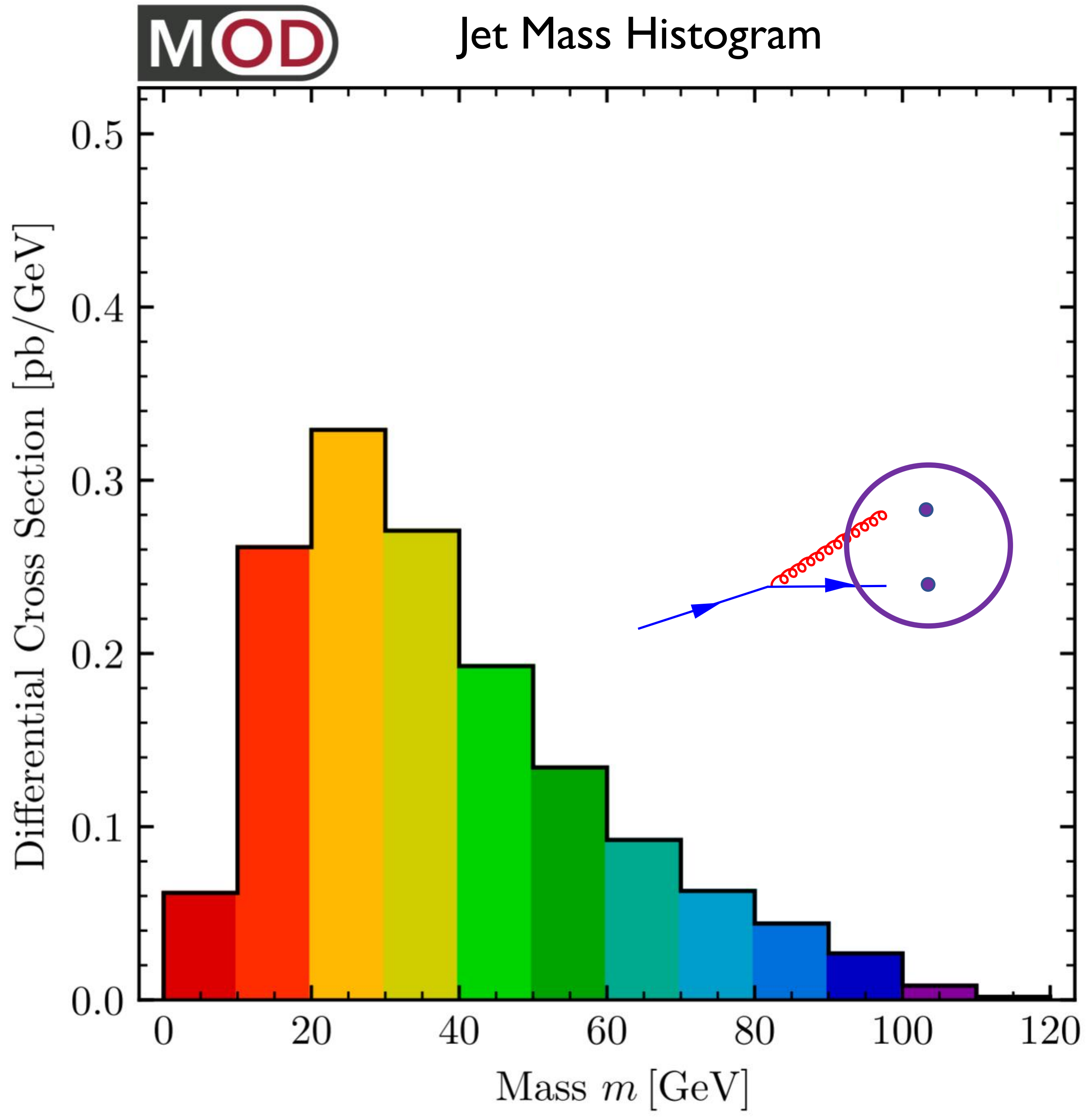
Most Representative Jets

Jet Mass: $m = \left(\sum_{i=1}^{M} p_i^{\mu}\right)^2$

Measures how "wide" the jet is.



Jet Mass Histogram



[Komiske, Mastandrea, **EMM**, Naik, Thaler, 1908.08542]

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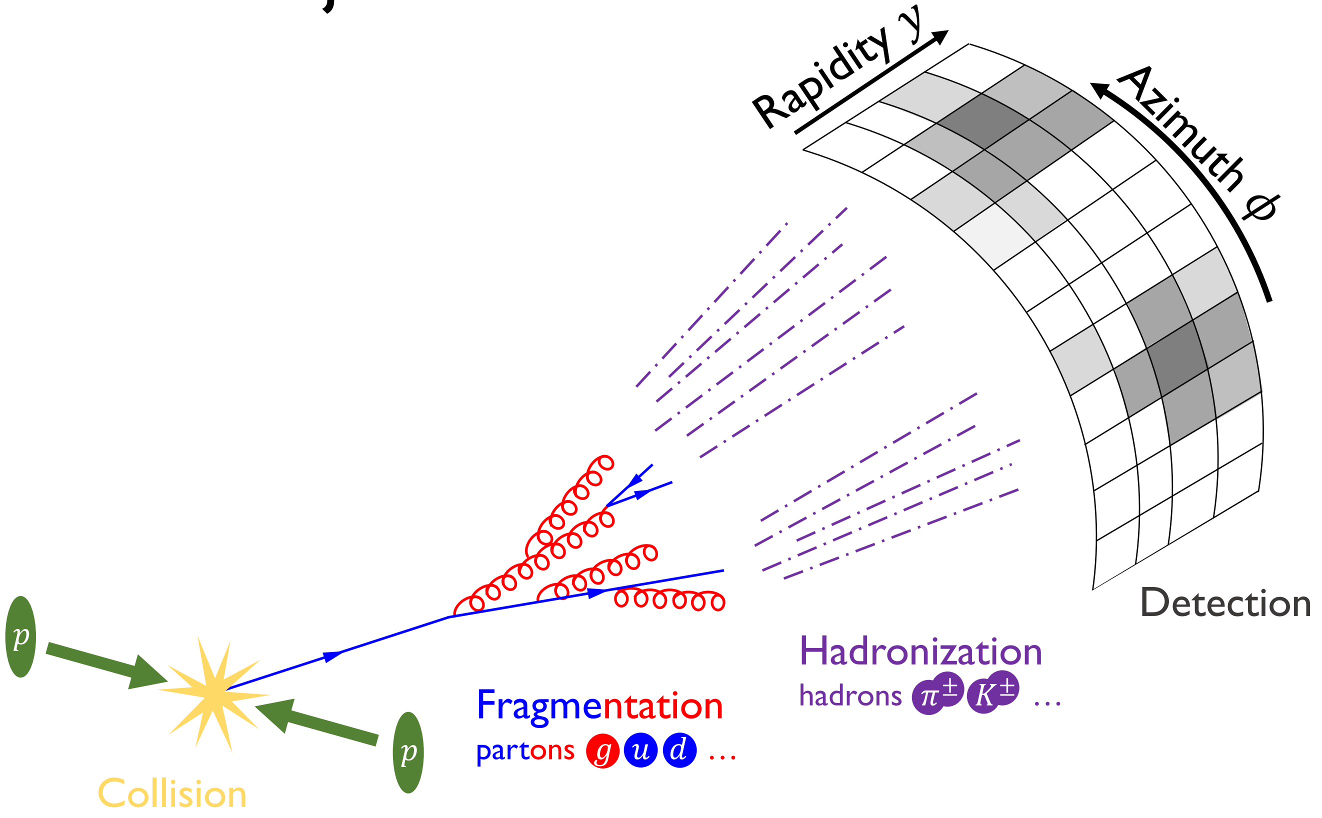
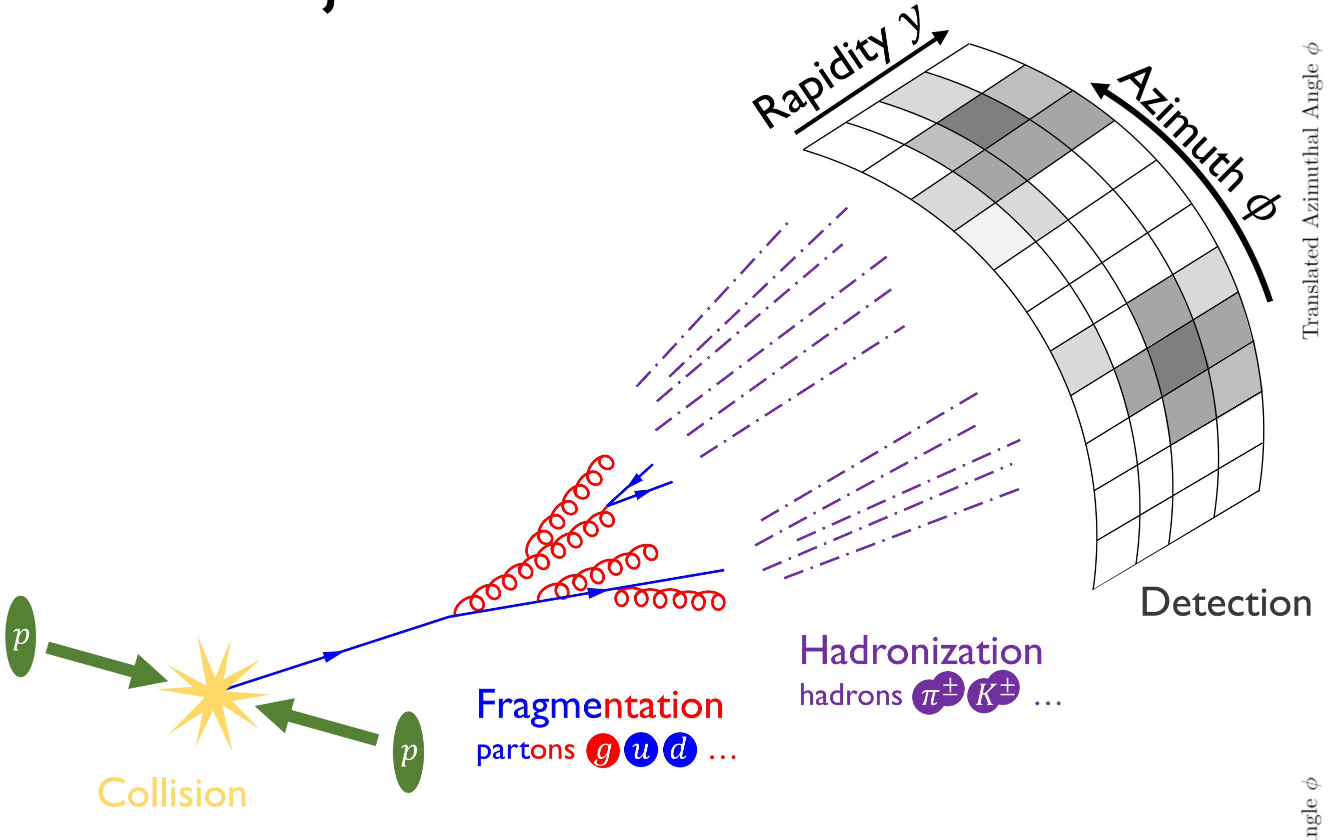


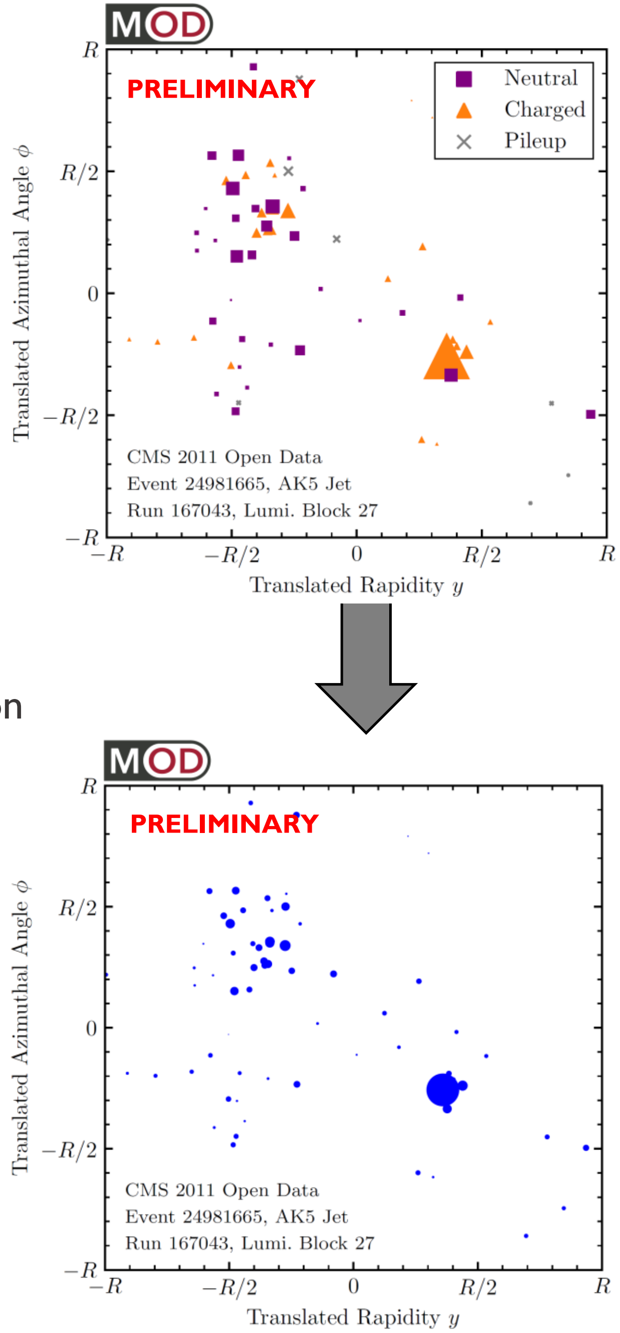
The *energy flow* (distribution of energy) is the information that is robust to:
fragmentation, hadronization, detector effects, …

[N.A. Sveshnikov, F.V. Tkachov, 9512370]
[F.V. Tkachov, 9601308]
[P.S. Cherzor, N.A. Sveshnikov, 9710349]

Energy flow ⇔ Infrared and Collinear (IRC) Safe information

When are two jets similar?





