Exploring The (Metric) Space of Collider Events

Harvard Particle Physics Lunch Talk

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Massachusetts Institute of Technology

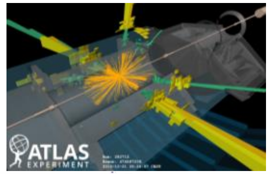
Joint work with Patrick Komiske and Jesse Thaler

[1902.02346]

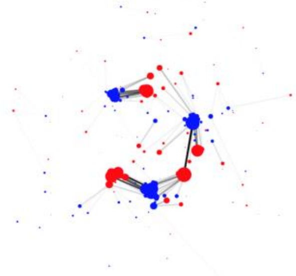
February 13, 2019

Outline

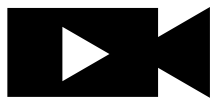
Part I
Introduction



When are two events similar?

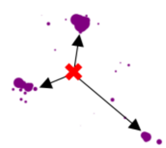


The Energy Mover's Distance

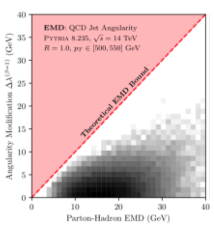


Movie Time

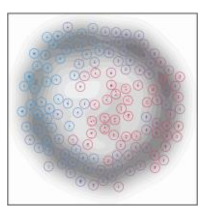
Part II
Applications



Observables



Quantifying event modifications



Exploring the Space of Events

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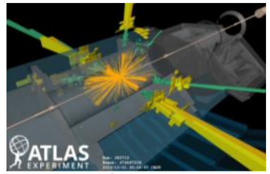

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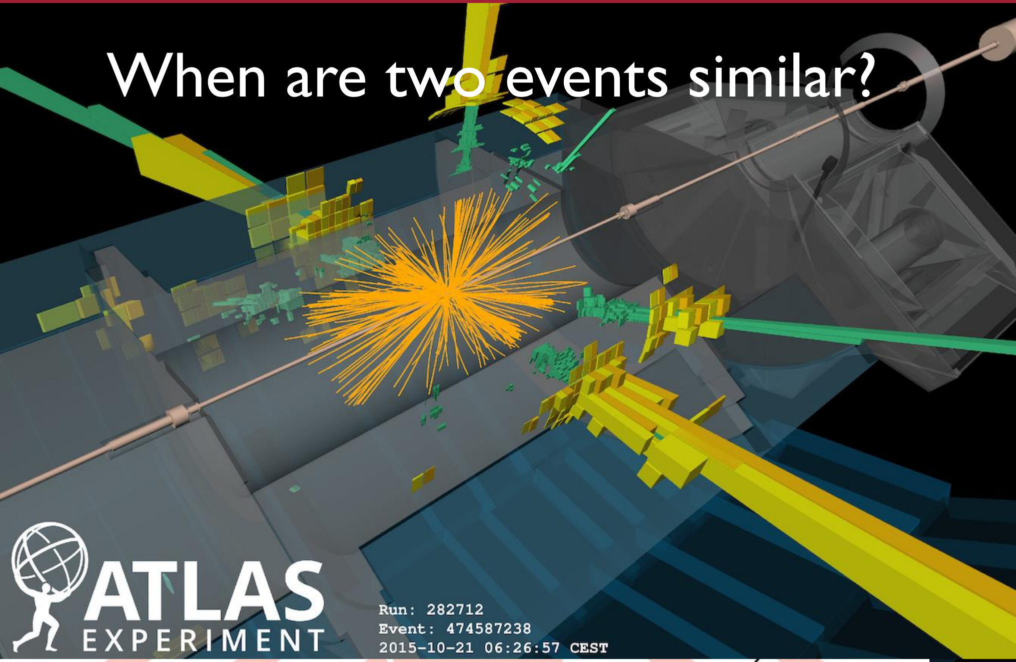
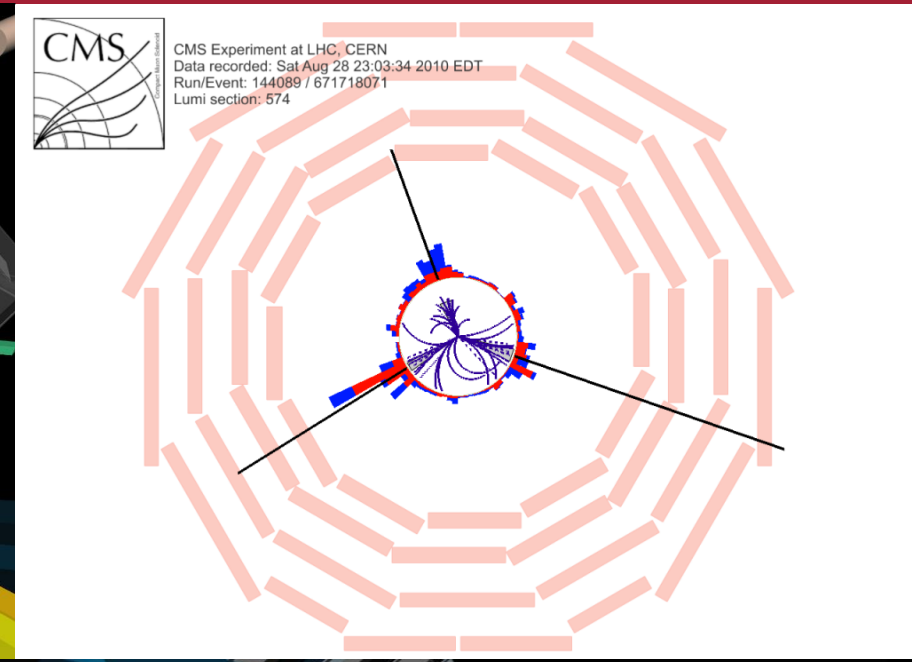
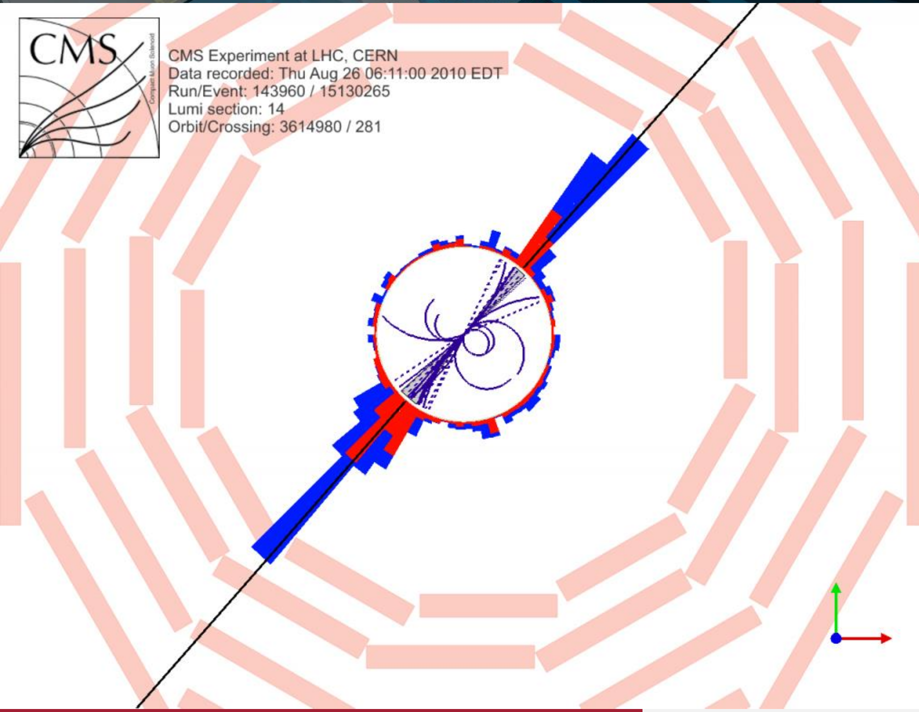
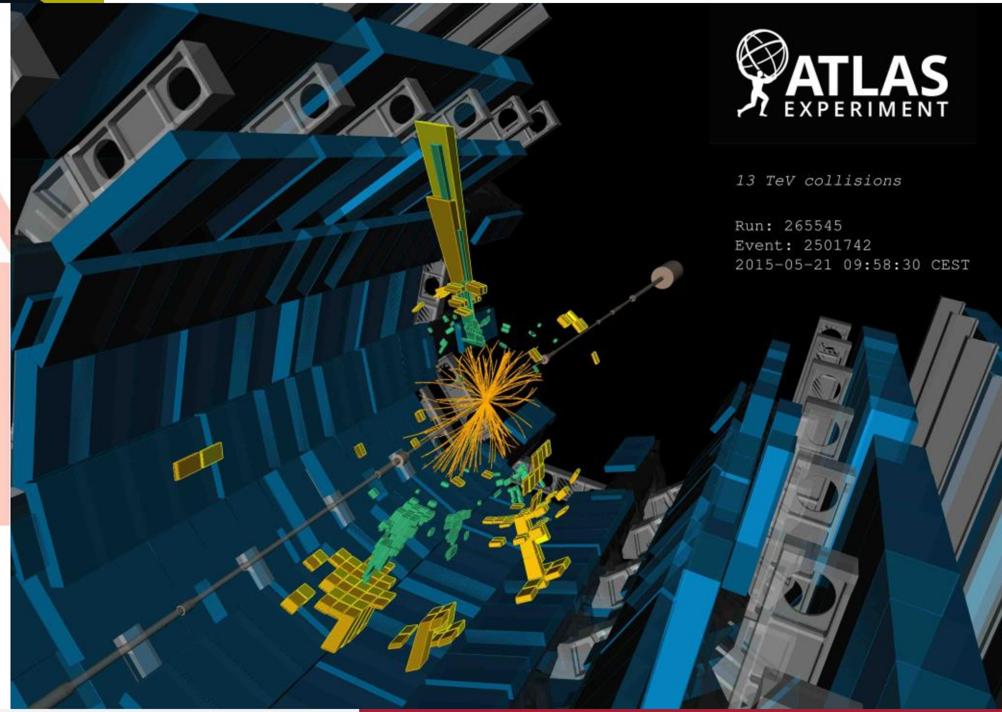


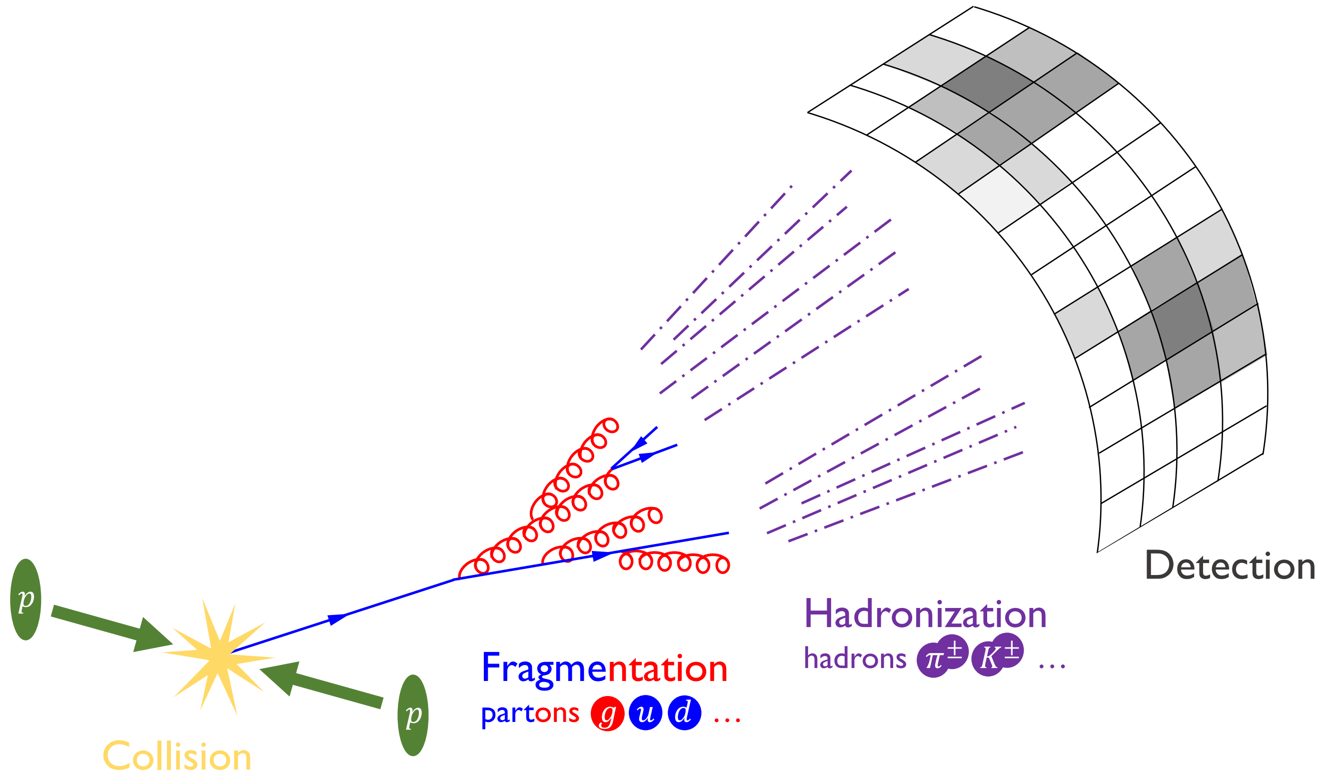






When are two collider events similar?

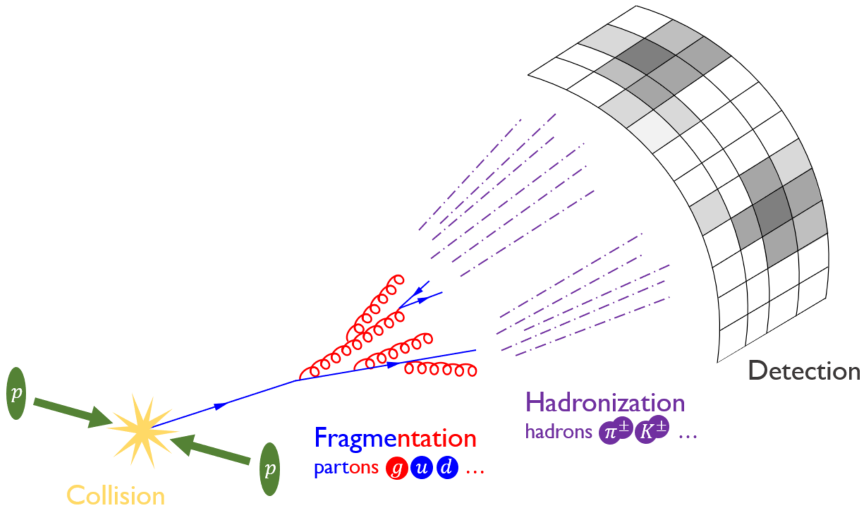
How an event gets its shape

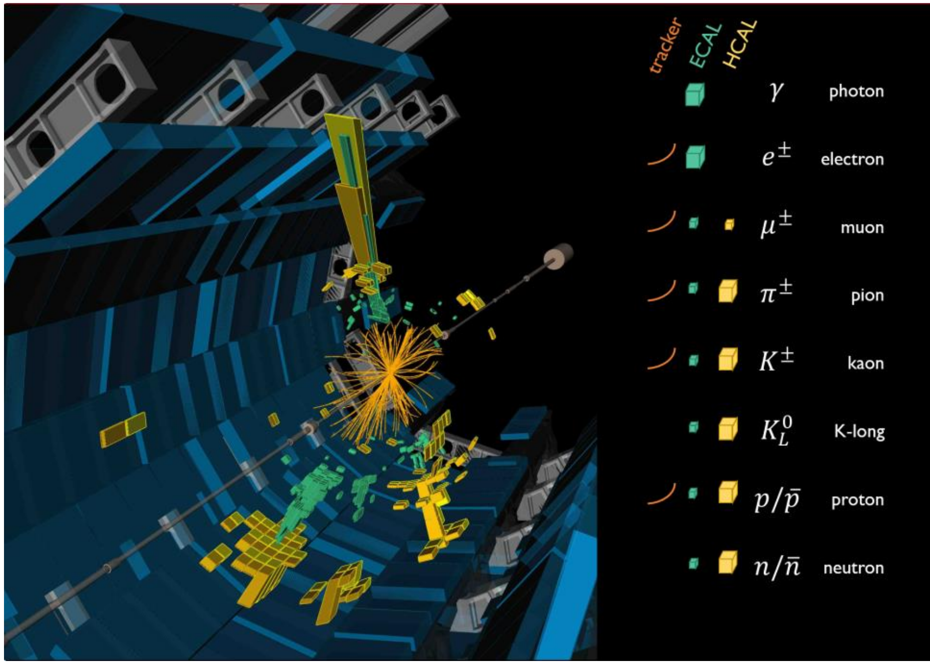


When are two collider events similar?

A collider event is…

Theoretically: very complicated | Experimentally: very complicated





However:

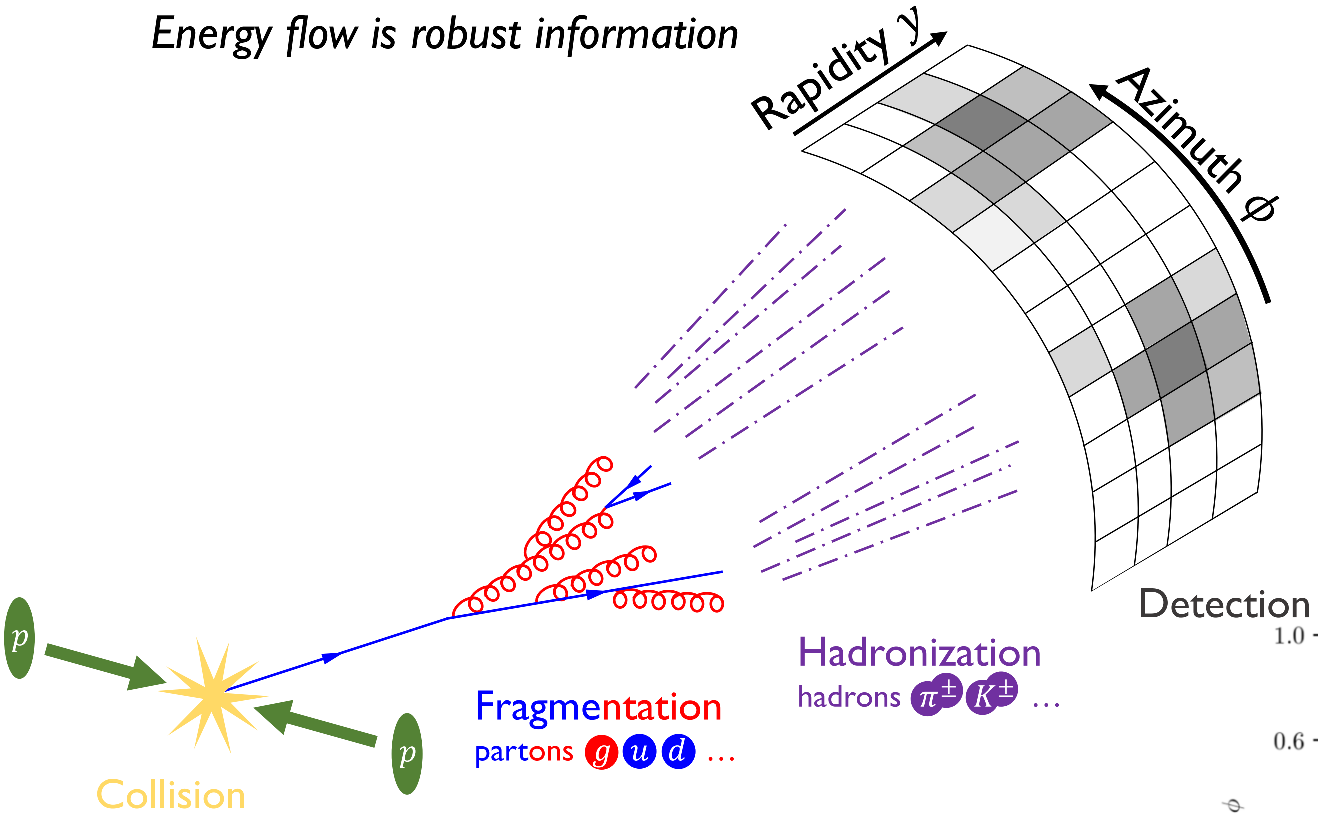
The *energy flow* (distribution of energy) is the information that is robust to: fragmentation, hadronization, detector effects, …

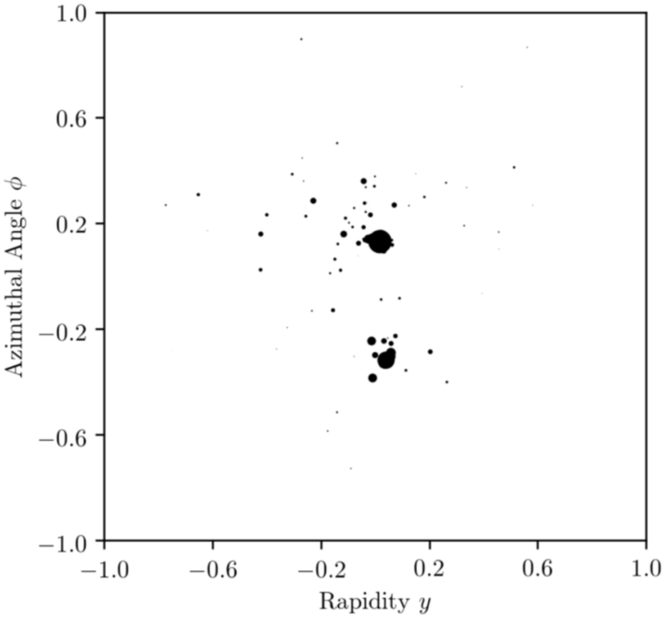
[N.A. Sveshnikov, F.V. Tkachov, 9512370]
[F.V. Tkachov, 9601308]
[P.S. Cherzor, N.A. Sveshnikov, 9710349]

Energy flow ⇔ Infrared and Collinear (IRC) Safe information

When are two collider events similar?

Energy flow is robust information





Treat events as distributions of energy: $\sum_{i=1}^{M} E_i \, \delta(\hat{p}_i)$

energy direction

Ignoring particle flavor, charge, multiplicity…

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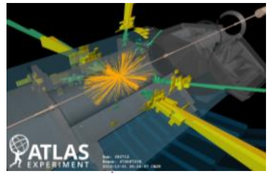

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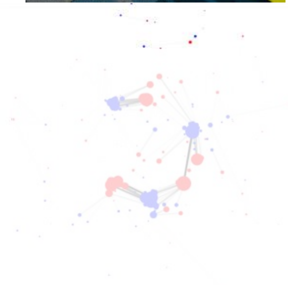

The Energy Mover's Distance

Movie Time

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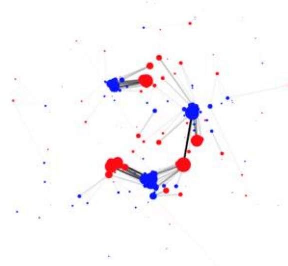

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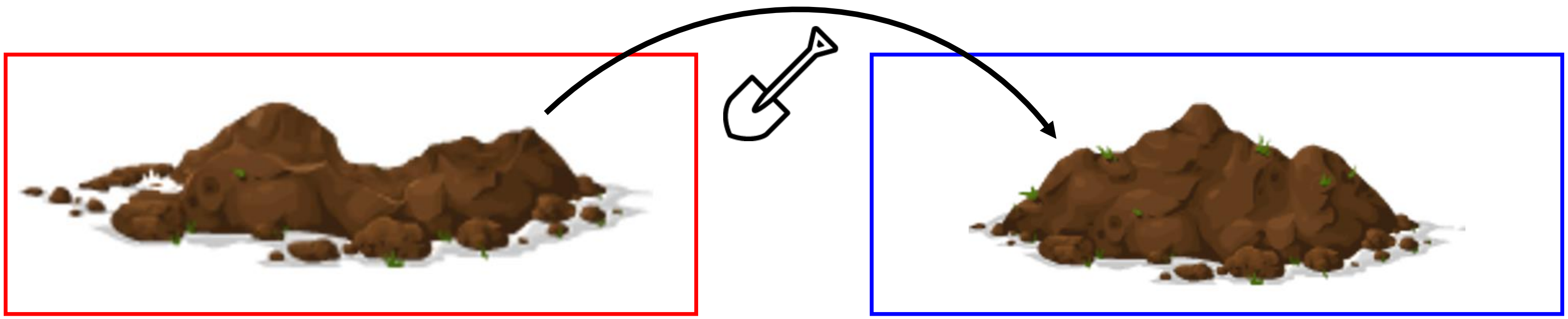
The Energy Mover's Distance

Review: The Earth Mover's Distance

Earth Mover's Distance: the minimum "work" (stuff x distance) to rearrange one pile of dirt into another

[S. Peleg, M. Werman, H. Rom]
[Y. Rubner, C. Tomasi, and L.J. Guibas]



Metric on the space of (normalized) distributions: *symmetric*, *non-negative*, *triangle inequality*