Treat jets as distributions of energy: $\mathcal{E}(\hat{n}) = \sum_{i=1}^{M} E_i \, \delta(\hat{n} - \hat{n}_i)$

energy direction

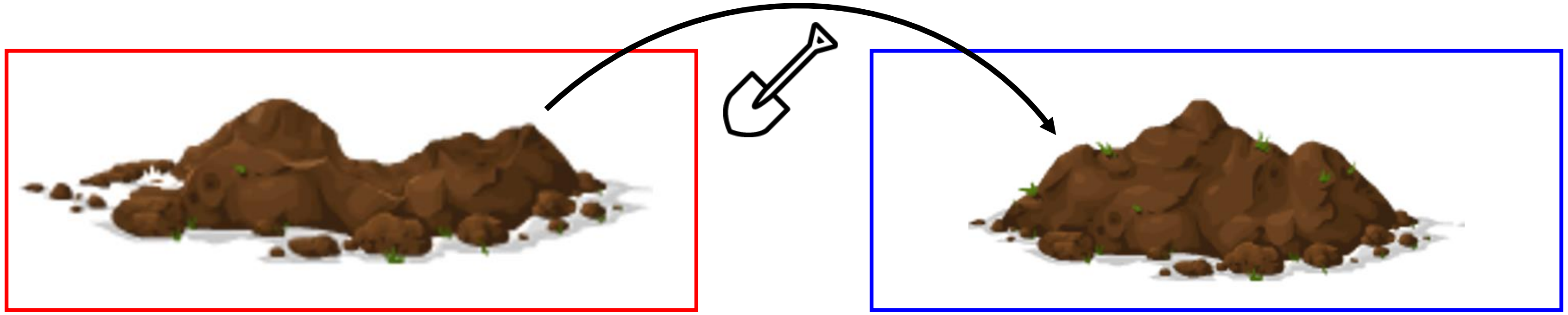
Ignoring particle flavor, charge…

The Energy Mover's Distance

Review: The Earth Mover's Distance

Earth Mover's Distance: the minimum "work" (stuff x distance) to rearrange one pile of dirt into another

[Peleg, Werman, Rom]
[Rubner, Tomasi, Guibas]



Metric on the space of (normalized) distributions: *symmetric, non-negative, triangle inequality*

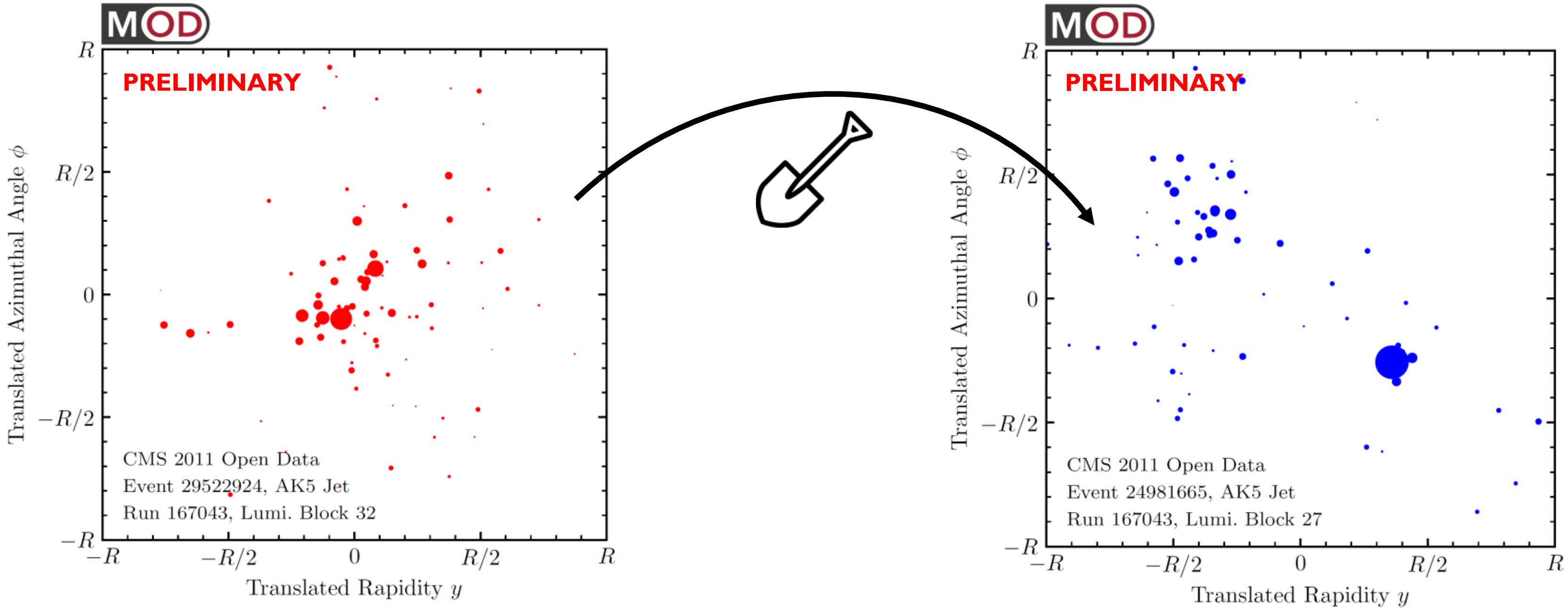
Distributions are close in EMD ⇔ their expectation values are close.

Also known as the 1-Wasserstein metric.

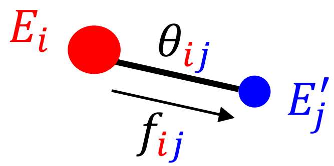
The Energy Mover's Distance

From Earth to Energy

Energy Mover's Distance: the minimum "work" (energy x angle) to rearrange one *event* (pile of energy) into another [Komiske, EMM, Thaler, 1902.02346]



$$\mathrm{EMD}(\mathcal{E},\mathcal{E}') = \min_{\{f\}} \sum_{i=1}^{M}\sum_{j=1}^{M'} f_{ij}\frac{\theta_{ij}}{R} + \left|\sum_{i=1}^{M} E_i - \sum_{j=1}^{M'} E'_j\right|$$



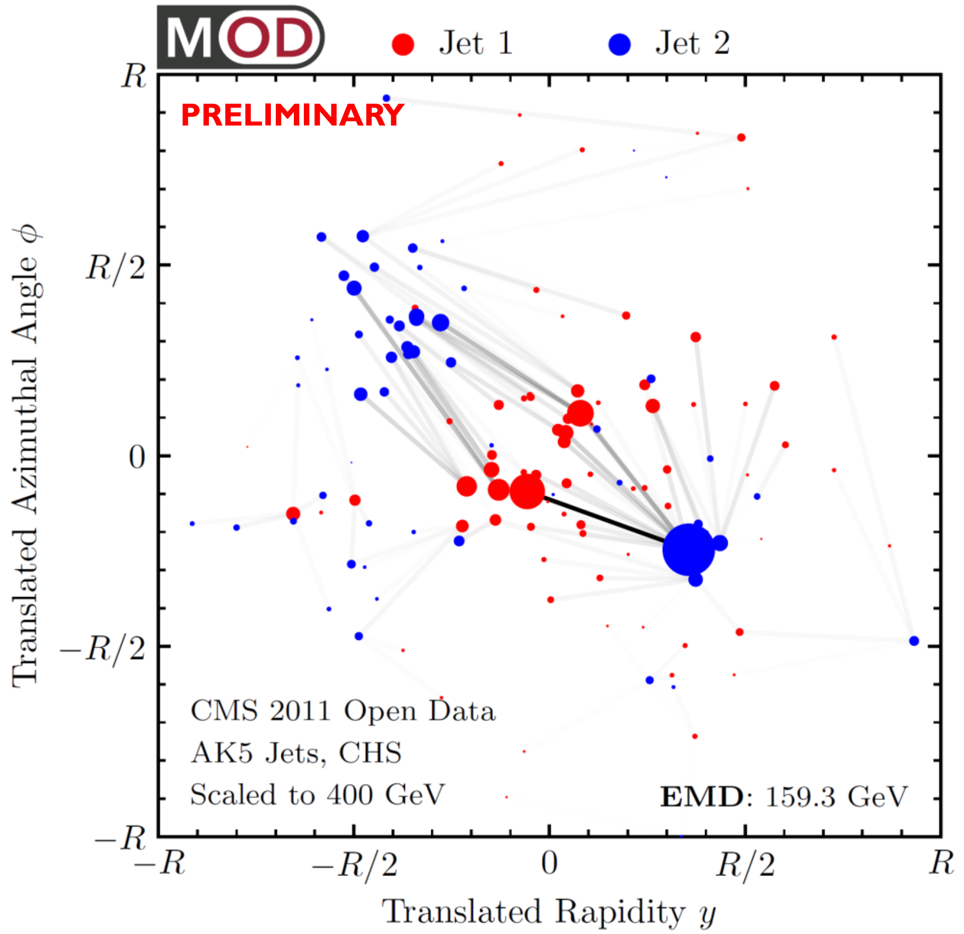
Difference in radiation pattern

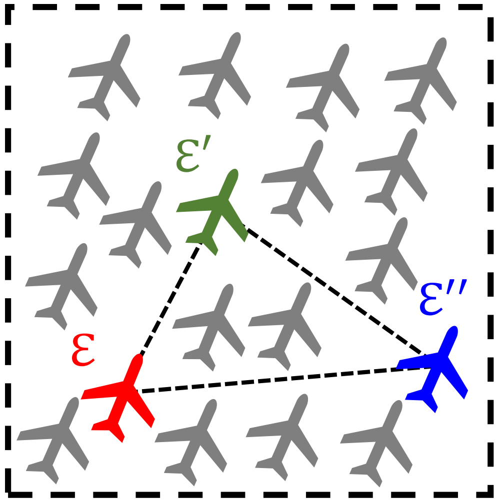
Difference in total energy

The Energy Mover’s Distance

From Earth to Energy

Energy Mover’s Distance: the minimum “work” (energy x angle) to rearrange one *event* (pile of energy) into another [Komiske, EMM, Thaler, 1902.02346]





$$\mathrm{EMD}(\mathcal{E}, \mathcal{E}') + \mathrm{EMD}(\mathcal{E}', \mathcal{E}'') \geq \mathrm{EMD}(\mathcal{E}, \mathcal{E}'')$$

EMD has dimensions of energy

True metric as long as $R \geq \frac{1}{2}\theta_{\max}$

$R \geq$ the jet radius, for conical jets

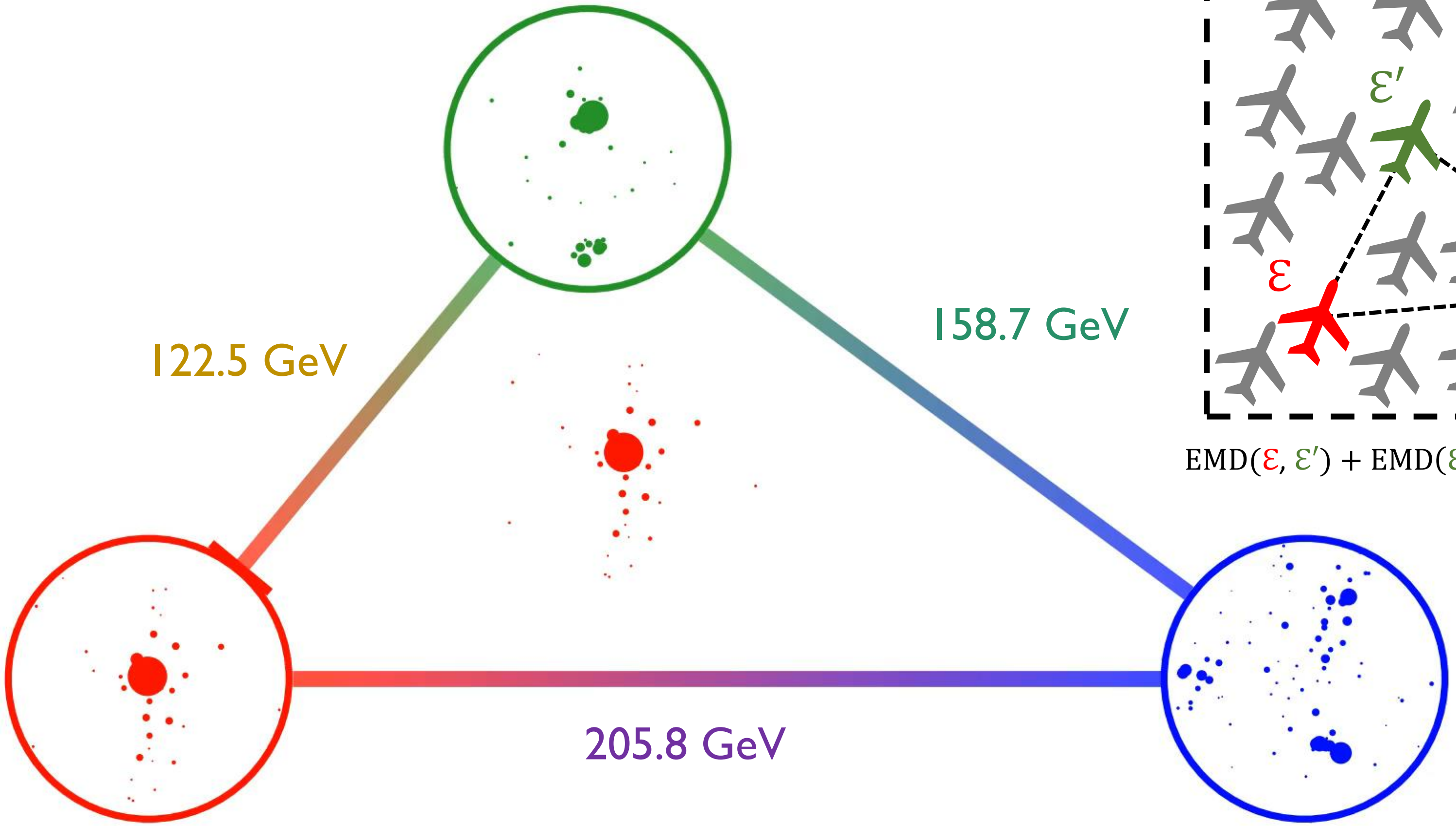
Solvable via Optimal Transport problem.