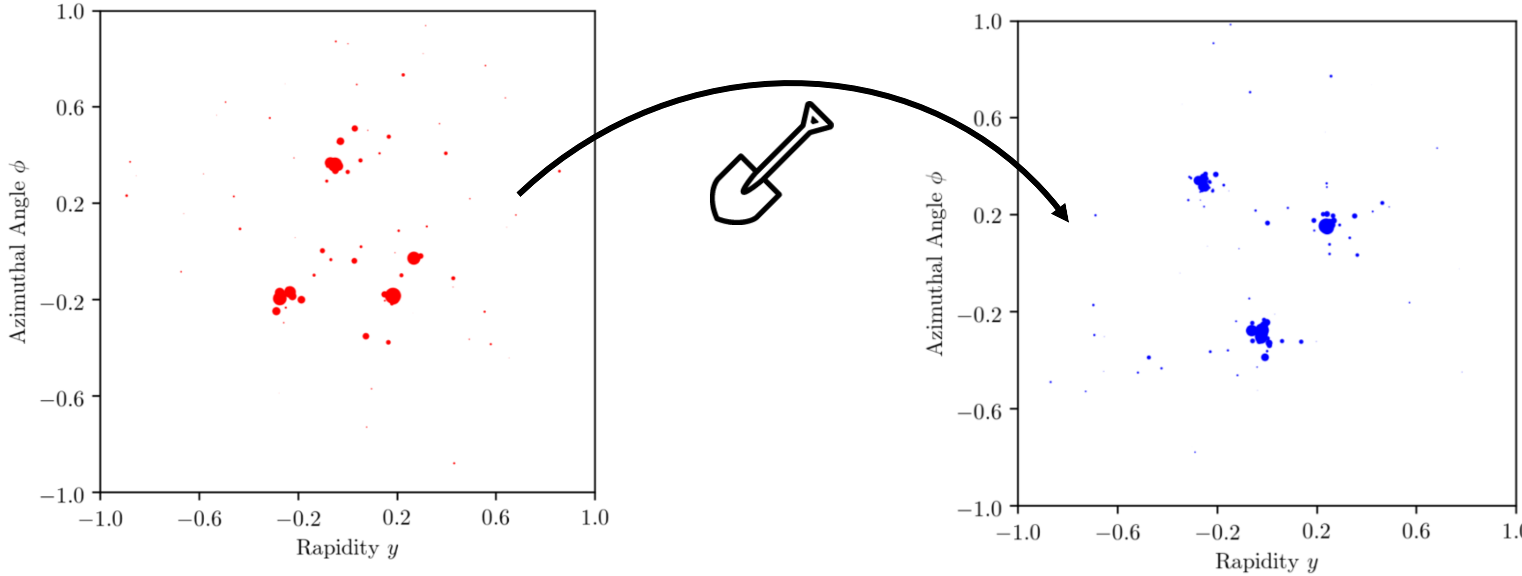
Distributions are close in EMD ⇔ their expectation values are close.

Also known as the 1-Wasserstein metric.

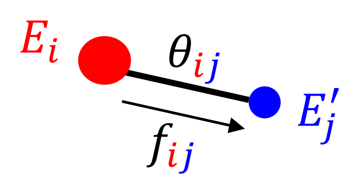
The Energy Mover's Distance

From Earth to Energy

Energy Mover's Distance: the minimum "work" (energy x angle) to rearrange one *event* (pile of energy) into another [P.T. Komiske, EMM, J. Thaler, 1902.02346]



$$\mathrm{EMD}(\mathcal{E},\mathcal{E}') = \min_{\{f\}} \sum_{i=1}^{M} \sum_{j=1}^{M'} f_{ij} \frac{\theta_{ij}}{R}$$



Difference in radiation pattern

The Energy Mover's Distance

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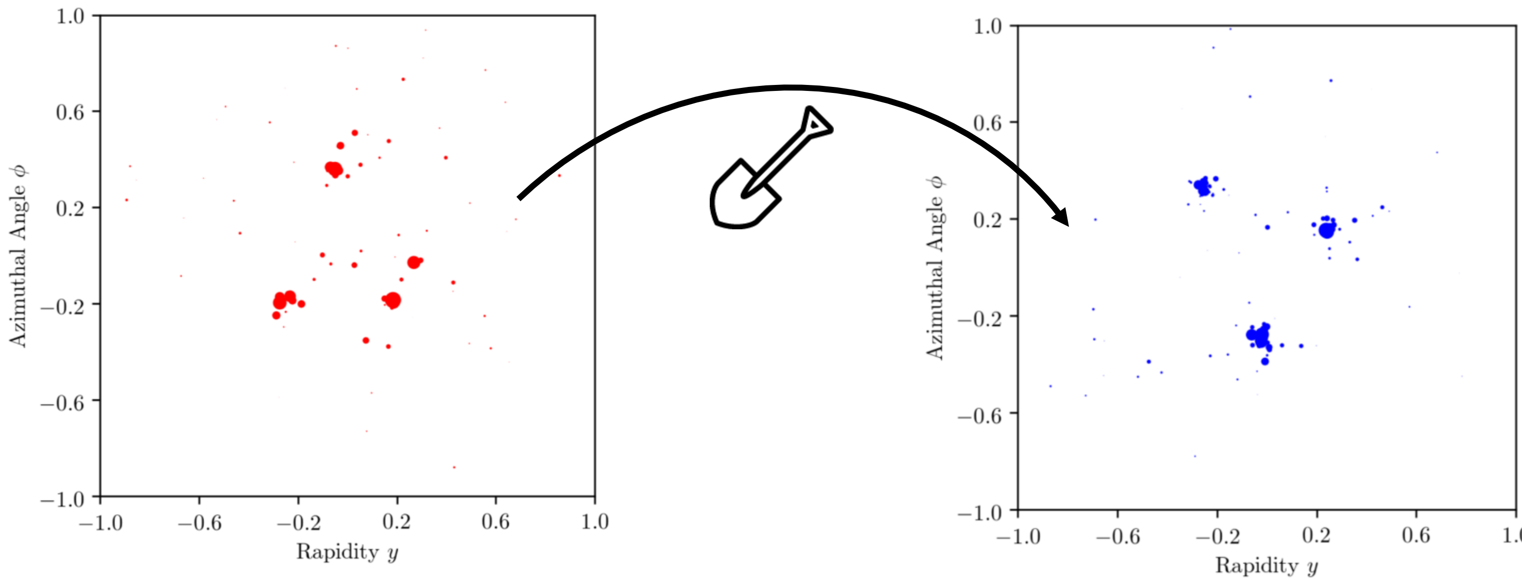
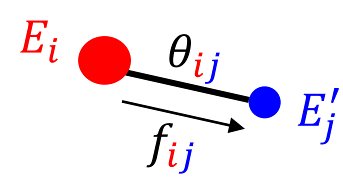


$$\mathrm{EMD}(\mathcal{E},\mathcal{E}') = \min_{\{f\}} \sum_{i=1}^{M}\sum_{j=1}^{M'} f_{ij}\frac{\theta_{ij}}{R} + \left|\sum_{i=1}^{M} E_i - \sum_{j=1}^{M'} E'_j\right|$$



Difference in radiation pattern

Difference in total energy

The Energy Mover's Distance

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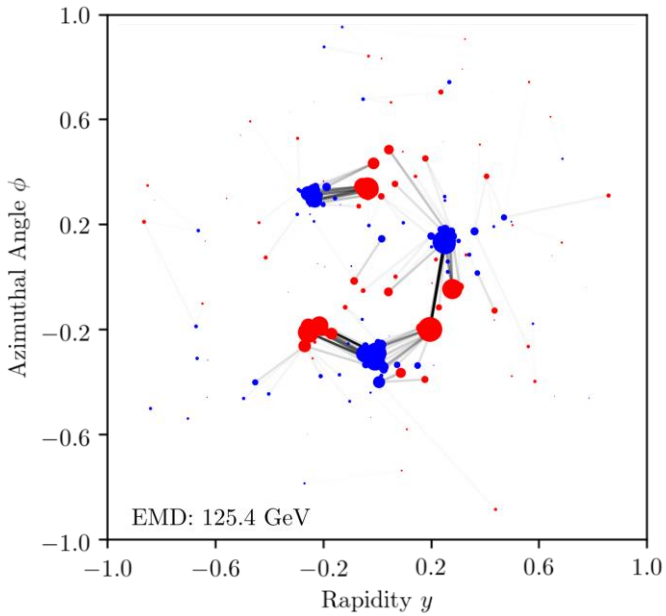


EMD has dimensions of energy

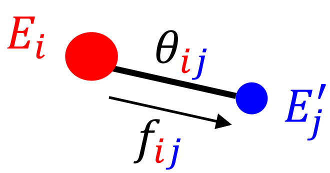
True metric as long as $R \geq \frac{1}{2}\theta_{\max}$

$R \geq$ the jet radius, for conical jets

Solvable via Optimal Transport problem.

~1ms to compute EMD for two jets with 100 particles.

$$\mathrm{EMD}(\mathcal{E}, \mathcal{E}') = \min_{\{f\}} \sum_{i=1}^{M} \sum_{j=1}^{M'} f_{ij} \frac{\theta_{ij}}{R} + \left| \sum_{i=1}^{M} E_i - \sum_{j=1}^{M'} E'_j \right|$$



Difference in radiation pattern

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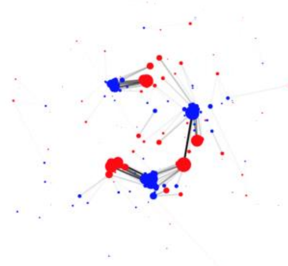

The Energy Mover's Distance

Work to rearrange one event into another.

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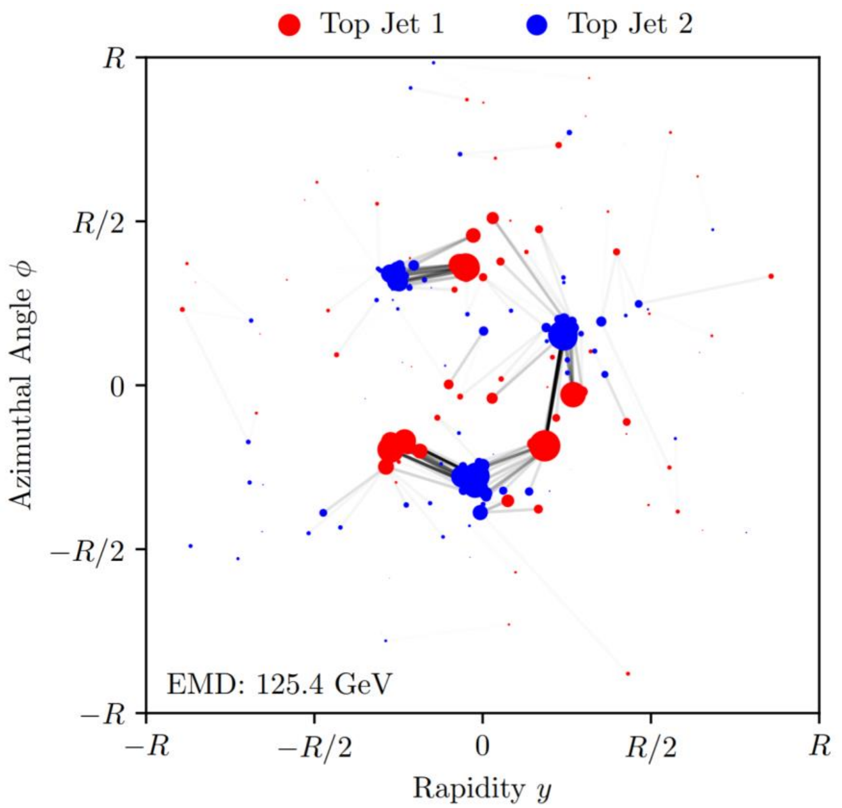
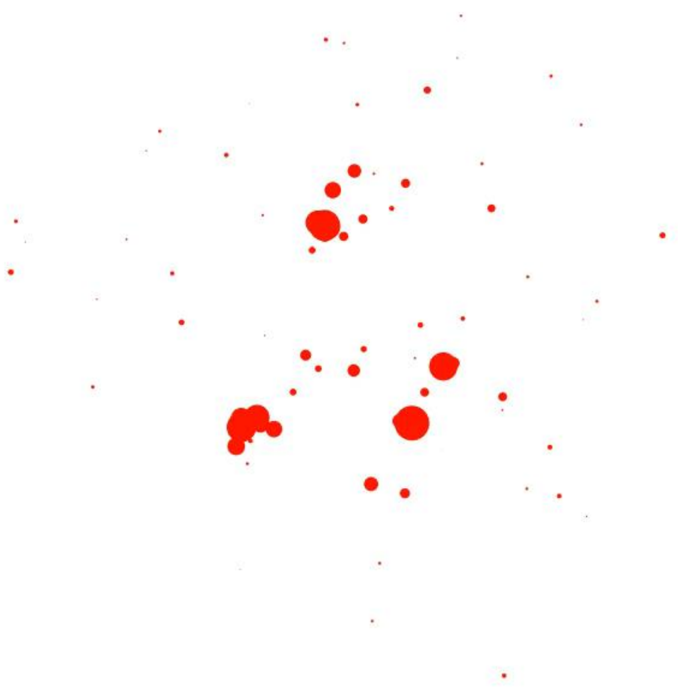
Movie Time: Visualizing the EMD

Taking a walk in the space of events





EMD is the cost of an optimal transport problem.

We also get the *shortest path* between the events.

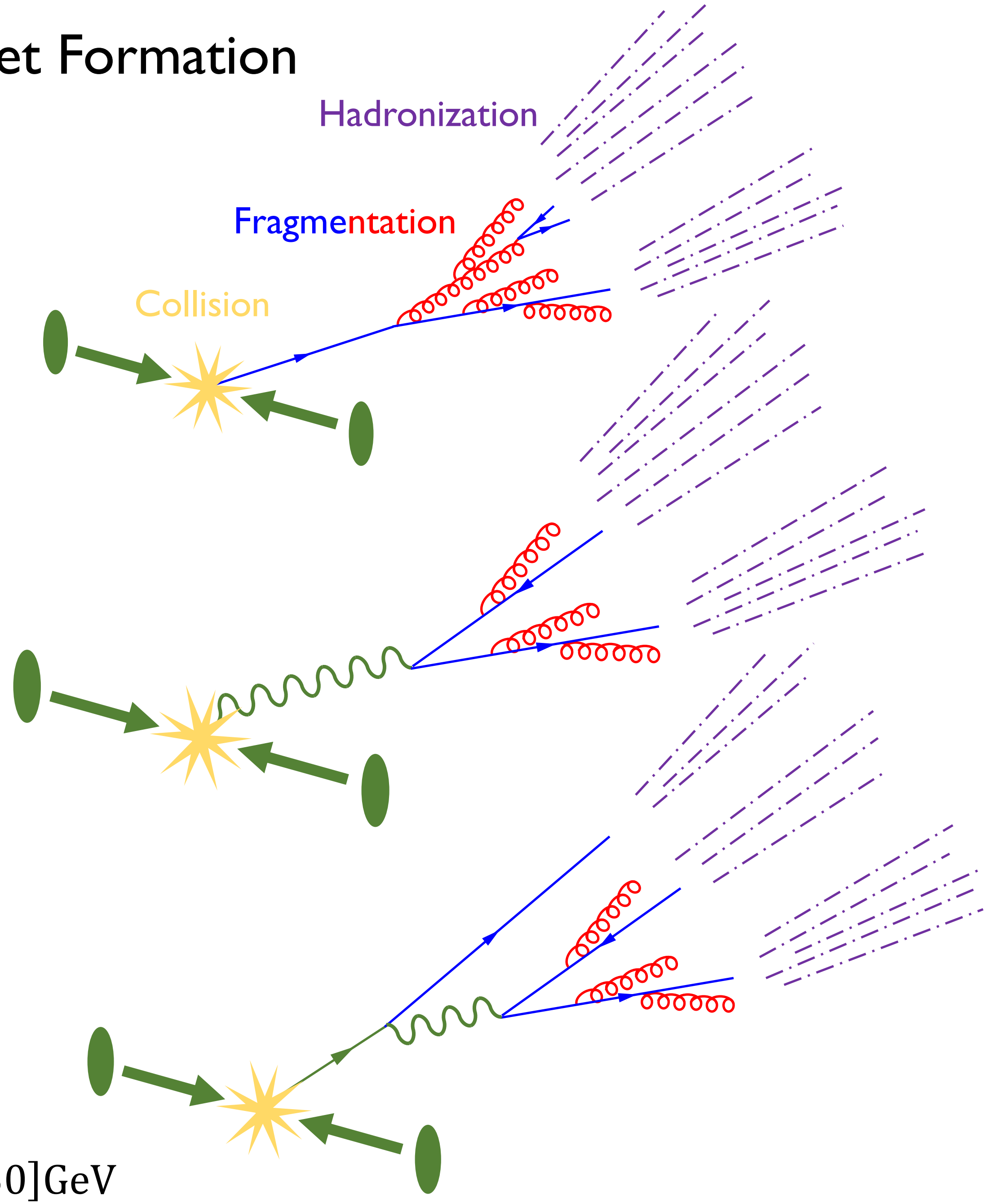
Interpolate along path to visualize!

Movie Time: Visualizing Jet Formation

QCD Jets

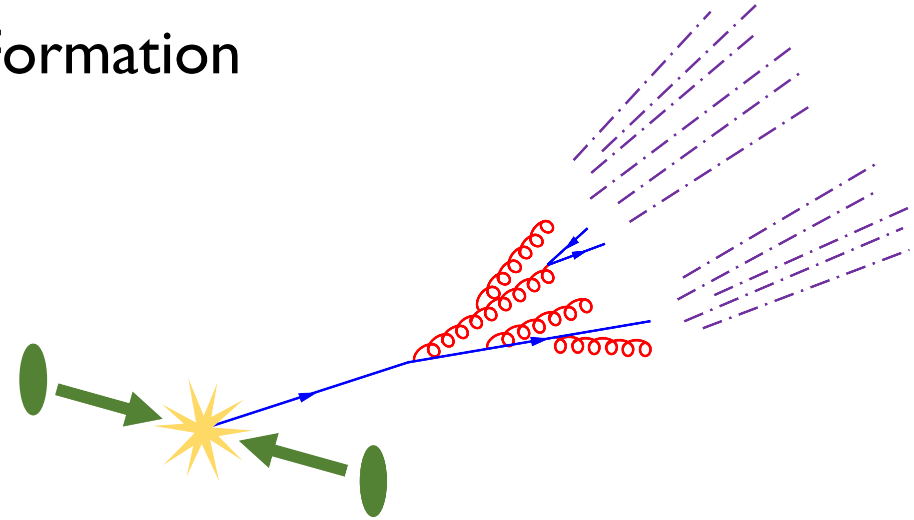
_W_ Jets

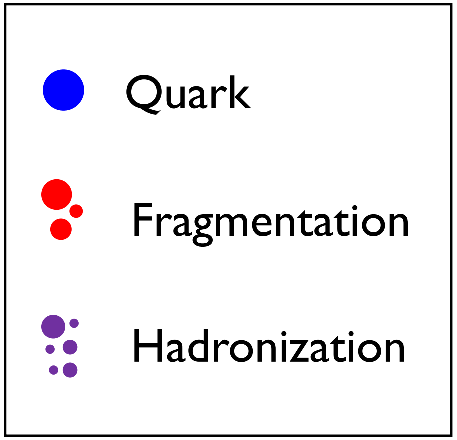
Top Jets



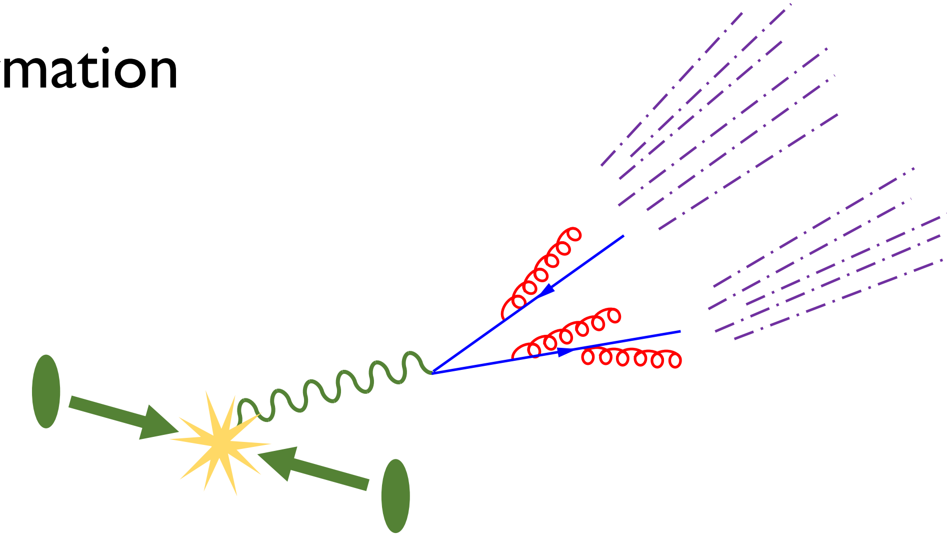
Pythia 8, $R = 1.0$ jets, $p_T \in [500,550]\mathrm{GeV}$

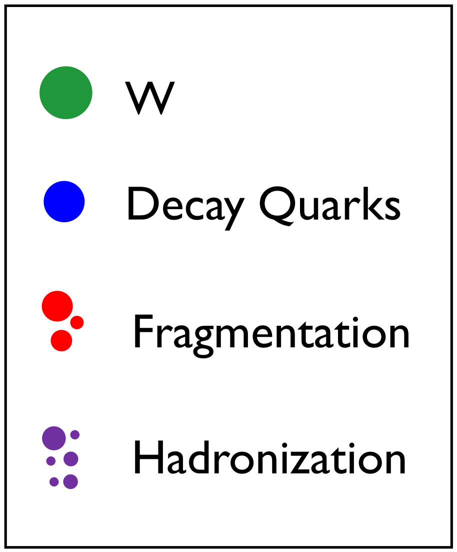
Movie Time: Visualizing QCD Jet Formation



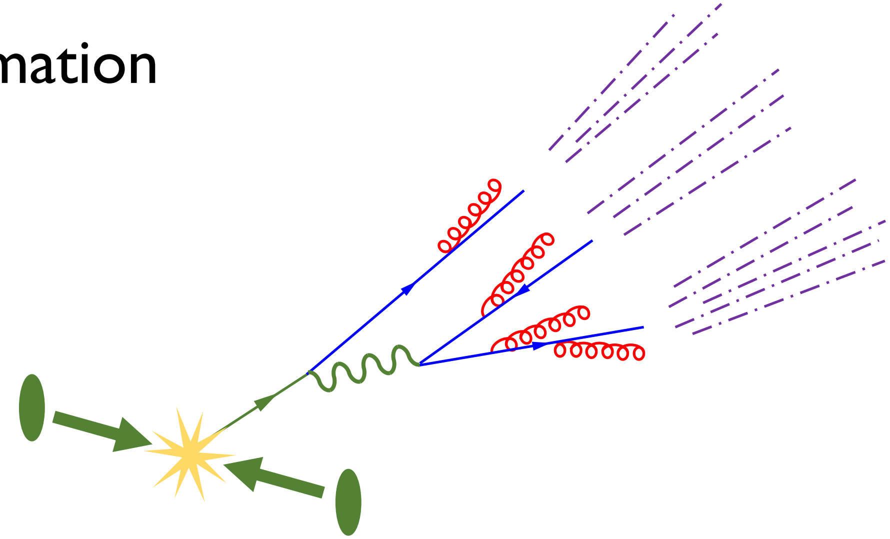


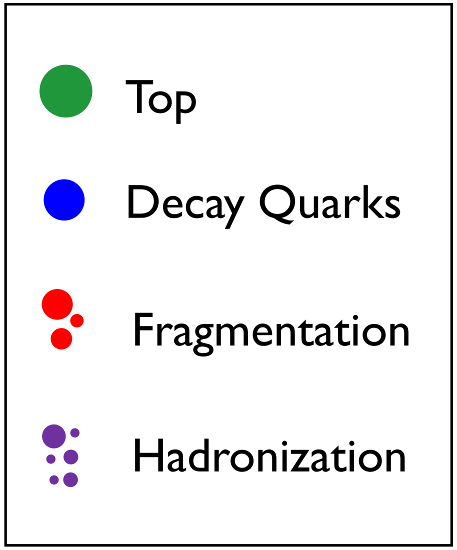
Movie Time: Visualizing W Jet Formation





Movie Time: Visualizing Top Jet Formation





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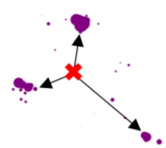