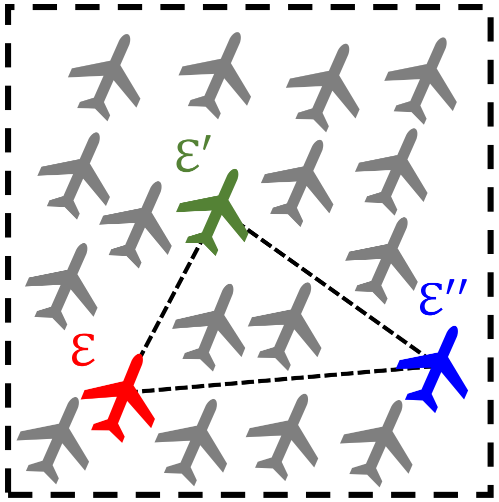
~1ms to compute EMD for two jets with 100 particles.

The Energy Mover's Distance

From Earth to Energy

Energy Mover's Distance: the minimum "work" (energy x angle) to rearrange one *event* (pile of energy) into another [Komiske, EMM, Thaler, 1902.02346]





$$\mathrm{EMD}(\mathcal{E},\mathcal{E}') + \mathrm{EMD}(\mathcal{E}',\mathcal{E}'') \geq \mathrm{EMD}(\mathcal{E},\mathcal{E}'')$$

https://energyflow.network

Energy Moving and IRC Safety

Events close in EMD are close in any infrared and collinear safe observable!

Additive IRC-safe observables:

Energy Mover's Distance

$$\boxed{\mathrm{EMD}(\mathcal{E}, \mathcal{E}') \geq \frac{1}{RL} \left|\mathcal{O}(\mathcal{E}) - \mathcal{O}(\mathcal{E}')\right|}$$

Difference in observable values

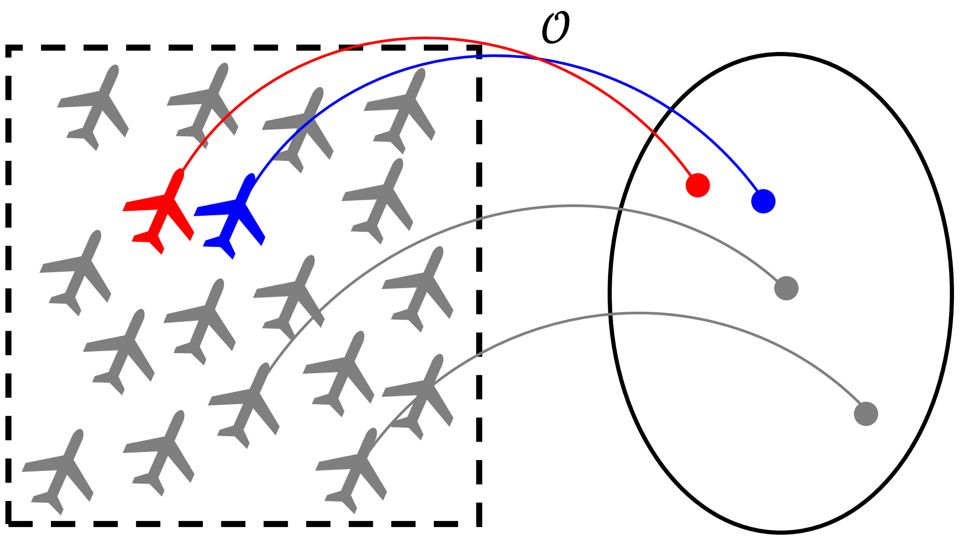
"Lipschitz constant" of Φ
i.e. bound on its derivative

$$\mathcal{O}(\mathcal{E}) = \sum_{i=1}^{M} E_i \, \Phi(\hat{n}_i)$$

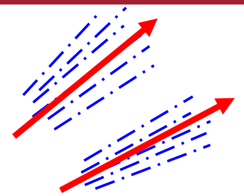
e.g. $\beta \geq 1$ jet angularities:

[Berger, Kucs, Sterman, 0303051]
[Larkoski, Thaler, Waalewijn, 1408.3122]

$$\left|\lambda^{(\beta)}(\mathcal{E}) - \lambda^{(\beta)}(\mathcal{E}')\right| \leq \beta \, \mathrm{EMD}(\mathcal{E}, \mathcal{E}')$$



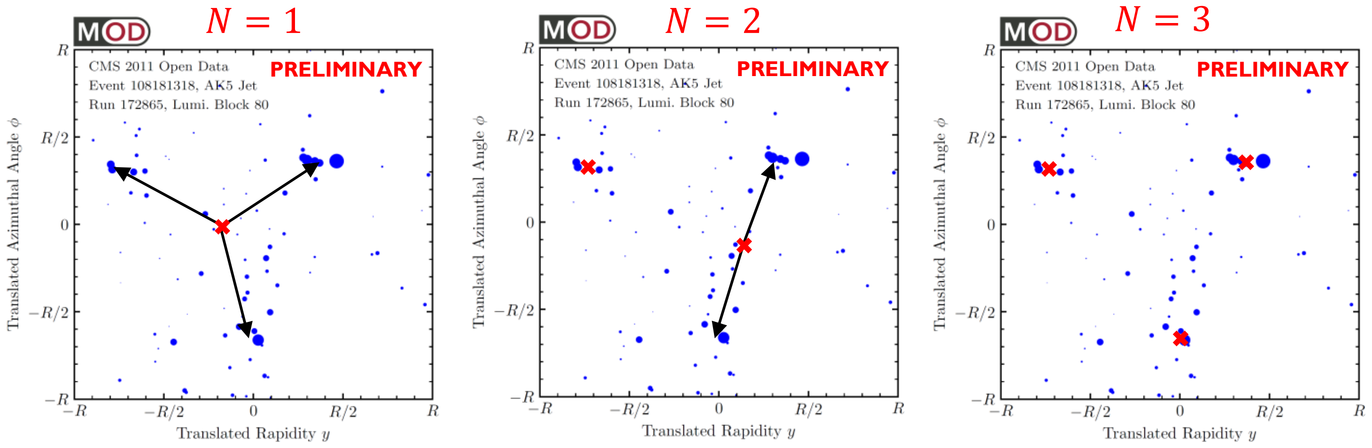
Old Observables in a New Language

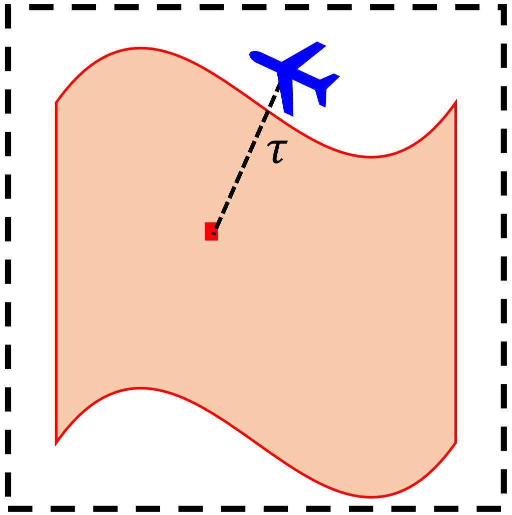


***N*-subjettiness** is the EMD between the event and the closest N-particle event.

$$\tau_N^{(\beta)}(\mathcal{E}) = \min_{N \text{ axes}} \sum_{i=1}^{M} E_i \min\{\theta_{1,i}^{\beta}, \theta_{2,i}^{\beta}, \dots, \theta_{N,i}^{\beta}\} \longrightarrow \tau_N(\mathcal{E}) = \min_{|\mathcal{E}'|=N} \mathrm{EMD}(\mathcal{E}, \mathcal{E}').$$

$\beta \geq 1$ is p-Wasserstein distance with $\mathrm{p} = \beta$.





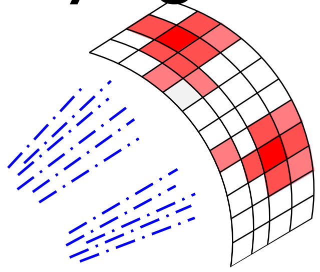
Geometry in the space of events

Thrust is the EMD between the event and two back-to-back particles.

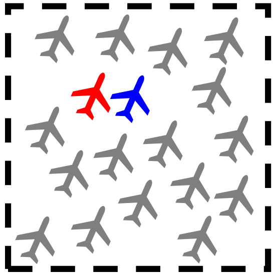
$$t(\mathcal{E}) = E - \max_{\hat{n}} \sum_i |\vec{p}_i \cdot \hat{n}| \longrightarrow t(\mathcal{E}) = \min_{|\mathcal{E}'|=2} \mathrm{EMD}(\mathcal{E}, \mathcal{E}')$$

with $\theta_{ij} = \hat{n}_i \cdot \hat{n}_j$, $\hat{n} = \vec{p}/E$

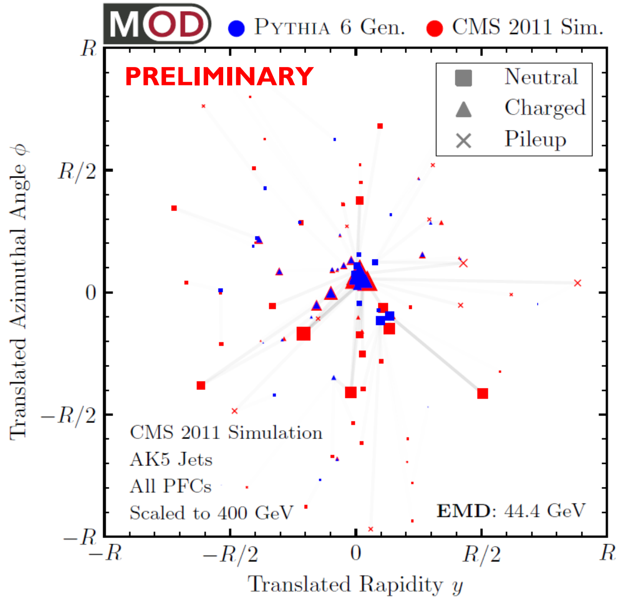
Quantifying Pileup and Detector Effects with EMD



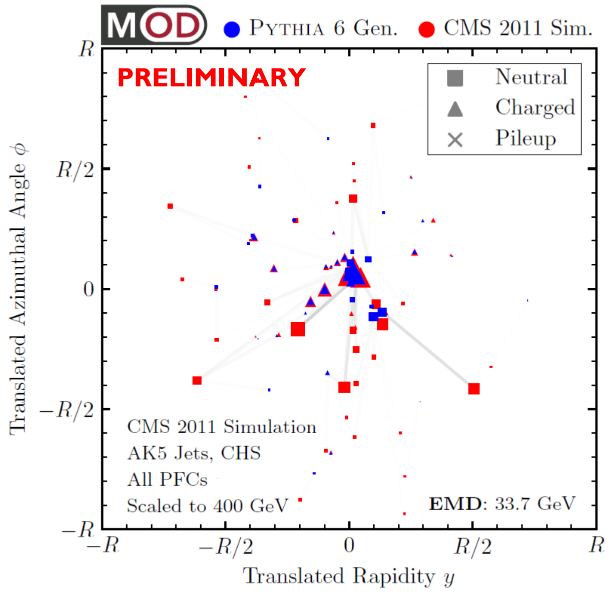
Gen./Sim. EMD universally quantifies pileup and detector effects.



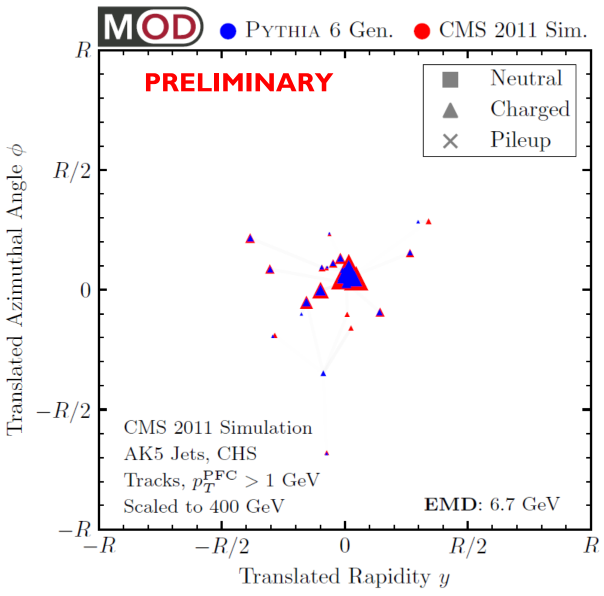
Gen./Sim. EMD: 44.4 GeV



Gen./Sim. EMD: 33.7 GeV



Gen./Sim. EMD: 6.7 GeV



+ charged hadron subtraction

+ Tracks only, $p_T^{\mathrm{PFC}} > 1$ GeV cut

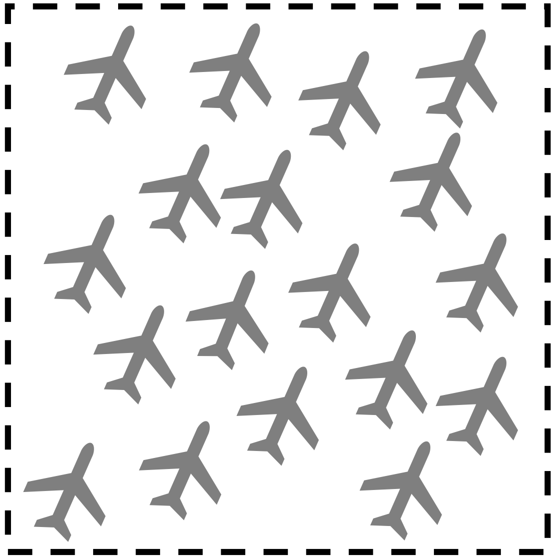
See extra slides for histograms. Can also quantify hadronization effects this way.