Observables

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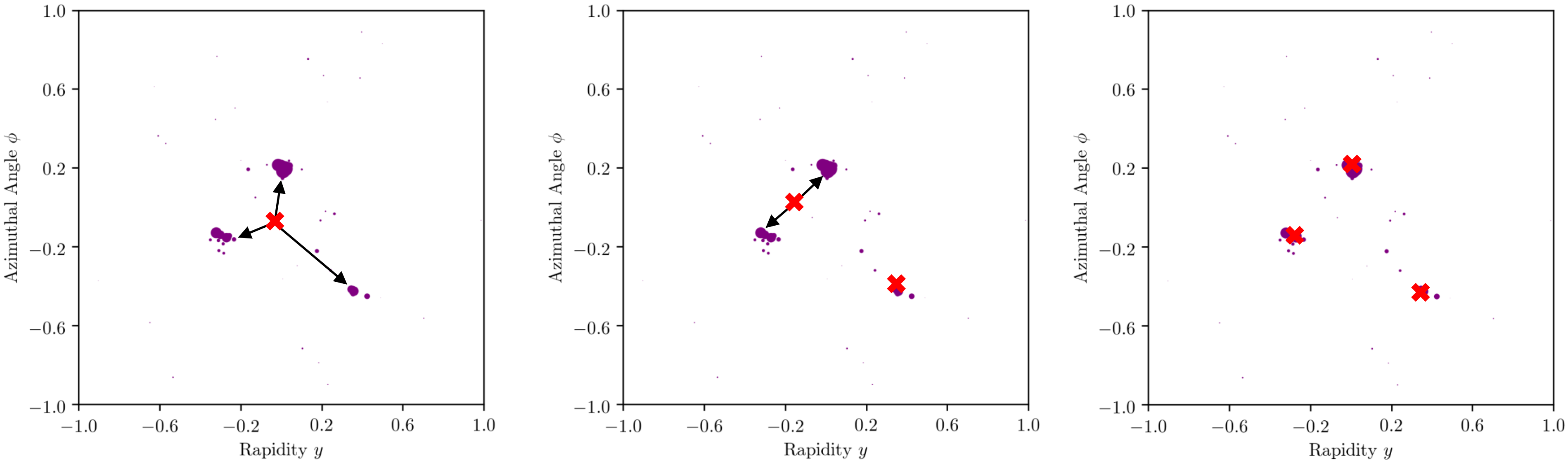
Observables

N-subjettiness: $$\tau_N^{(\beta)} = \sum_{i=1}^{M} E_i \min_{N \text{ axes}} \{\theta_{1,k}^{\beta}, \theta_{2,k}^{\beta}, \dots, \theta_{N,k}^{\beta}\}$$

[J. Thaler, K. Van Tilburg, 1011.2268]
[J. Thaler, K. Van Tilburg, 1108.2701]

measures how well jet energy is aligned into N subjets

$\tau_1/E \gg 0$ $\tau_1/E > \tau_2/E \gg 0$ $\tau_3/E \simeq 0$



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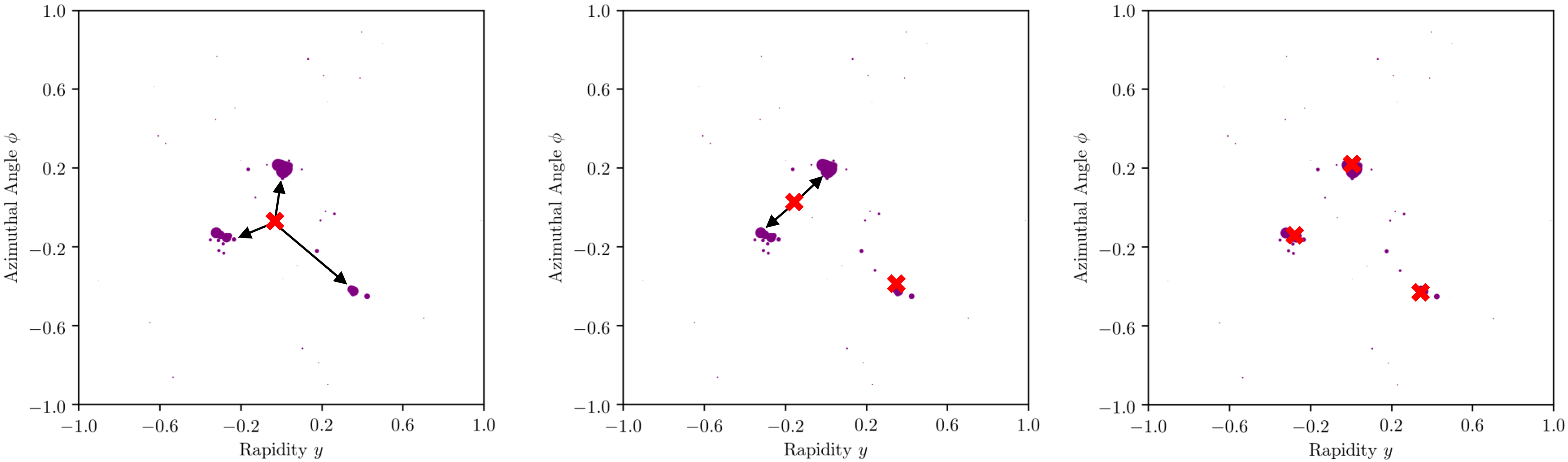


N-subjettiness is the EMD between the event and the closest N-particle event.

$$\tau_N(\mathcal{E}) = \min_{|\mathcal{E}'|=N} \text{EMD}(\mathcal{E}, \mathcal{E}').$$

$\beta \neq 1$ corresponds to other p-Wasserstein distances with $p = \beta$.

Observables

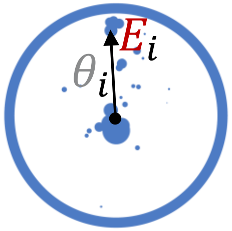
Getting quantitative

Take any additive IRC-safe observable: $\mathcal{O}(\mathcal{E}) = \sum_{i=1}^{M} E_i \, \Phi(\hat{p}_i)$

e.g. jet angularities: $\lambda^{(\beta)} = \sum_{i=1}^{M} E_i \, \theta_i^{\beta}$

[C. Berger, T. Kucs, and G. Sterman, 0303051]

[A. Larkoski, J. Thaler, and W. Waalewijn, 1408.3122]



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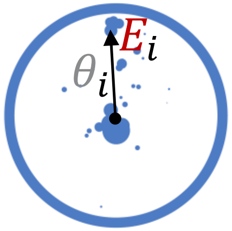

[C. Berger, T. Kucs, and G. Sterman, 0303051]
[A. Larkoski, J. Thaler, and W. Waalewijn, 1408.3122]

Via the Kantorovich-Rubinstein dual formulation of EMD:

Earth Mover's Distance

Difference in observable values

$$\mathrm{EMD}(\mathcal{E}, \mathcal{E}') \geq \frac{1}{RL} \left| \sum_{i=1}^{M} E_i \Phi(\hat{p}_i) - \sum_{j=1}^{M'} E'_j \, \Phi(\hat{p}_i) \right| = \frac{1}{RL} \, |\mathcal{O}(\mathcal{E}) - \mathcal{O}(\mathcal{E}')|$$

"Lipschitz constant" of Φ
i.e. bound on its derivative

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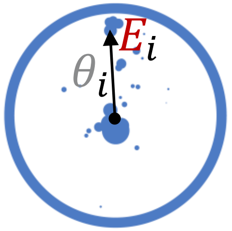

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"Lipschitz constant" of Φ
i.e. bound on its derivative

For $\beta \geq 1$ jet angularities, $L = \beta / R$ over the jet cone, so:

$$|\lambda^{(\beta)}(\mathcal{E}) - \lambda^{(\beta)}(\mathcal{E}')| \leq \beta\ \mathrm{EMD}(\mathcal{E}, \mathcal{E}')$$

The EMD provides a robust upper bound to any modifications of these observables.

Observables

Key idea: Energy-weighted angular structures contain all the IRC-safe information.

$$\frac{1}{RL}\left|\sum_{i=1}^{M} E_i \Phi(\hat{p}_i) - \sum_{j=1}^{M'} E_j' \,\Phi(\hat{p}_i)\right| \leq \mathrm{EMD}(\mathcal{E}, \mathcal{E}')$$

Theorem: Any infrared and collinear safe observable $\mathcal{O}$ can be approximated arbitrarily well as:

$$\mathcal{O}(p_1, \ldots, p_M) = F\left(\sum_{i=1}^{M} E_i \,\vec{\Phi}(\hat{p}_i)\right)$$

for some Φ: $\mathbb{R}^2 \to \mathbb{R}^\ell$ and F: $\mathbb{R}^\ell \to \mathbb{R}$ and sufficiently large ℓ.

[M. Zaheer, *et al.*, 1703.06114]
[P.T. Komiske, EMM, J. Thaler, 1810.05165]

Events close in EMD are close in all infrared and collinear safe information!

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When they have similar distributions of energy

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Work to rearrange one event into another.

Movie Time

Visualize energy movement and jet formation.

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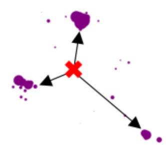

Observables

Conceptually rich connections to EMD.

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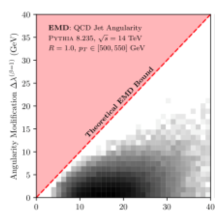
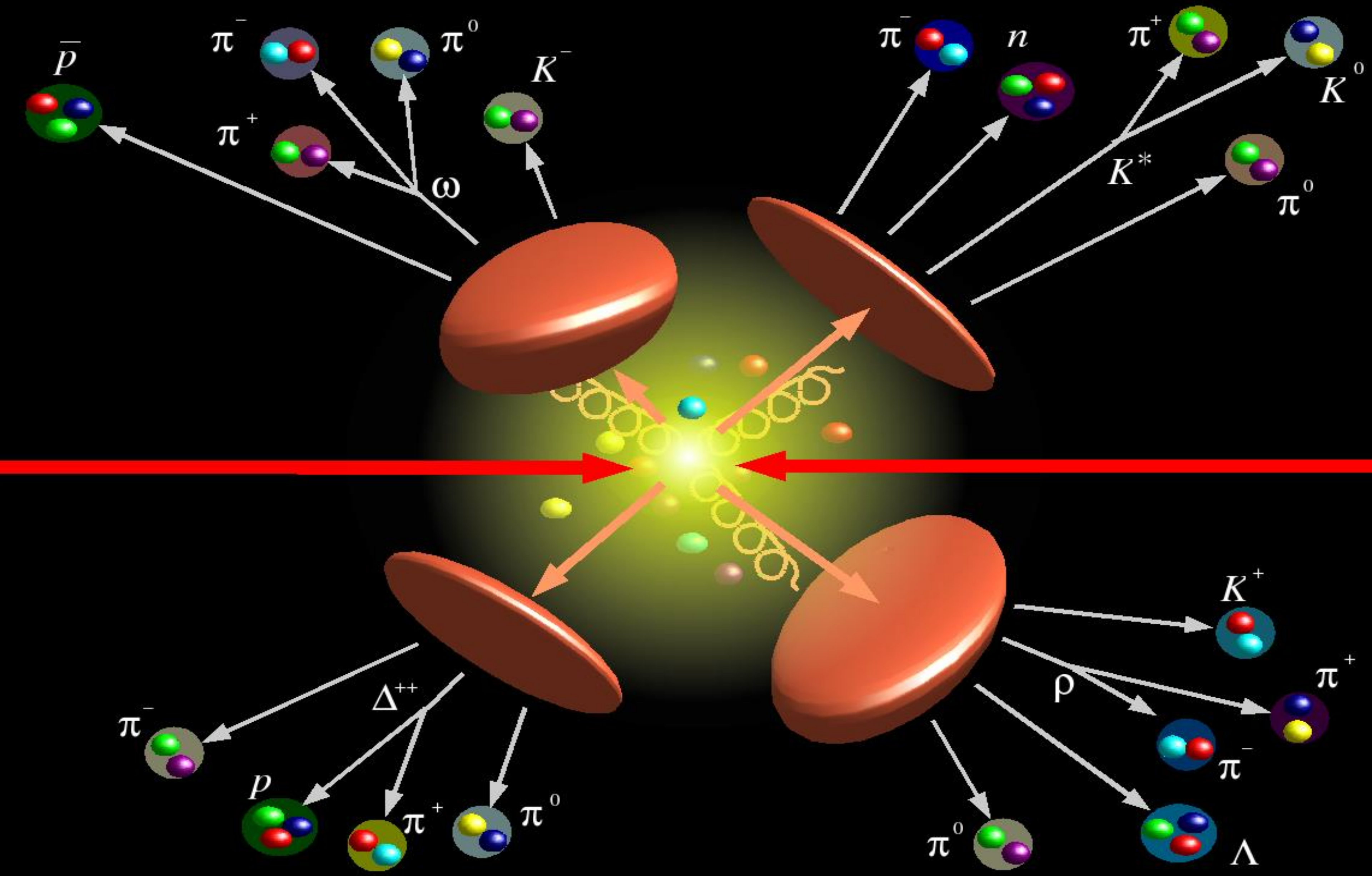


Quantifying event modifications

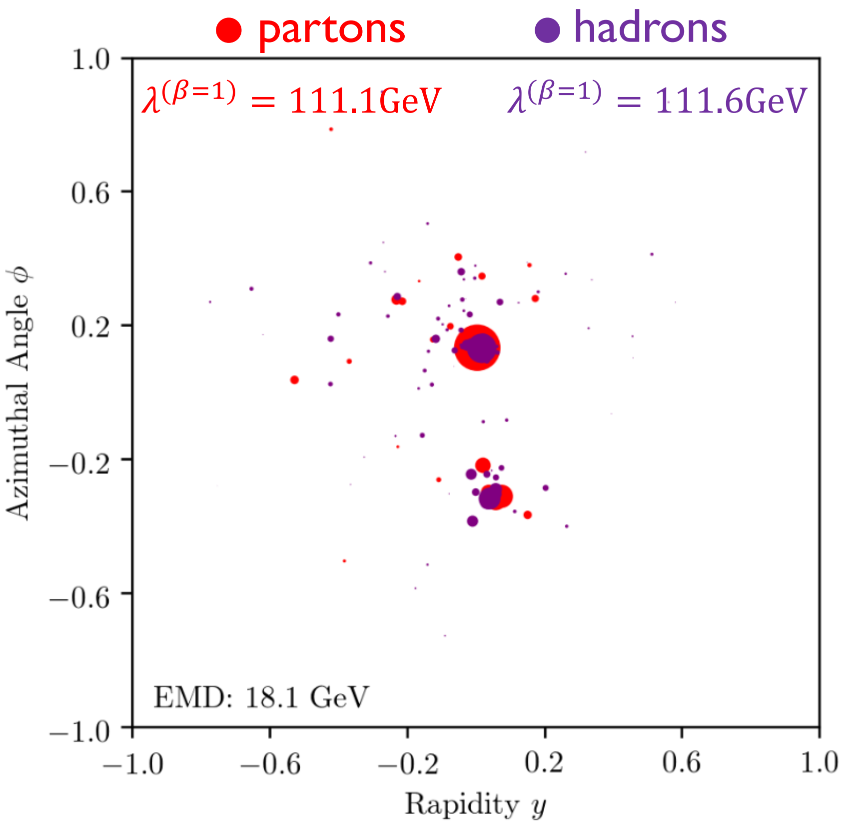
Exploring the Space of Events

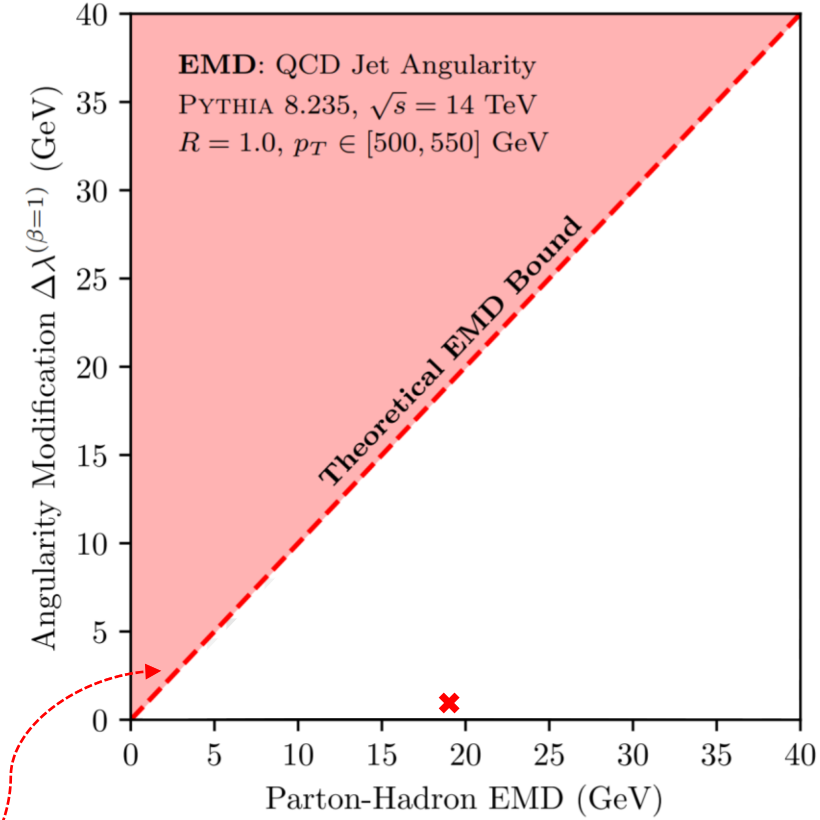
Quantifying event modifications: Hadronization



Quantifying event modifications: Hadronization

$$\lambda^{(\beta=1)} = \sum_{i=1}^{M} E_i \, \theta_i$$





$$\mathcal{E} = \mathcal{E}_{\text{partons}}$$

$$\mathcal{E}' = \mathcal{E}_{\text{hadrons}}$$

$$\left|\lambda^{(\beta=1)}(\mathcal{E}) - \lambda^{(\beta=1)}(\mathcal{E}')\right| \leq \text{EMD}(\mathcal{E}, \mathcal{E}')$$

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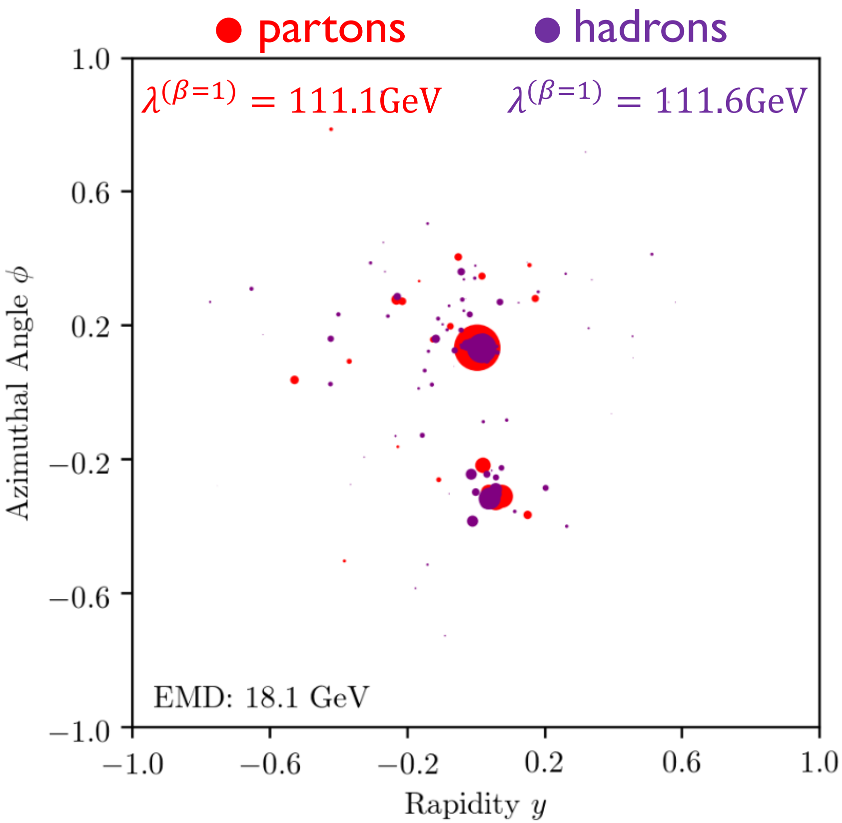

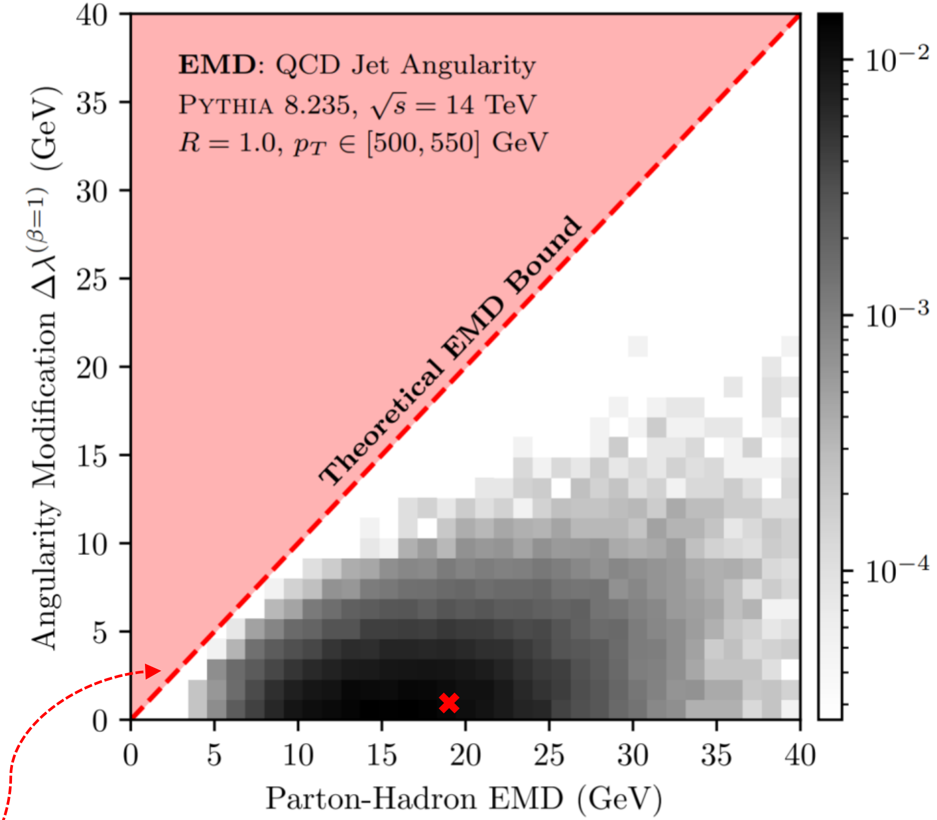

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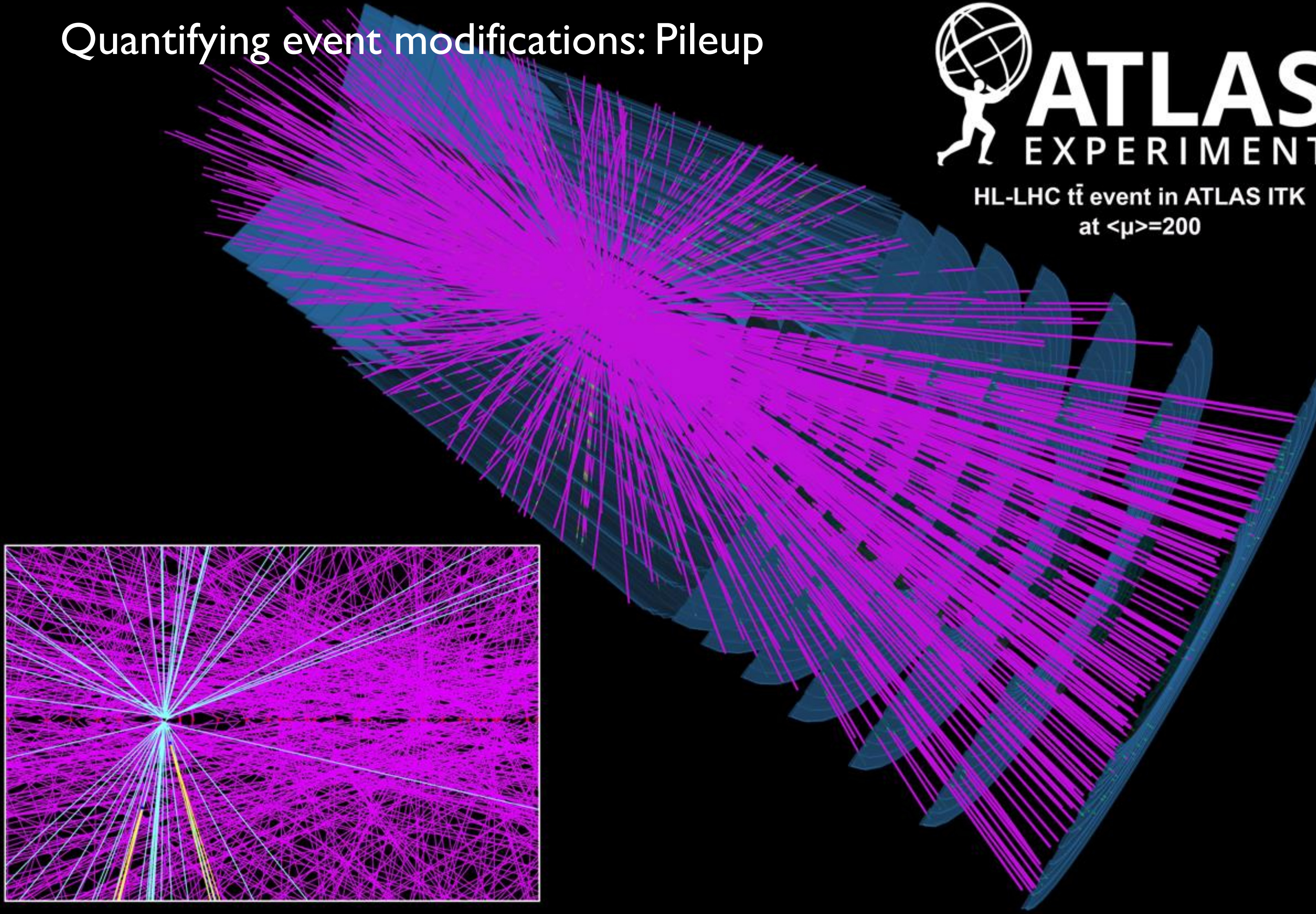
Quantifying event modifications: Pileup