Exploring the Space of Jets: Visualizing the Manifold

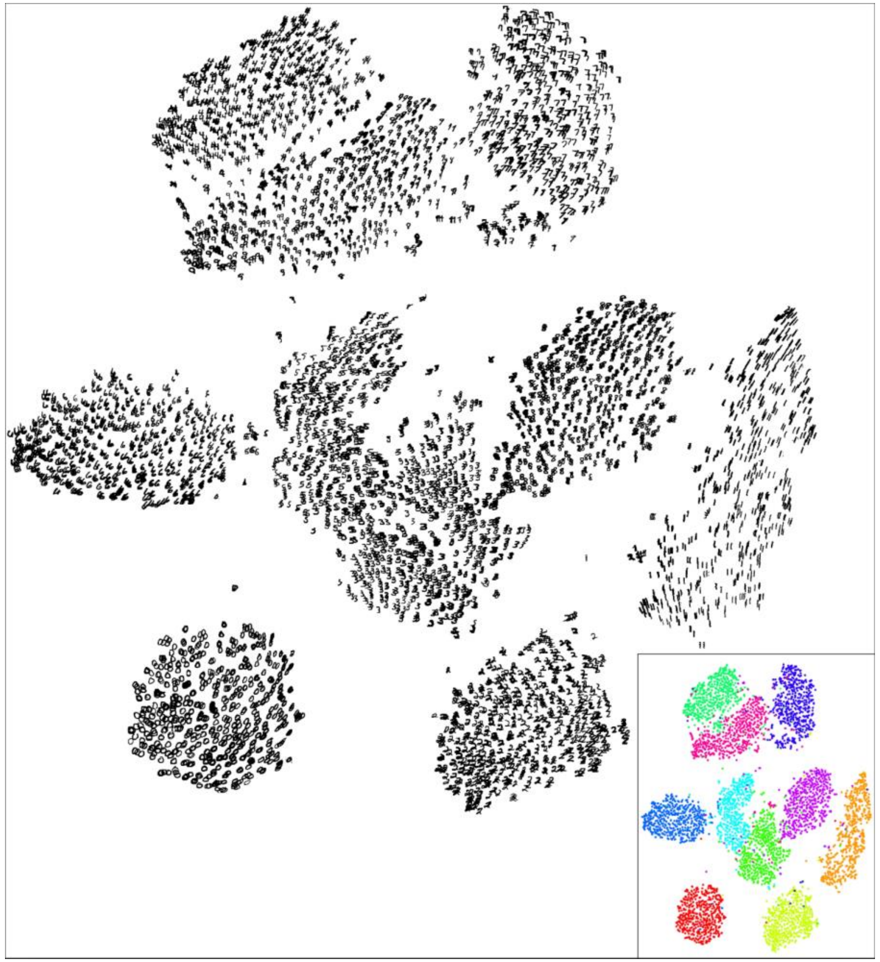
Visualize the space of events with t-Distributed Stochastic Neighbor Embedding (t-SNE).

[L. van der Maaten, G. Hinton]

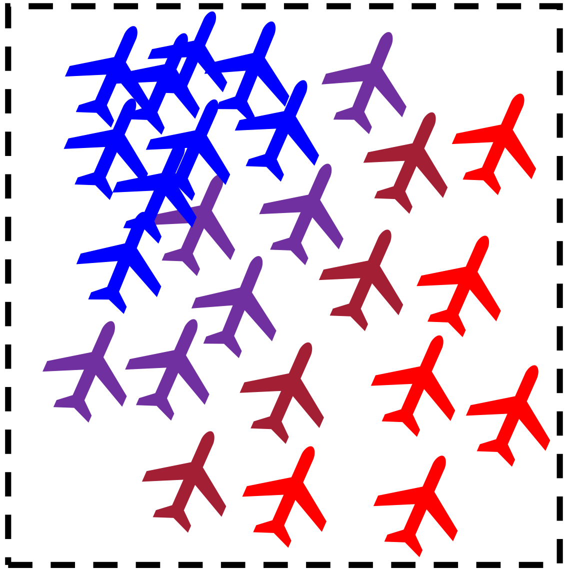
Finds an embedding into a low-dimensional manifold that respects distances.



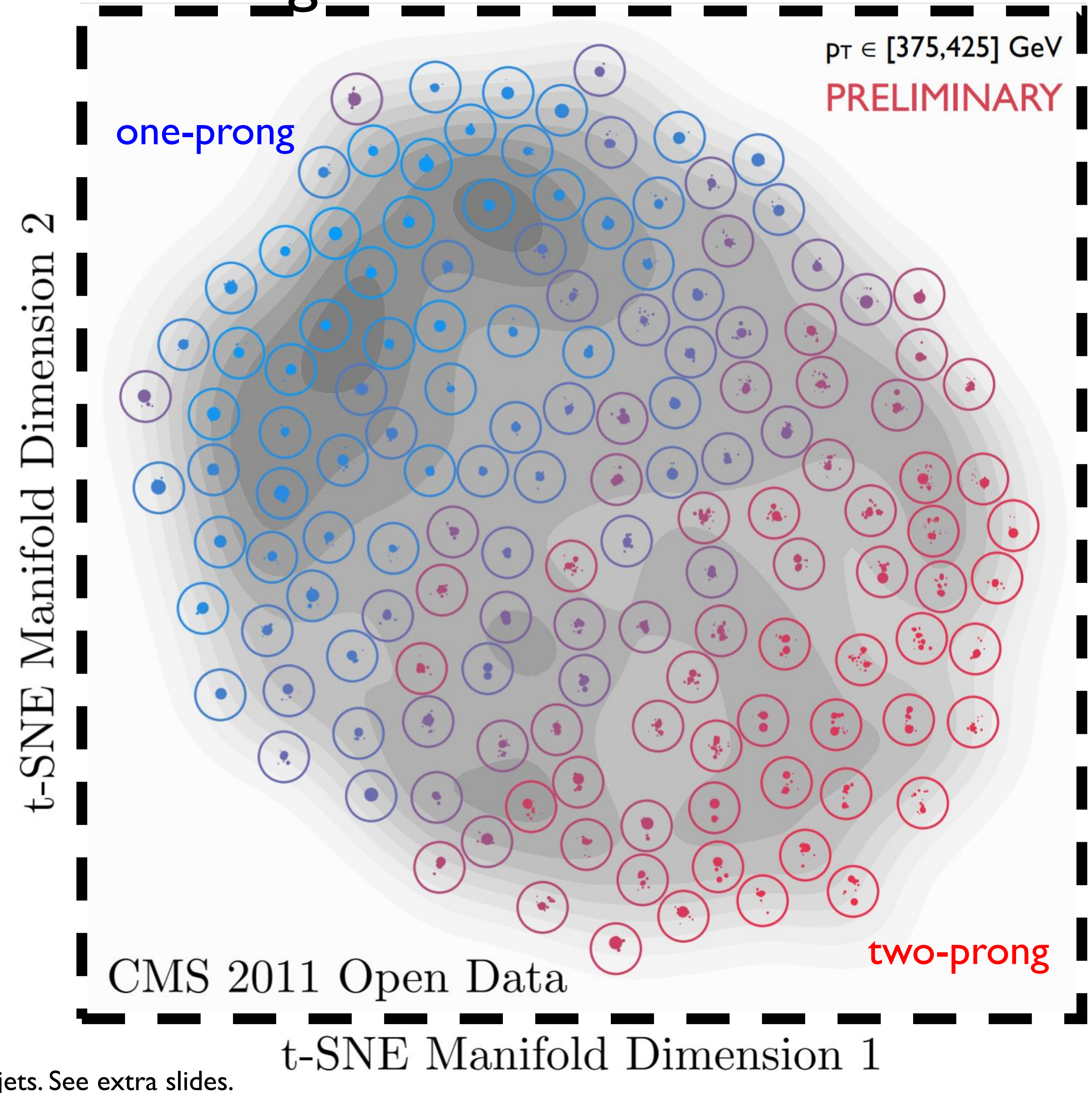
What does the space of jets look like?



Exploring the Space of Jets: Visualizing the Manifold



What does the space of jets look like?



Quantify (and calculate?) dimension of the space of jets. See extra slides.

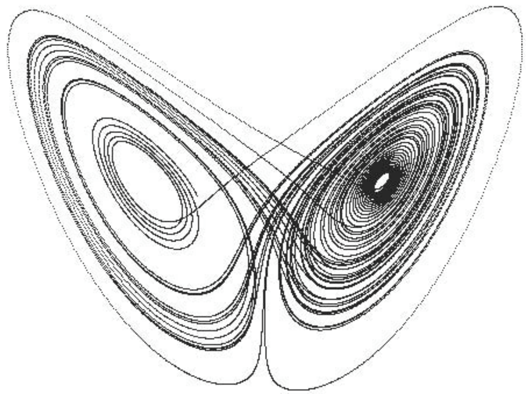
Exploring the Space of Jets: Correlation Dimension

VOLUME 50, NUMBER 5 PHYSICAL REVIEW LETTERS 31 JANUARY 1983

Characterization of Strange Attractors

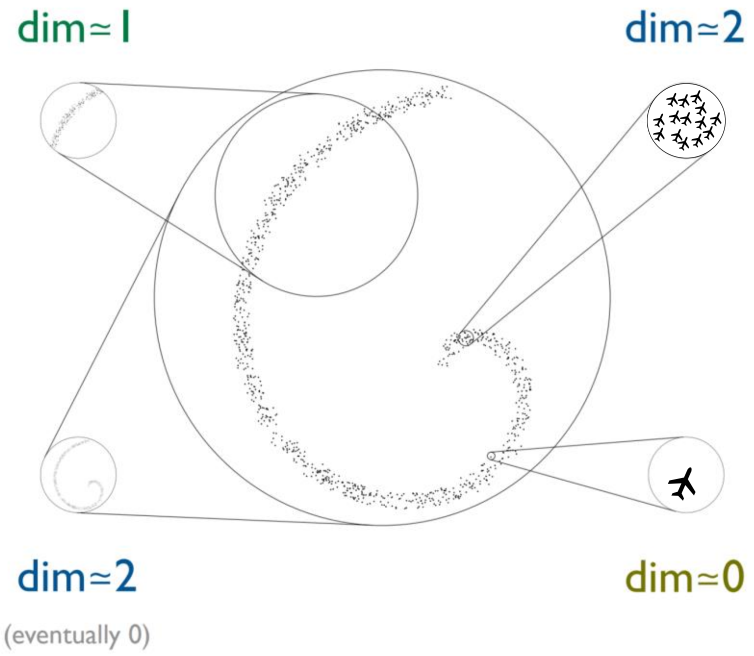
Peter Grassberger[(a)] and Itamar Procaccia
Chemical Physics Department, Weizmann Institute of Science, Rehovot 76100, Israel
(Received 7 September 1982)

A new measure of strange attractors is introduced which offers a practical algorithm to determine their character from the time series of a single observable. The relation of this new measure to fractal dimension and information-theoretic entropy is discussed.



Intuition:

$$N_{\substack{\text{neighboring}\\ \text{points}}}(r) \propto r^{\text{dim}} \quad \Longrightarrow \quad \text{dim}(r) = r\frac{\partial}{\partial r}\ln N_{\text{neighbors}}(r)$$



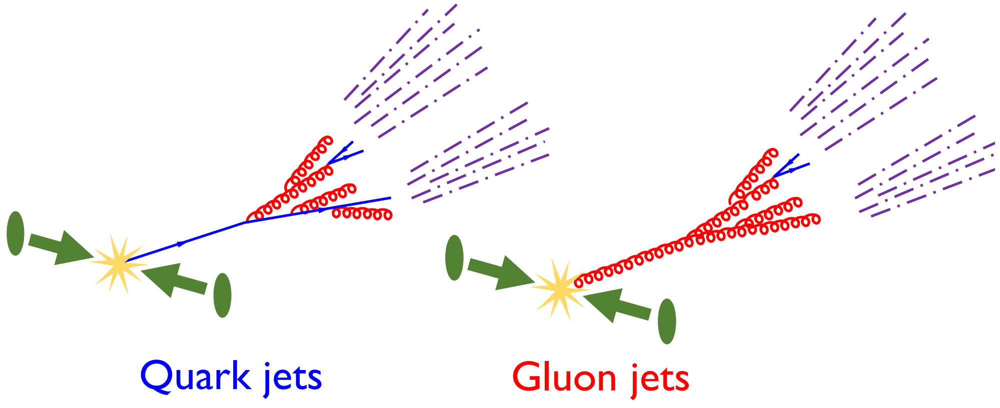
Correlation dimension:

$$\boxed{\text{dim}(Q) = Q\frac{\partial}{\partial Q}\ln \sum_{i=1}^{N}\sum_{j=1}^{N}\Theta[\text{EMD}(\mathcal{E}_i,\mathcal{E}_j) < Q]}$$

Energy scale Q dependence

Count neighbors in ball of radius Q

Exploring the Space of Jets: Correlation Dimension

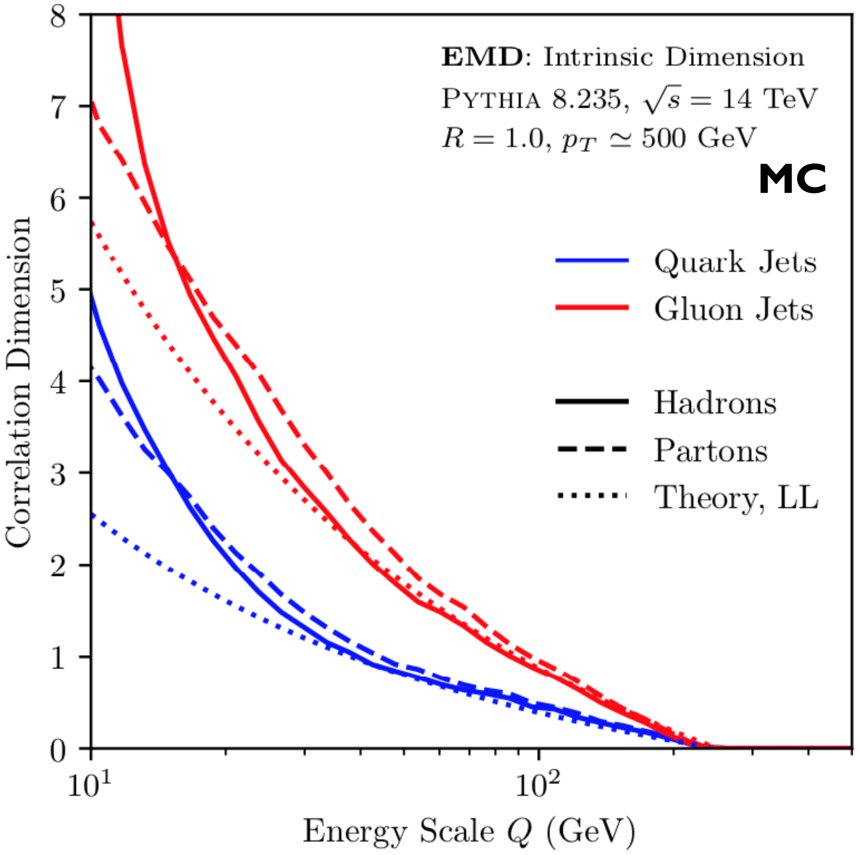


$$\text{At LL: } \dim_{q/g}(Q) = -\frac{8\alpha_s C_{q/g}}{\pi} \ln \frac{Q}{p_T/2}$$

+ 1-loop running of α_s

$C_q = C_F = \frac{4}{3}$

$C_g = C_A = 3$

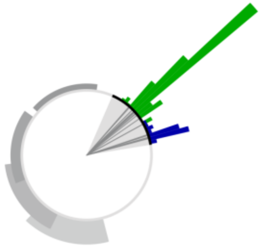


Dimension blows up at low energies.