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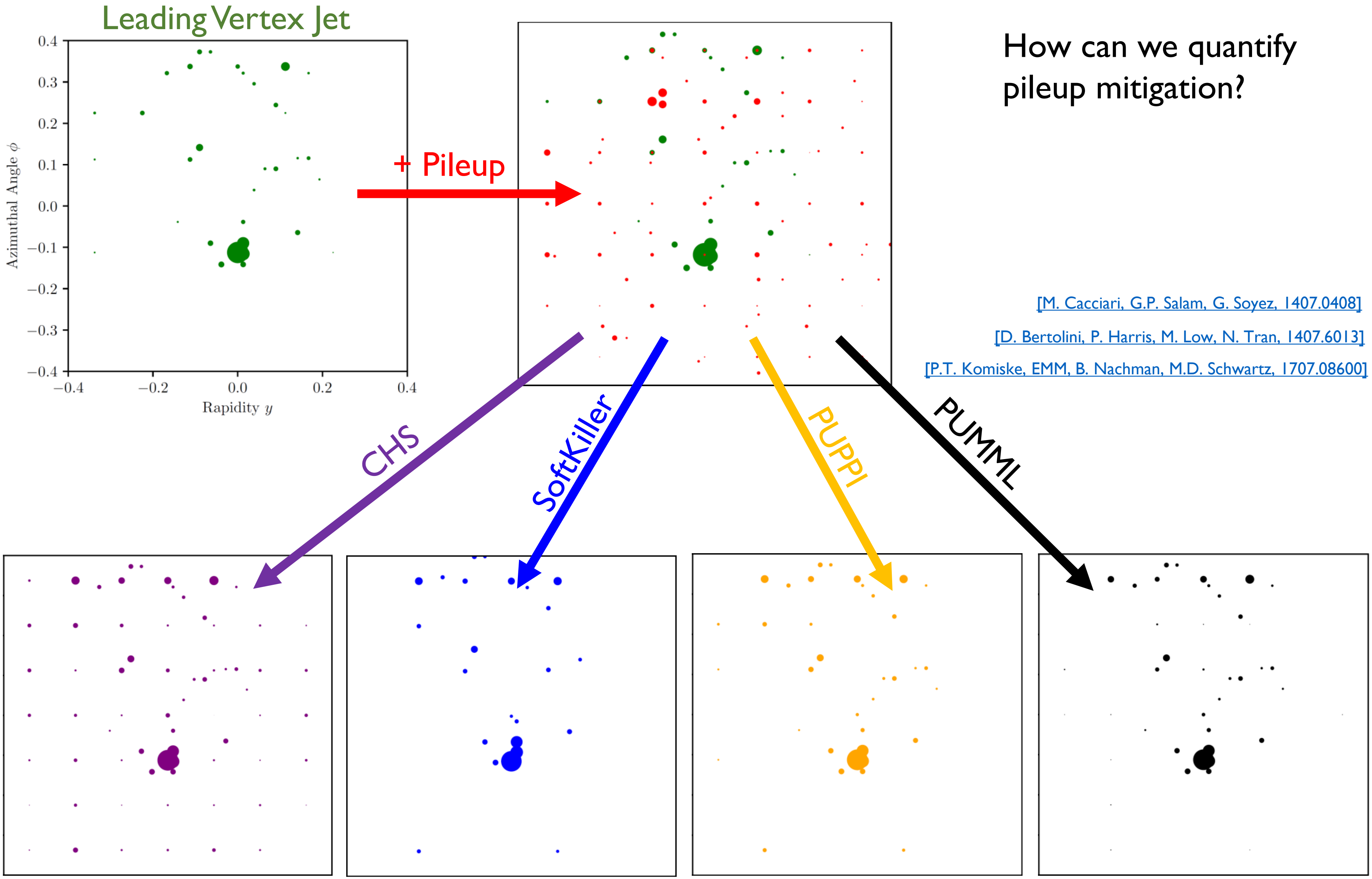


How can we quantify pileup mitigation?

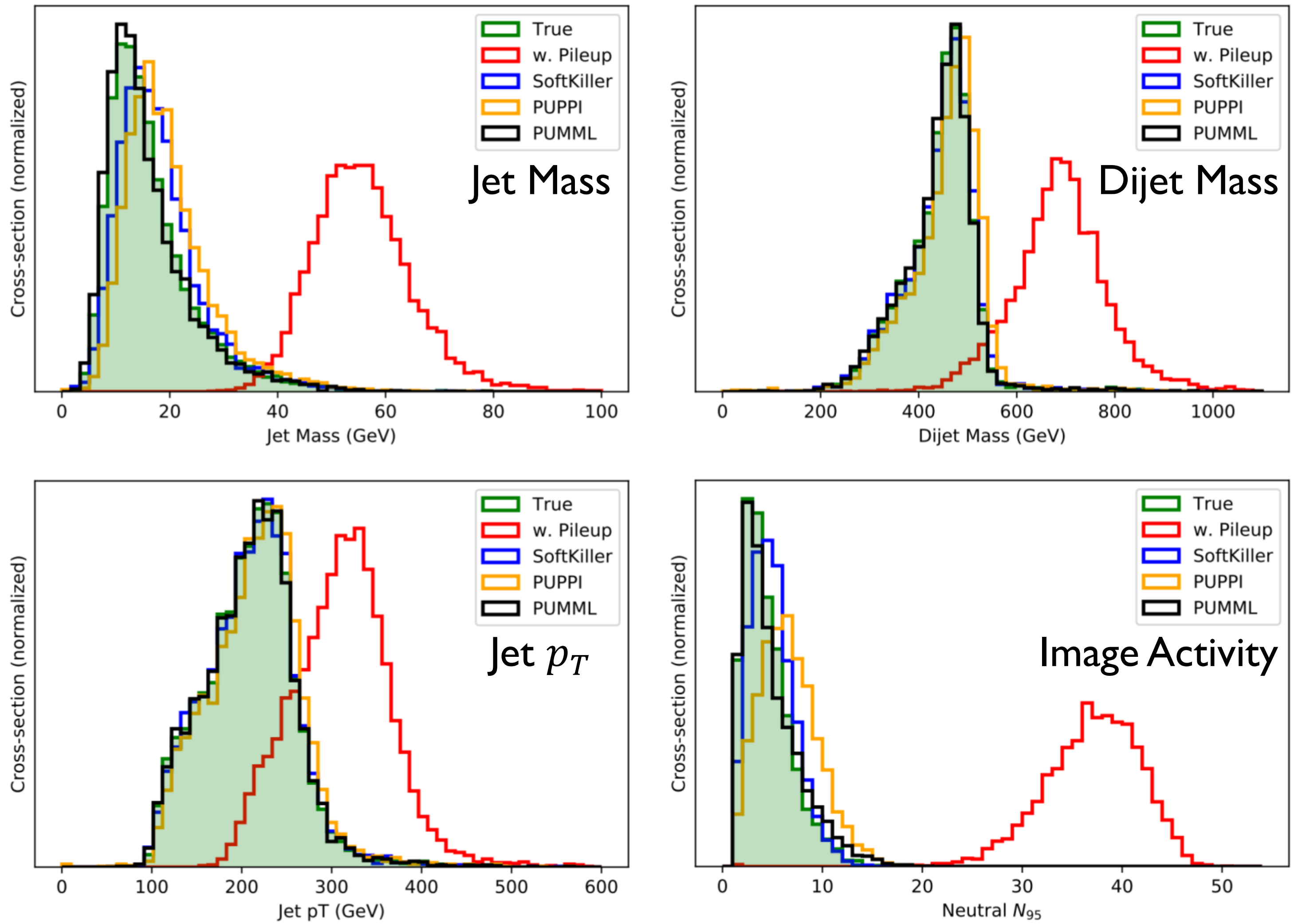
[M. Cacciari, G.P. Salam, G. Soyez, 1407.0408]

[D. Bertolini, P. Harris, M. Low, N. Tran, 1407.6013]

[P.T. Komiske, EMM, B. Nachman, M.D. Schwartz, 1707.08600]

Quantifying event modifications: Pileup

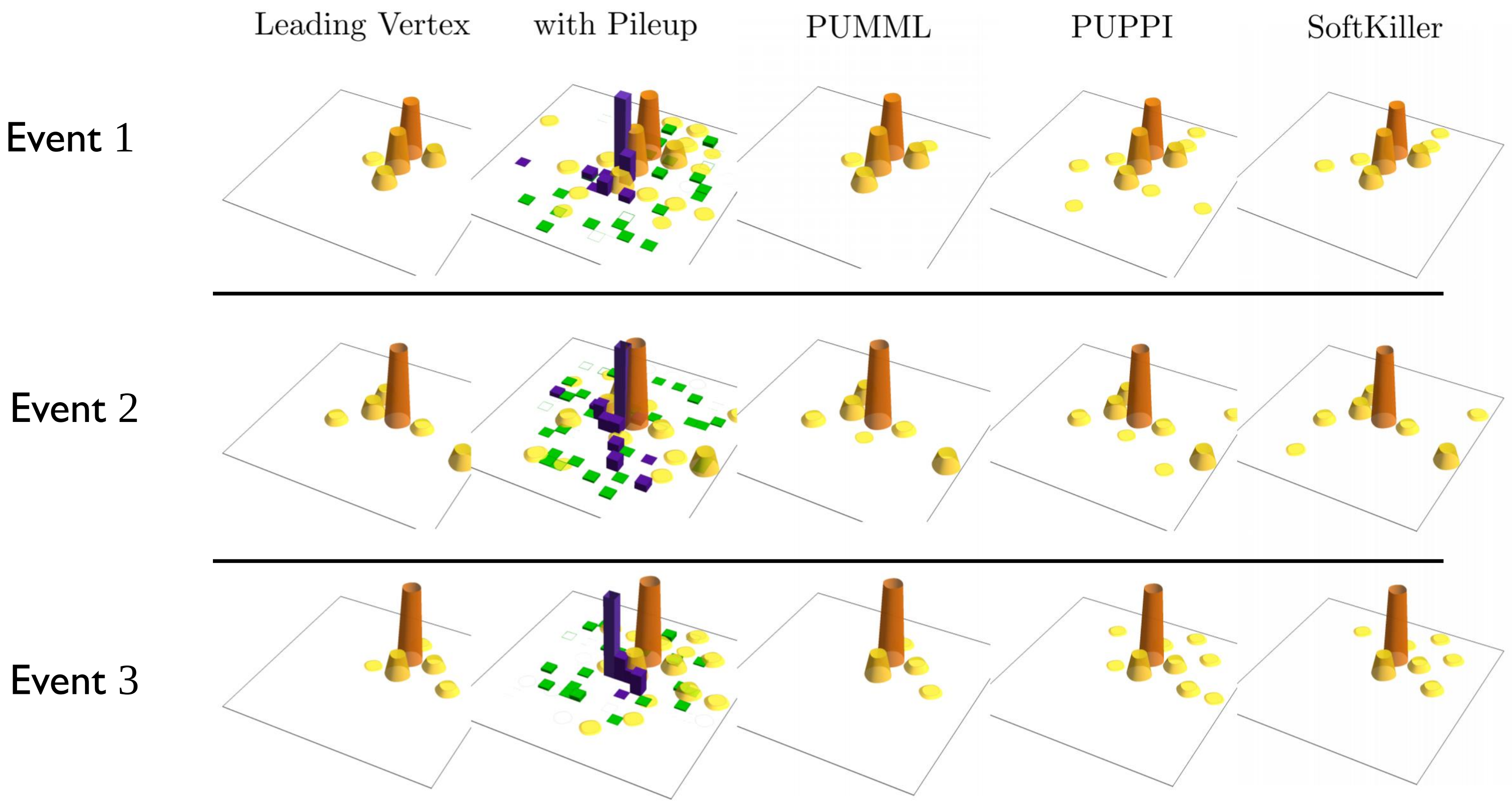
Compare on a collection of observables?