Jets are "more than fractal"

CMS Open Data

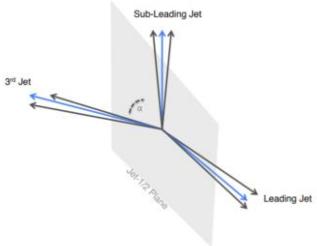
Many exciting physics applications with the CMS Open Data already.



Exposing the QCD splitting function

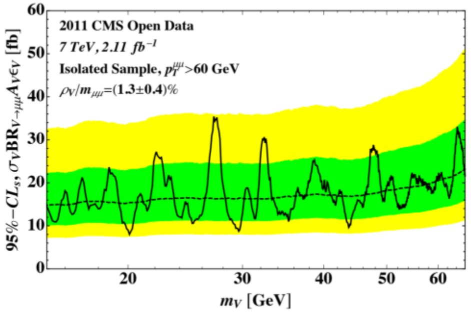
[Tripathee, Xue, Larkoski, Marzani, Thaler, 1704.05842]

[Larkoski, Marzani, Thaler, Tripathee, Xue, 1704.05066]



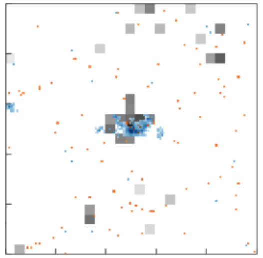
Looking for parity violation in jets

[Lester, Schott, 1904.11195]



Searching for dimuon resonances

[Cesarotti, Soreq, Strassler, Thaler, Xue, 1902.04222]



Analyzing collision data with deep learning techniques

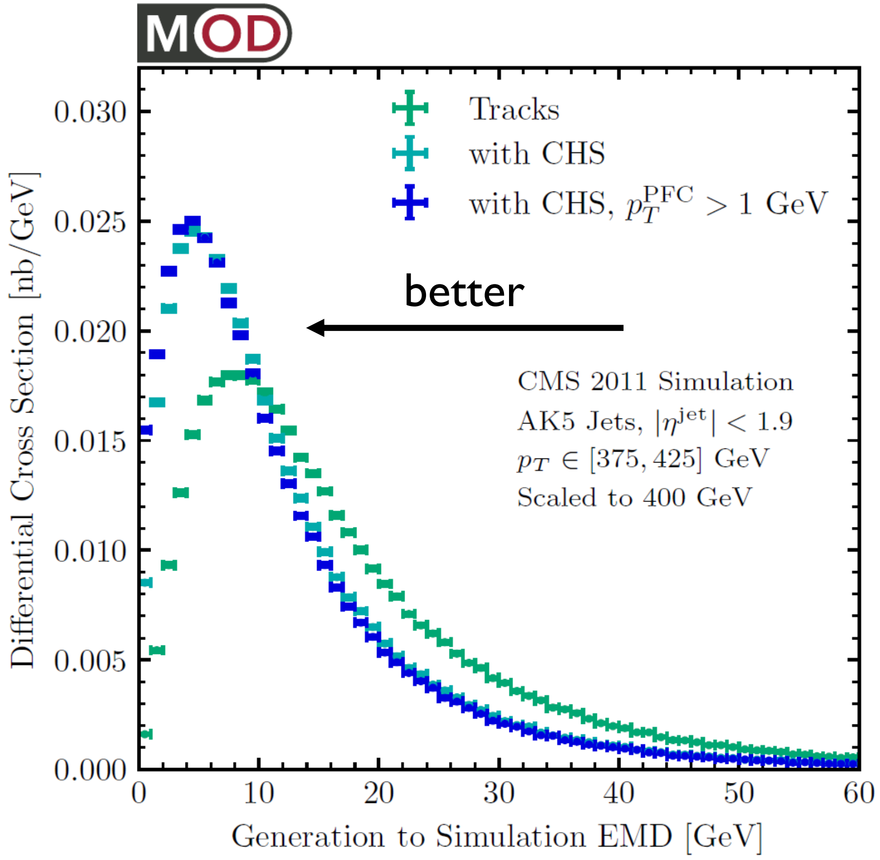
[Madrazo, Cacha, Iglesias, de Lucas, 1708.07034]

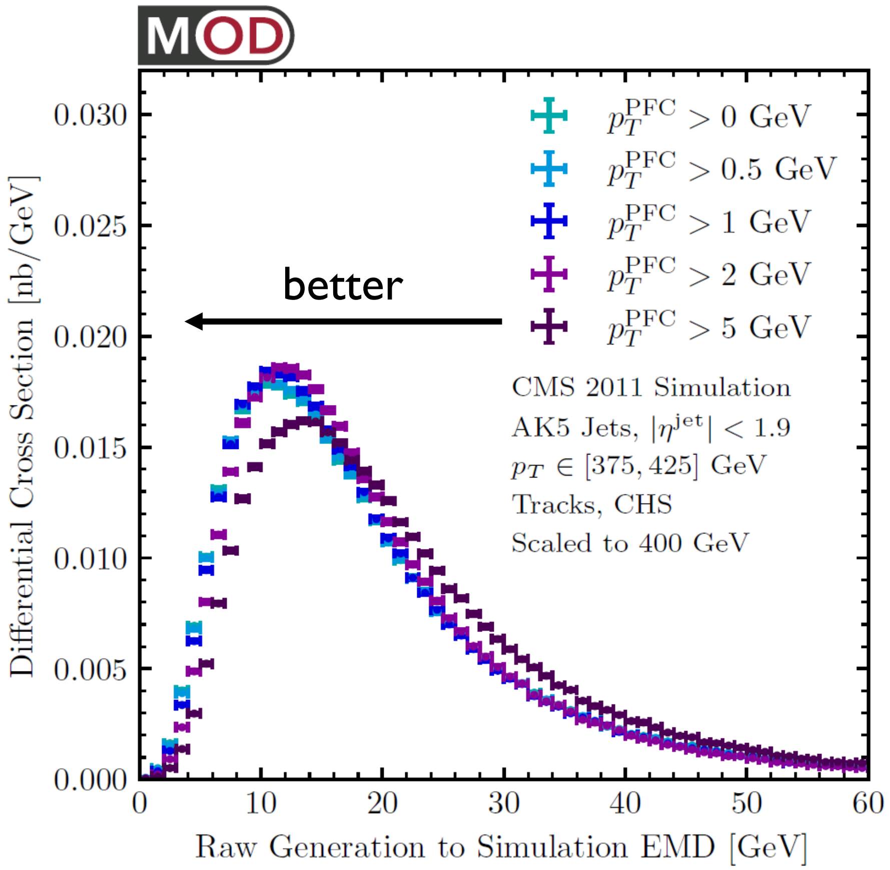
[Andrews, Paulini, Gleyzer, Poczos, 1807.11916]

[Andrews, et al., 1902.08276]

Quantifying Pileup and Detector Effects with EMD

Gen./Sim. EMD universally quantifies pileup mitigation and detector effects.

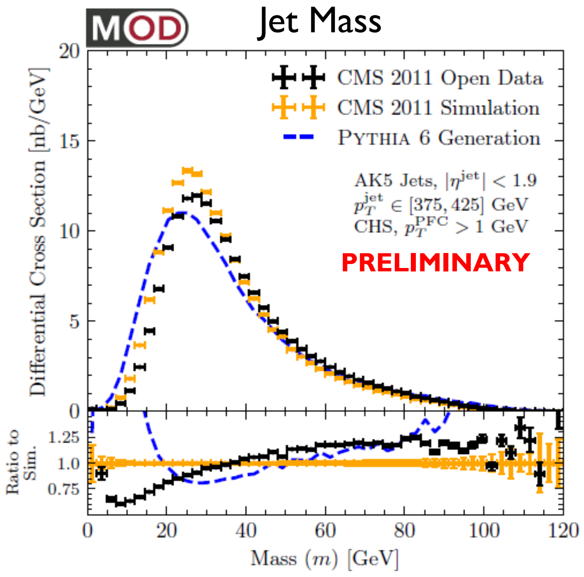


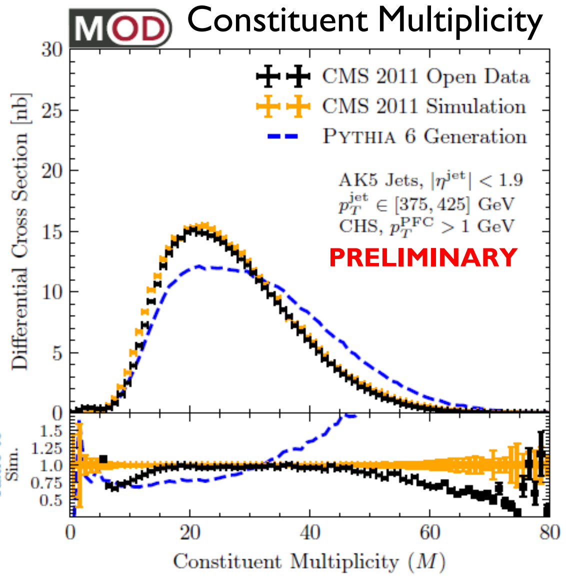


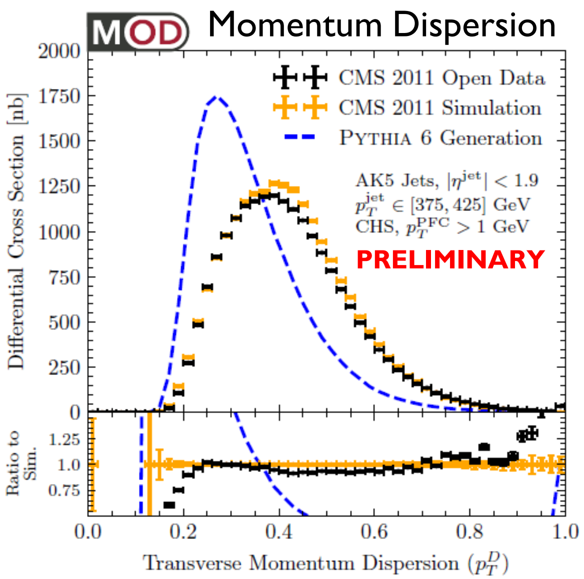
Jet Substructure Observables

Study jet substructure at truth and detector level.

$$m^2 = \left(\sum_{i \in \mathrm{Jet}} p_i^\mu\right)^2 \qquad M = \sum_{i \in \mathrm{Jet}} 1 \qquad p_T^D = \frac{\sum_{i \in \mathrm{Jet}} p_{T,i}^2}{\left(\sum_{i \in \mathrm{Jet}} p_{T,i}\right)^2}$$

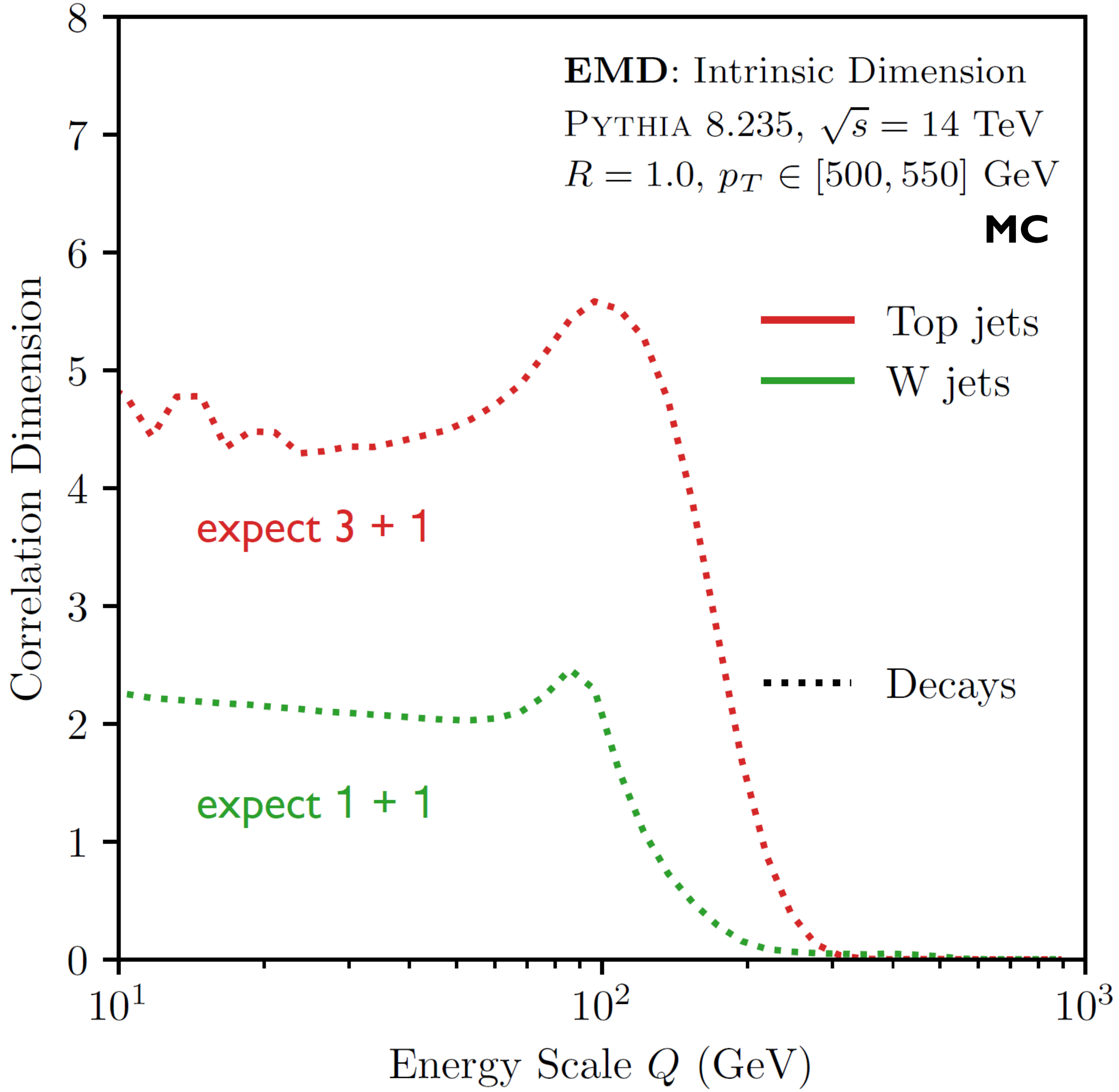


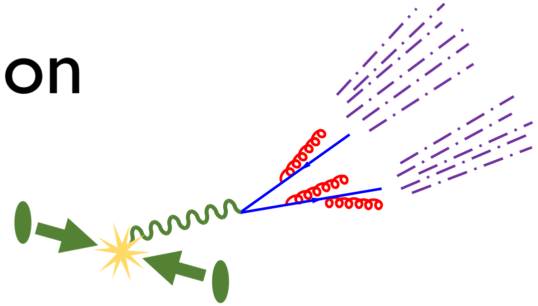




Similar to: [Larkoski, Marzani, Thaler, Tripathee, Xue, 1704.05066]

Exploring the Space of Jets: Correlation Dimension





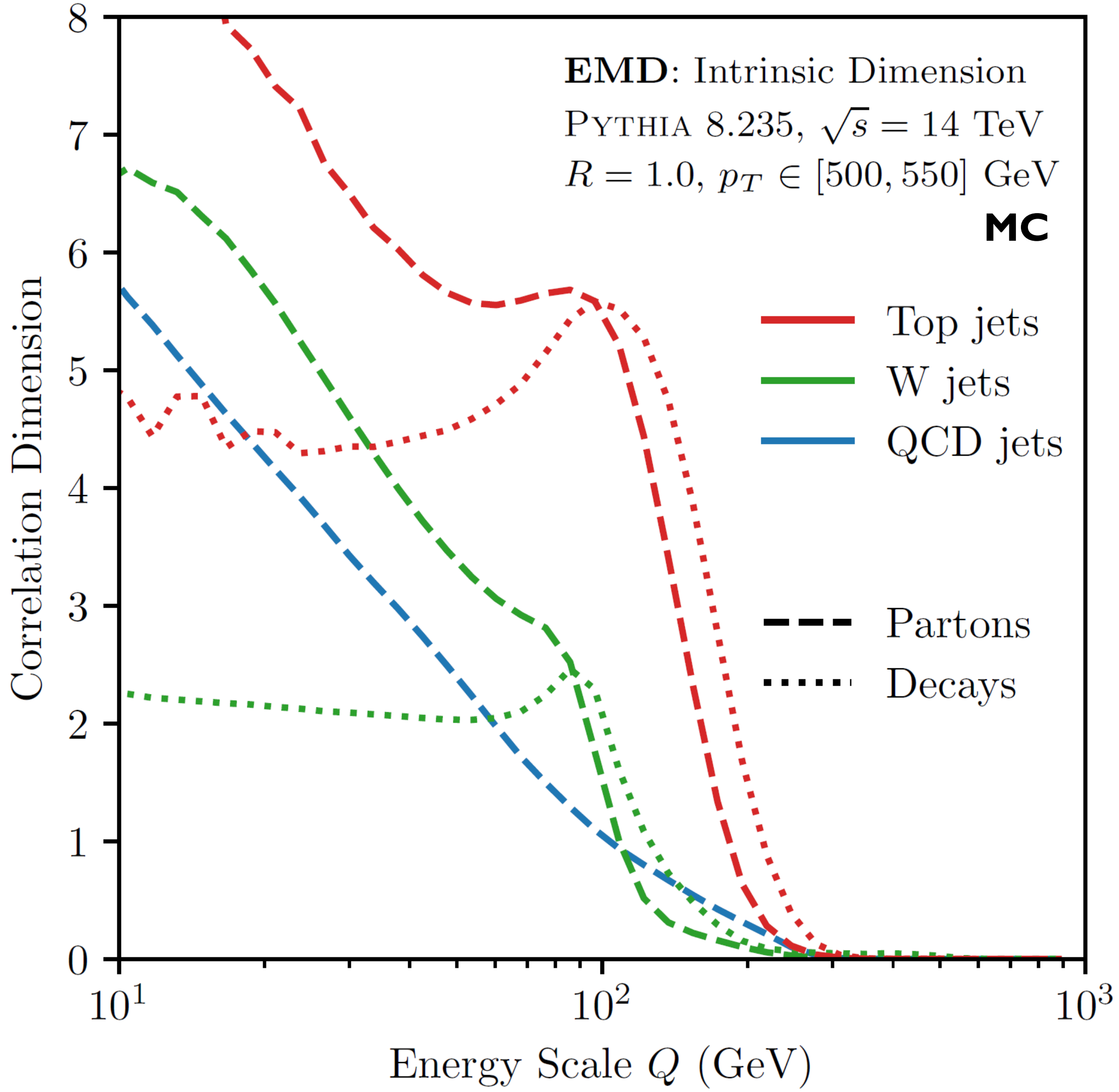
QCD jets are simplest.

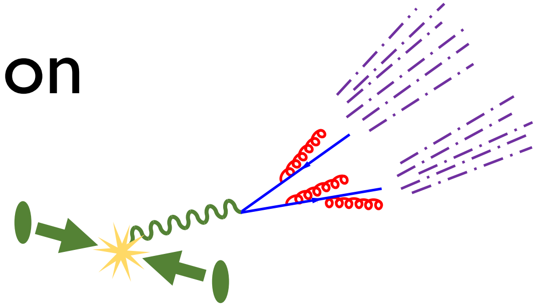
W jets are more complicated.

Top jets are most complex.

"Decays" have ~constant dimension.

Exploring the Space of Jets: Correlation Dimension





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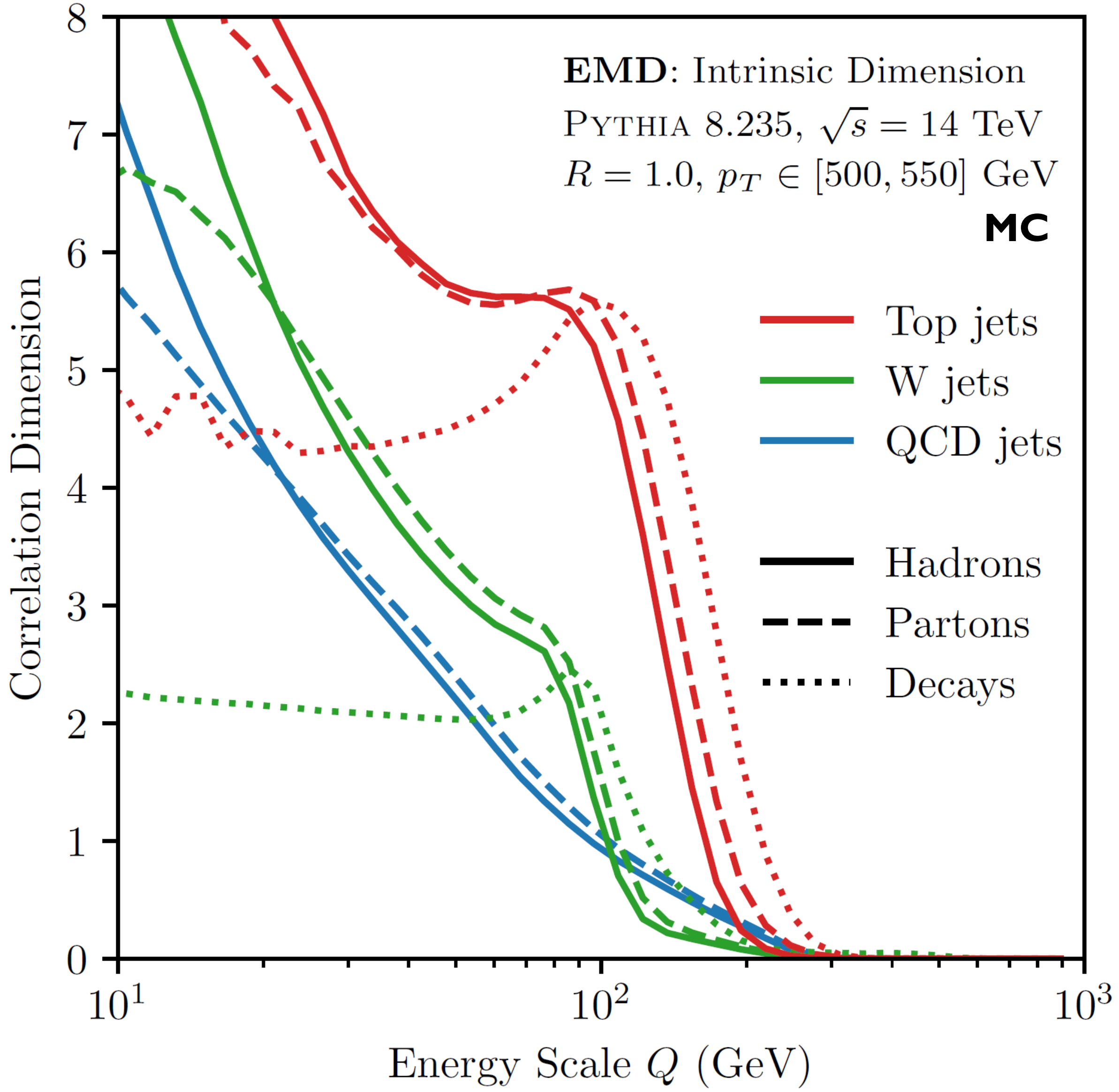
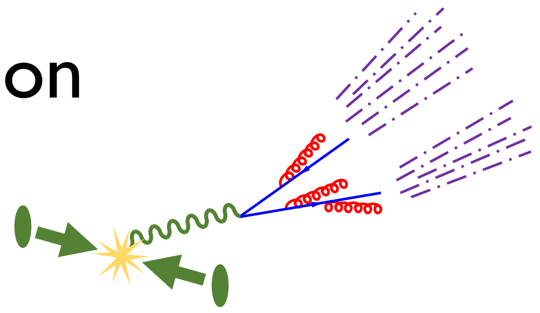
Exploring the Space of Jets: Correlation Dimension





QCD jets are simplest.

W jets are more complicated.

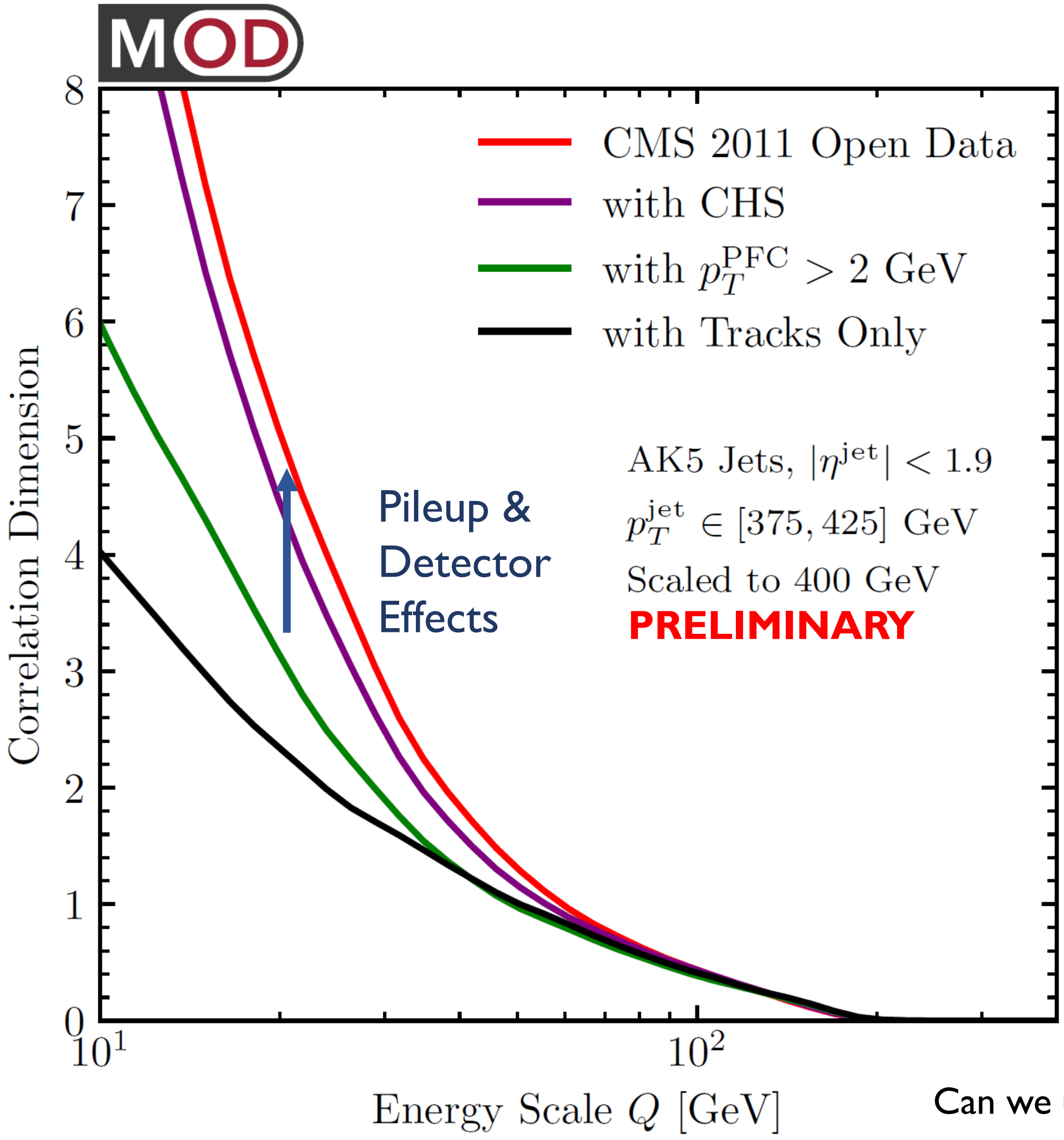
Top jets are most complex.

"Decays" have ~constant dimension.

Fragmentation becomes more complex at lower energy scales.

Hadronization becomes relevant at scales around 20 GeV.

Exploring the Space of Jets: Correlation Dimension

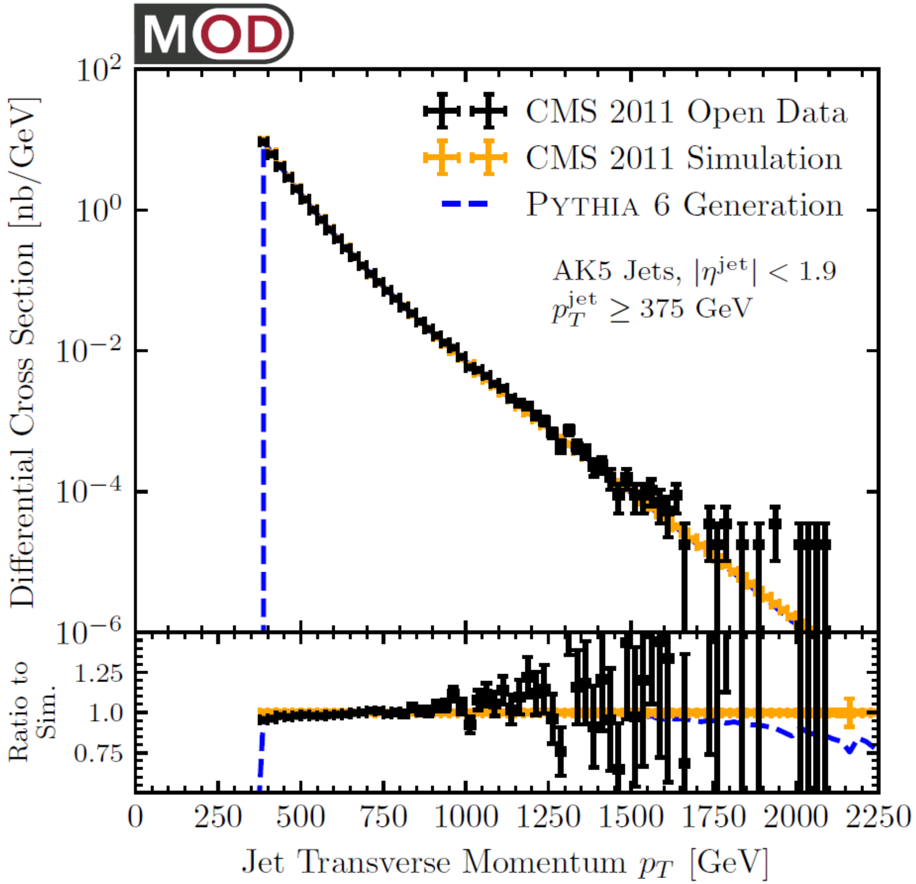


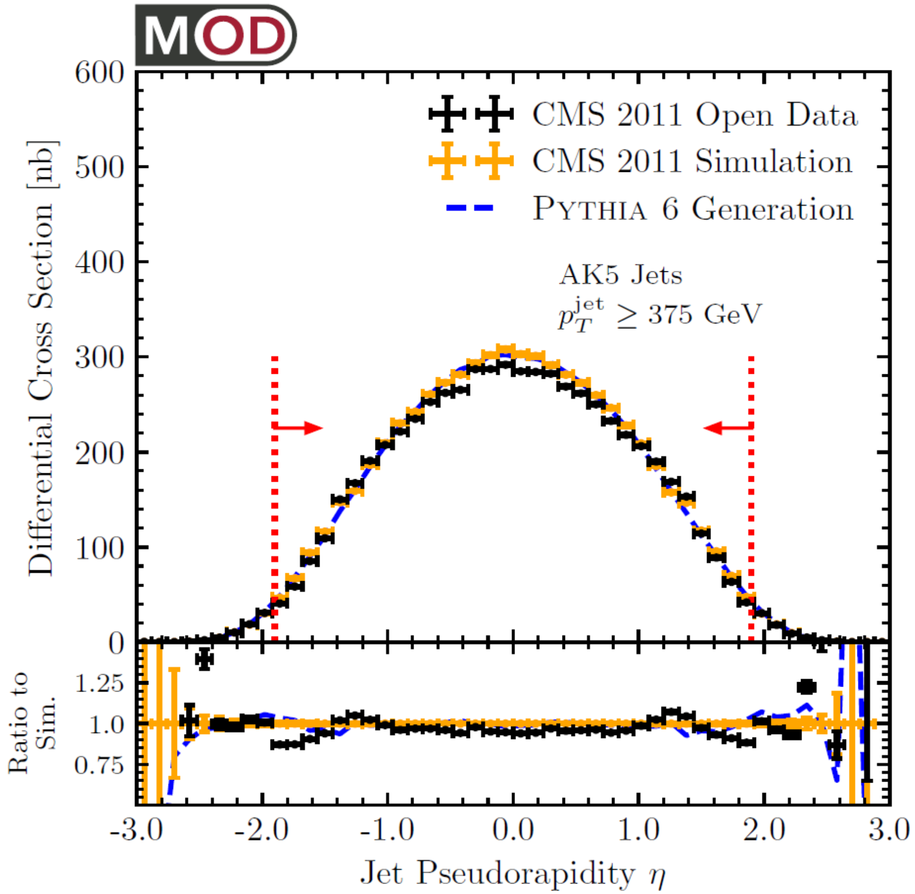
Dimension blows up at low energies.

Jets are "more than fractal"

Can we understand this analytically?

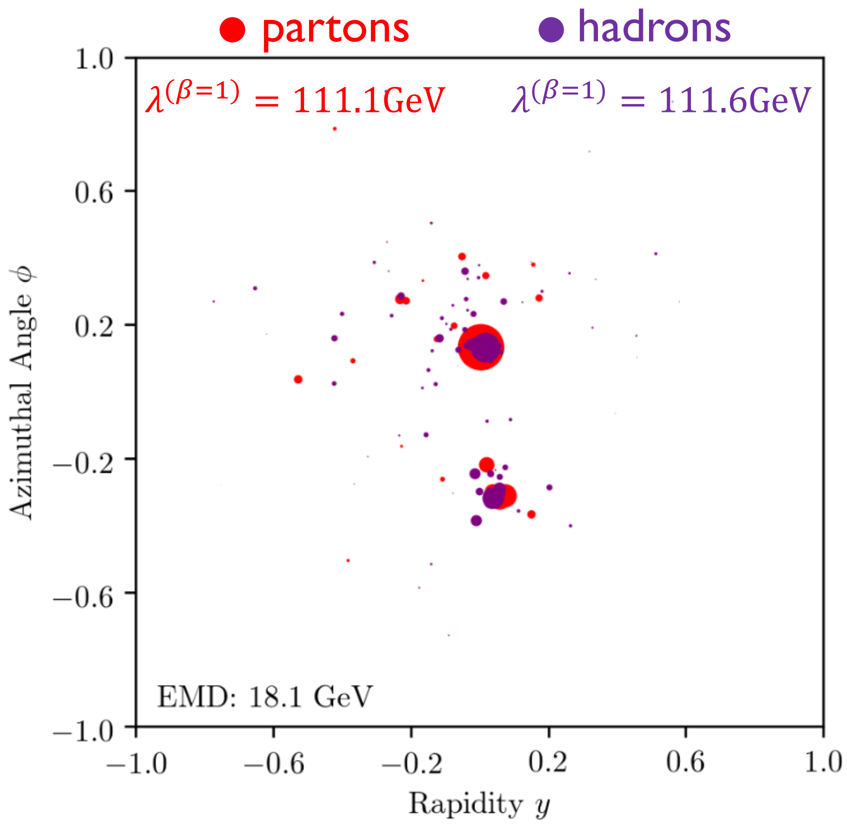
Jet Kinematic Distributions

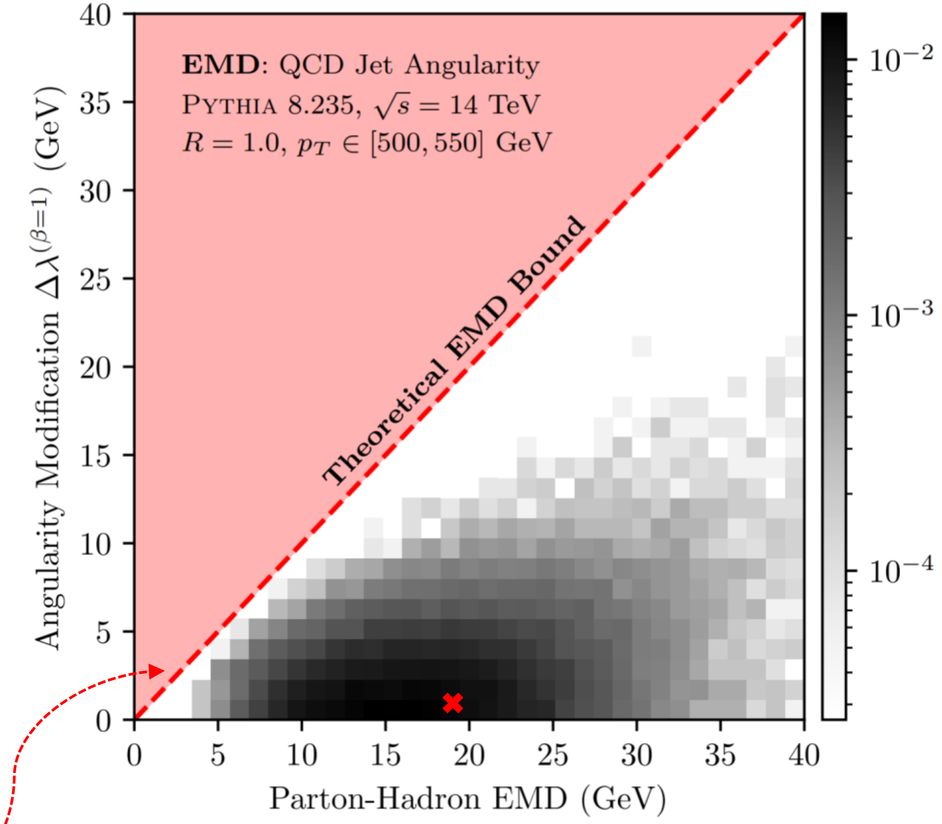




Quantifying event modifications: Hadronization

$$\lambda^{(\beta=1)} = \sum_{i=1}^{M} E_i\,\theta_i$$





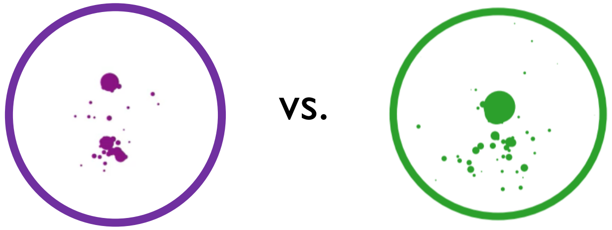
$\mathcal{E} = \mathcal{E}_{\text{partons}}$

$\mathcal{E}' = \mathcal{E}_{\text{hadrons}}$

$$\boxed{\left|\lambda^{(\beta=1)}(\mathcal{E}) - \lambda^{(\beta=1)}(\mathcal{E}')\right| \leq \text{EMD}(\mathcal{E}, \mathcal{E}')}$$

Exploring the Space of Events: Jet Classification

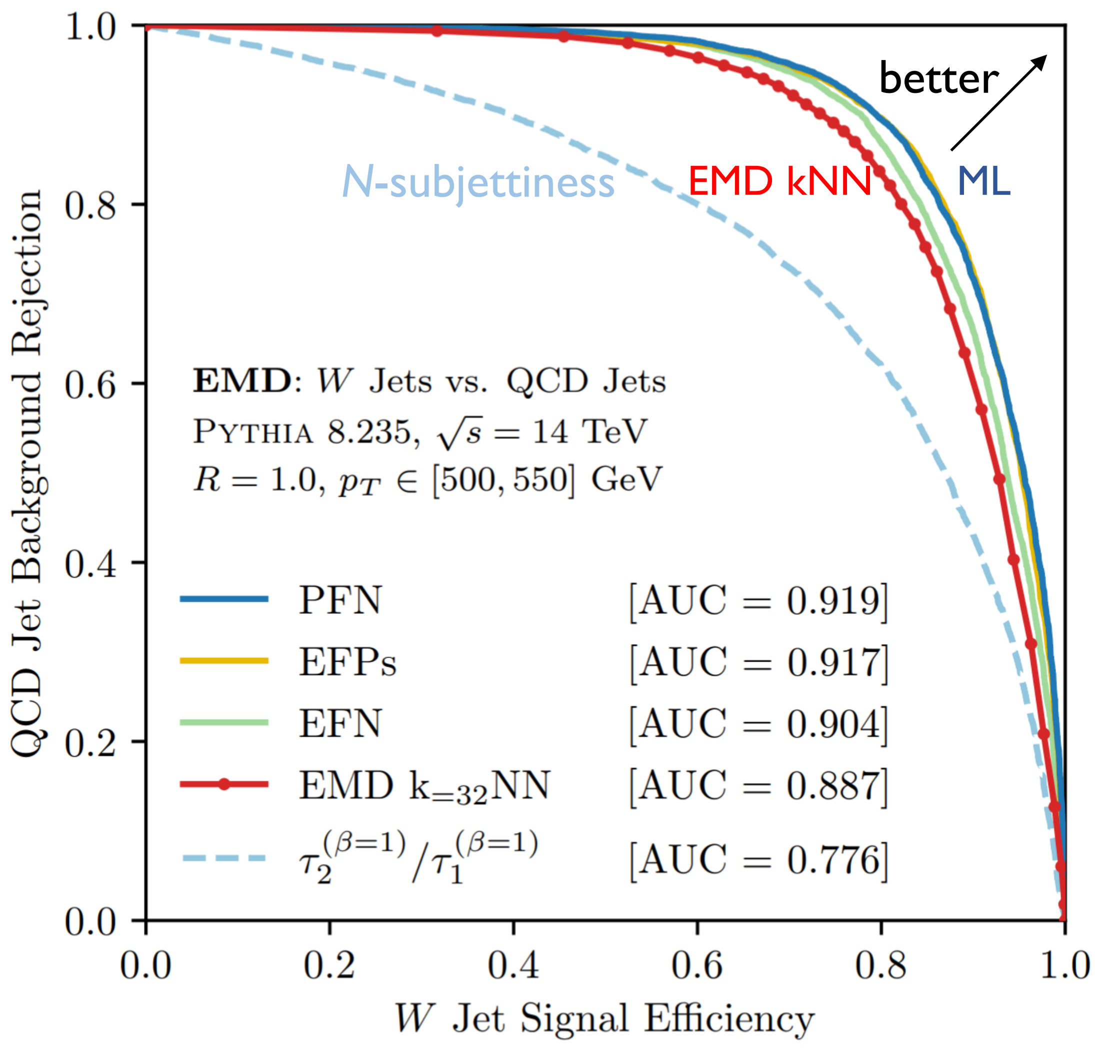
Classify *W* jets vs. QCD jets



Look at a jet's nearest neighbors (kNN) to predict its class.

Optimal IRC-safe classifier with enough data.

Nearing performance of ML.



Exploring the Space of Events

Use EMD as a measure of event similarity

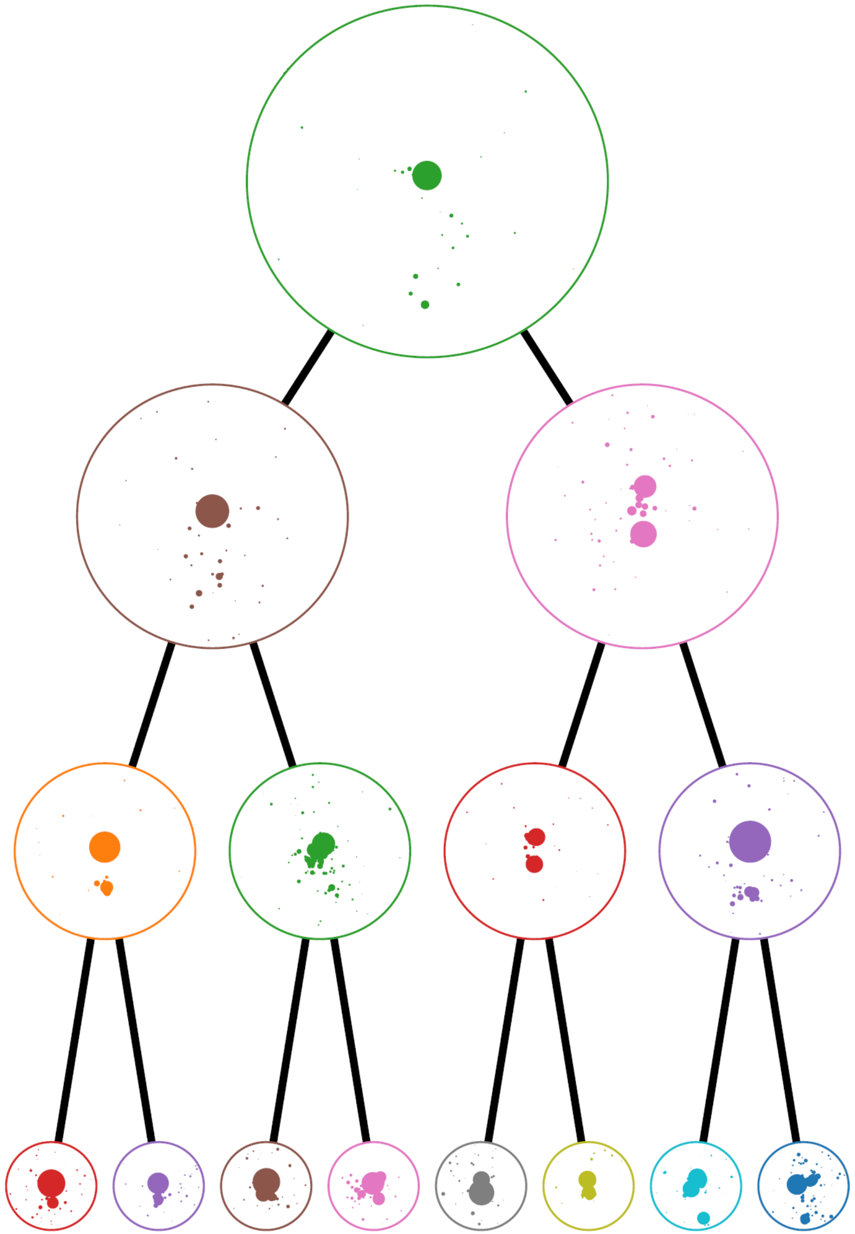
Unsupervised clustering algorithms can be used to cluster events

Jets are clusters of particles
???? are clusters of jets

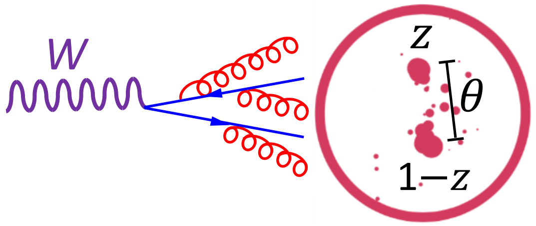
VP Tree: O(log(N)) neighbor query time

Much more to explore.

Vantage Point (VP) Tree



Exploring the Space of Events: W jets



W jets are 2-pronged:

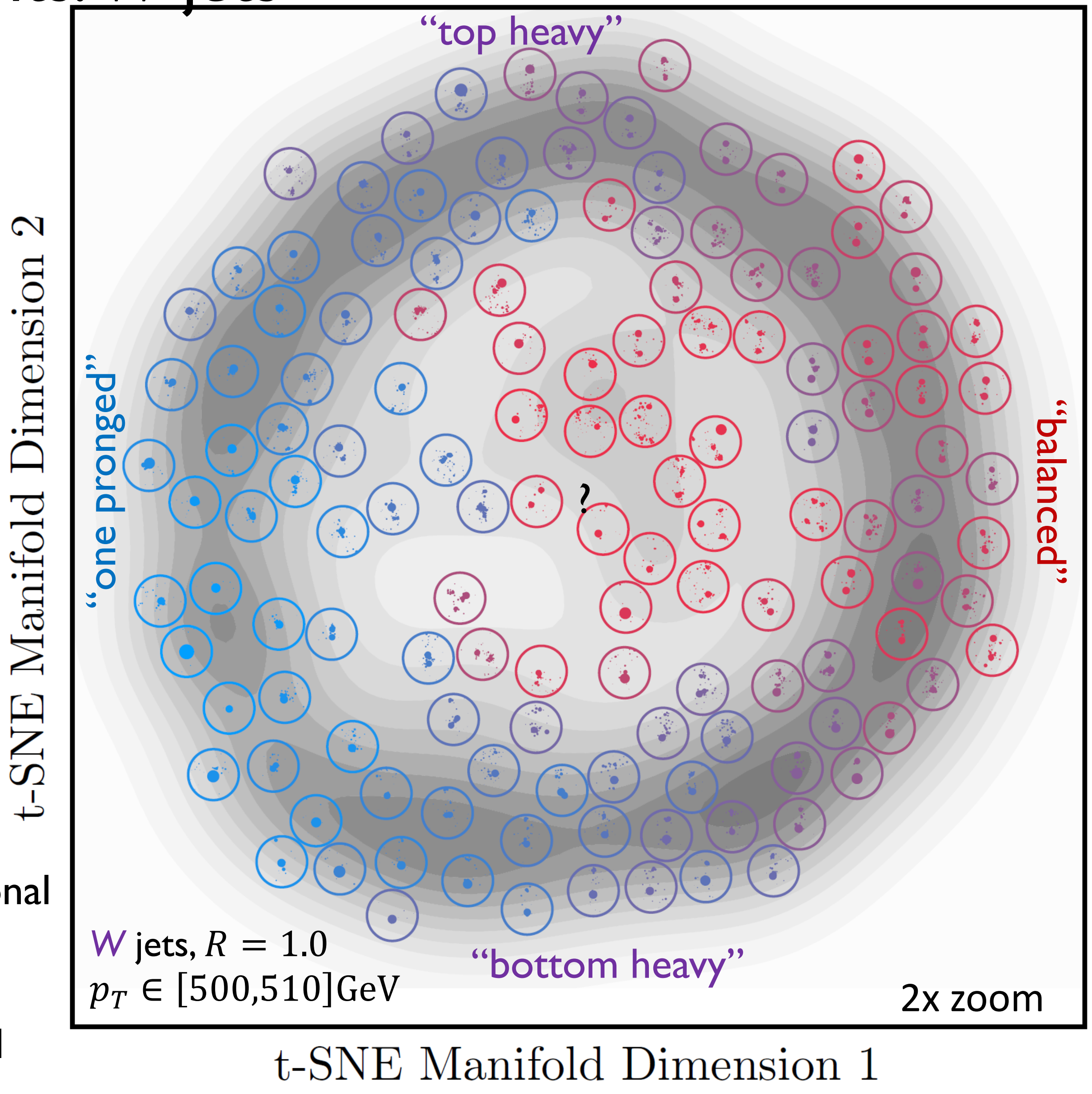
z: Energy Sharing of Prongs
θ: Angle between Prongs
φ: Azimuthal orientation

Constrained by W mass:

$$z(1-z)\theta^2 = \frac{p_{\mu J}^2}{p_T^2} = \frac{m_W^2}{p_T^2}$$

Hence we expect a **two**-dimensional space of W jets.

After φ rotation: **one**-dimensional



Exploring the Space of Jets: Correlation Dimension

Sketch of leading log (one emission) calculation:

$$\dim_{q/g}(Q) = Q\frac{\partial}{\partial Q}\ln \sum_{i=1}^{N}\sum_{j=1}^{N}\Theta[\mathrm{EMD}(\mathcal{E}_i,\mathcal{E}_j) < Q]$$

$$= Q\frac{\partial}{\partial Q}\ln \Pr\,[\mathrm{EMD} < Q]$$

$$= Q\frac{\partial}{\partial Q}\ln \Pr\,[\lambda^{(\beta=1)} < Q;\, C_{q/g} \to 2\,C_{q/g}]$$

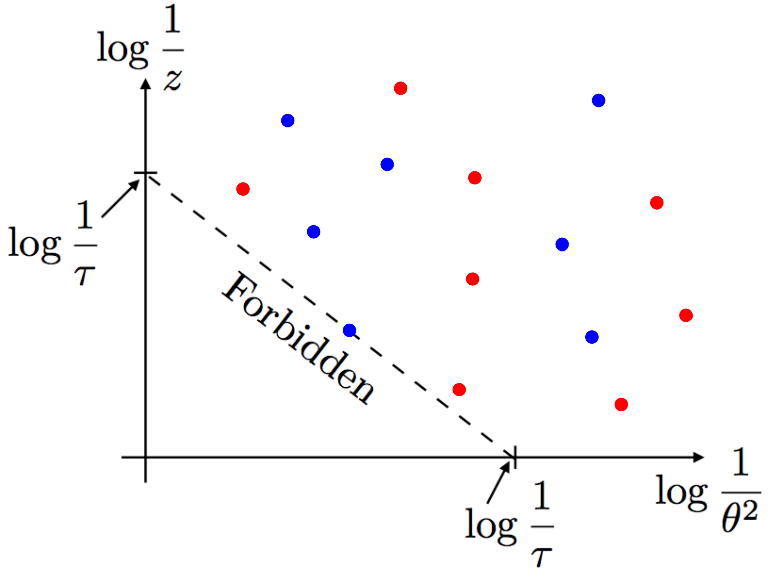
$$= Q\frac{\partial}{\partial Q}\ln \exp\left(-\frac{4\alpha_s C_{q/g}}{\pi}\ln^2\frac{Q}{p_T/2}\right)$$

$$= -\frac{8\alpha_s C_{q/g}}{\pi}\ln\frac{Q}{p_T/2}$$

+ 1-loop running of α_s

$$C_q = C_F = \frac{4}{3}$$

$$C_g = C_A = 3$$



[A. Larkoski, 1709.06195]