Requires ad hoc choices of observables.

Quantifying event modifications: Pileup

Compare calorimeter images pixel by pixel?

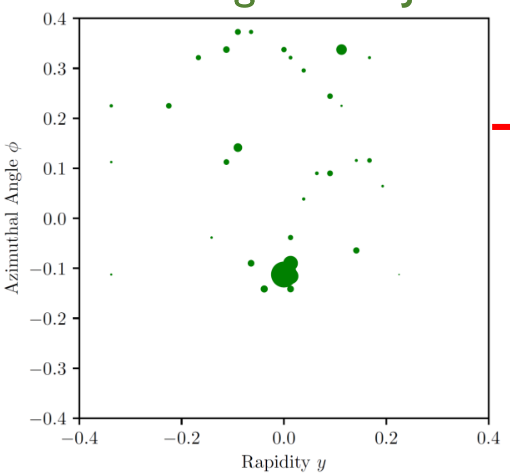


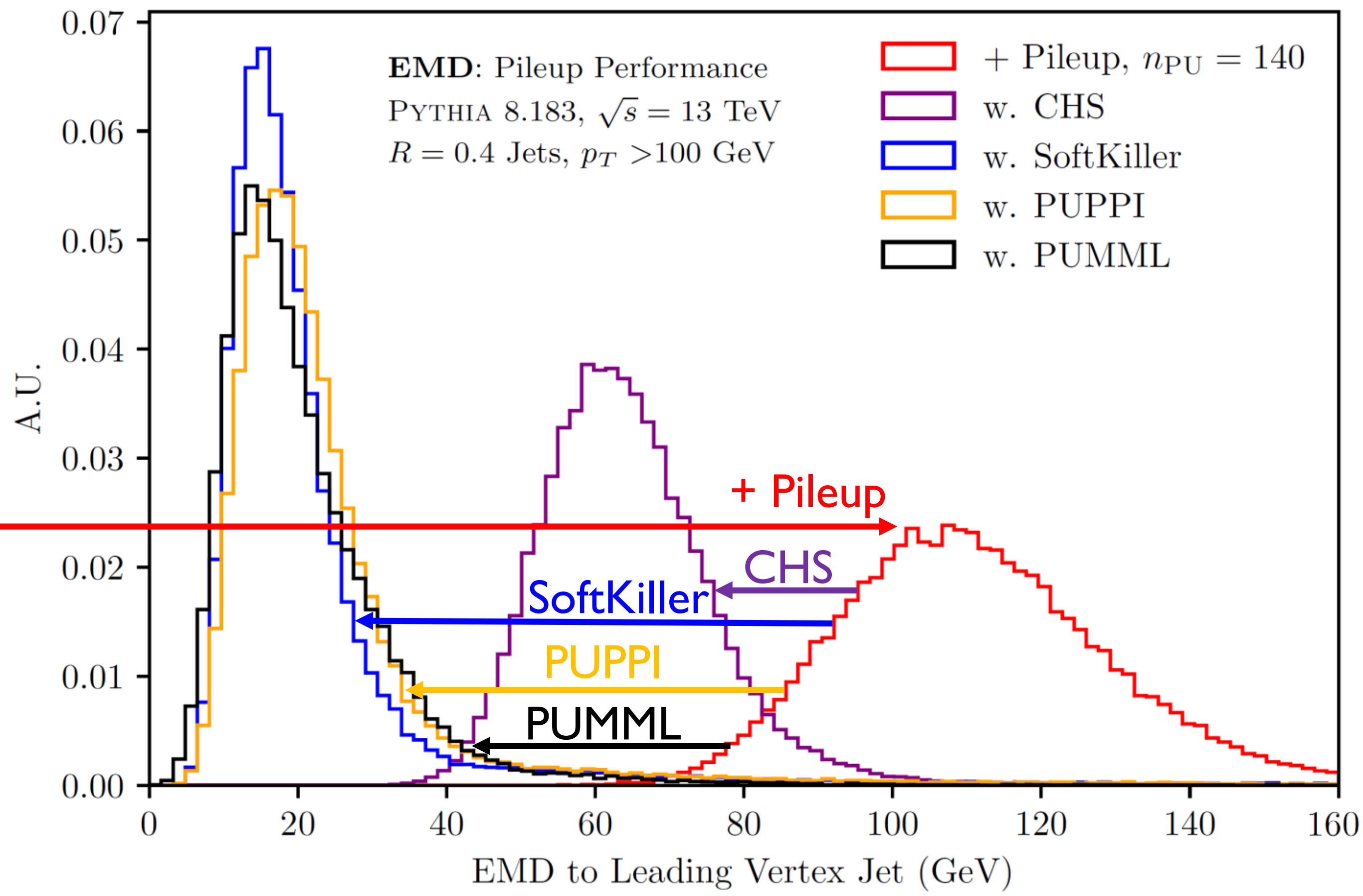
Discontinuous under physically-sensible single-pixel perturbations.
Undesirable behavior with increasing resolution.

Quantifying event modifications: Pileup

Measure pileup mitigation performance with EMD!

Leading Vertex Jet





Guarantees performance on IRC safe observables.
Stable under physically-sensible perturbations.
Train to optimize EMD with machine learning?

Outline

Part I
Introduction

When are two events similar?

When they have similar distributions of energy

The Energy Mover's Distance

Work to rearrange one event into another.

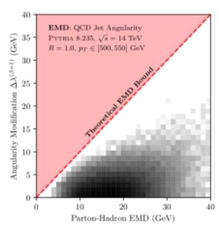
Movie Time

Visualize energy movement and jet formation.

Part II
Applications

Observables

Conceptually rich connections to EMD.



Quantifying event modifications

Hadronization, pileup, detector effects

Exploring the Space of Events

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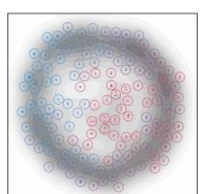
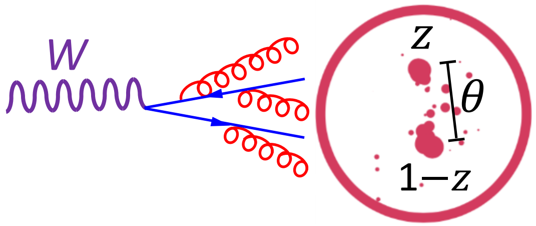


Exploring the Space of Events

Exploring the Space of Events: W jets



W jets are 2-pronged:

z: Energy Sharing of Prongs
θ: Angle between Prongs
φ: Azimuthal orientation

Constrained by W mass:

$$z(1-z)\theta^2 = \frac{p_{\mu J}^2}{p_T^2} = \frac{m_W^2}{p_T^2}$$

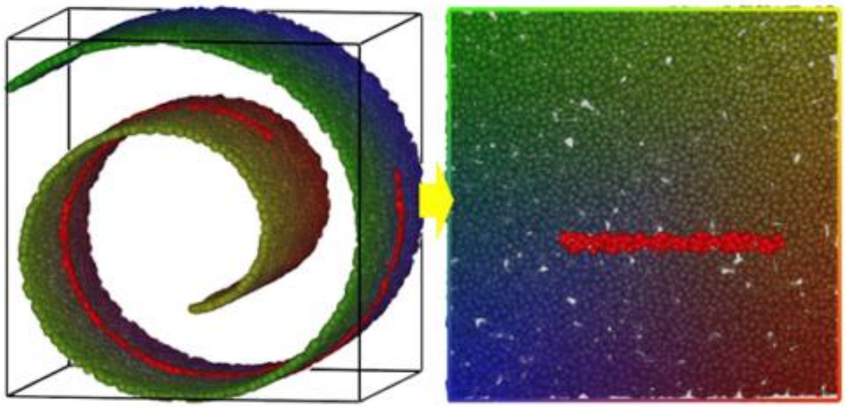
Hence we expect a **two**-dimensional space of W jets.

After φ rotation: **one**-dimensional

Visualize the space of events with t-Distributed Stochastic Neighbor Embedding (t-SNE).

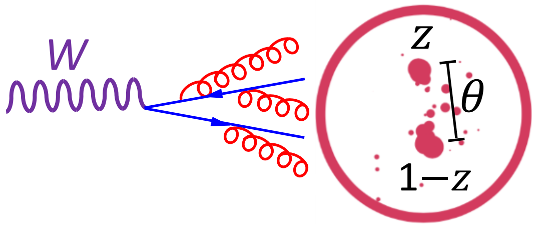
[L. van der Maaten, G. Hinton]

Finds an embedding into a low-dimensional manifold that respects distances.



Src: http://web-ext.u-aizu.ac.jp/~shigeo/home.html

Exploring the Space of Events: *W* jets



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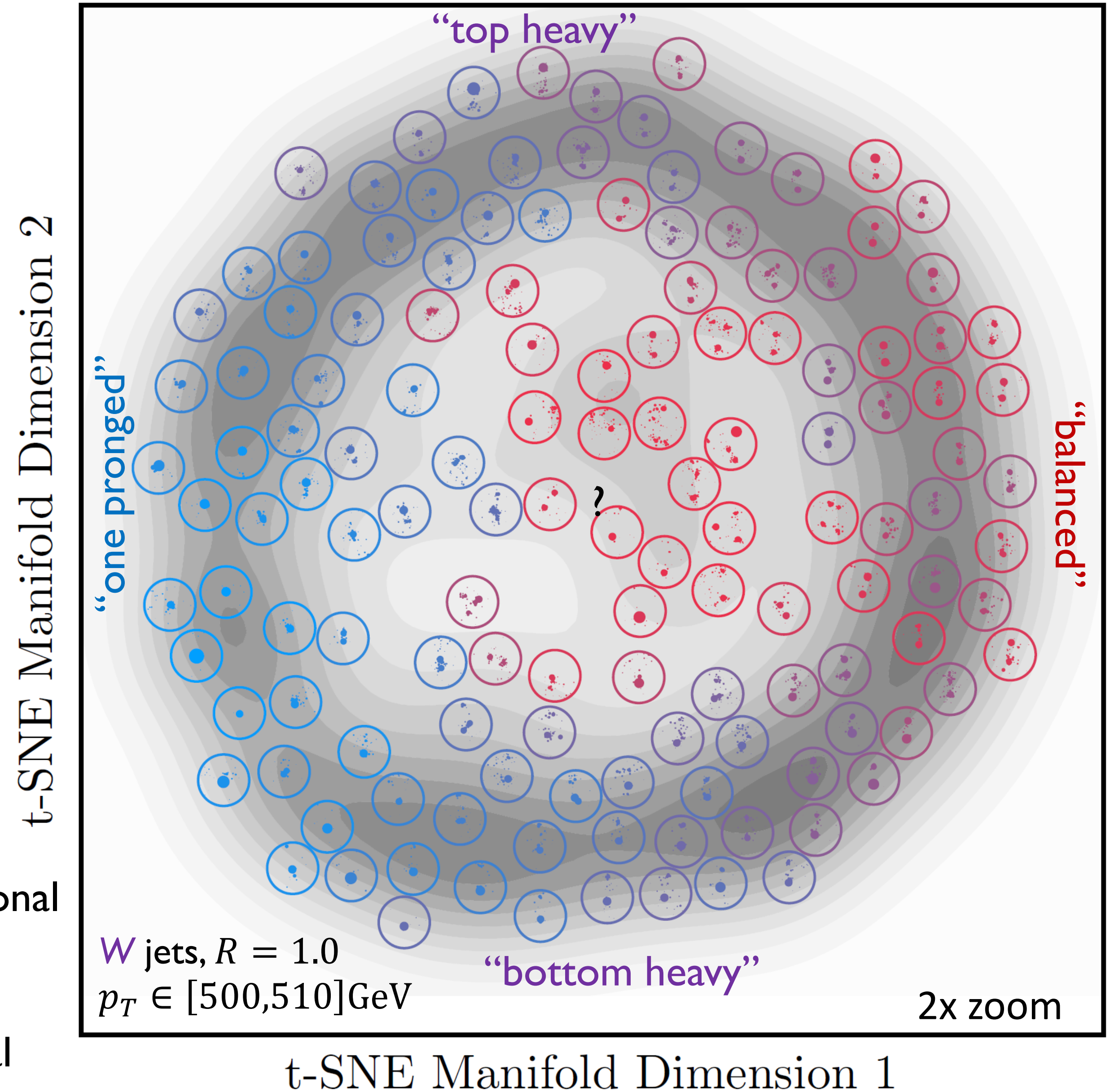


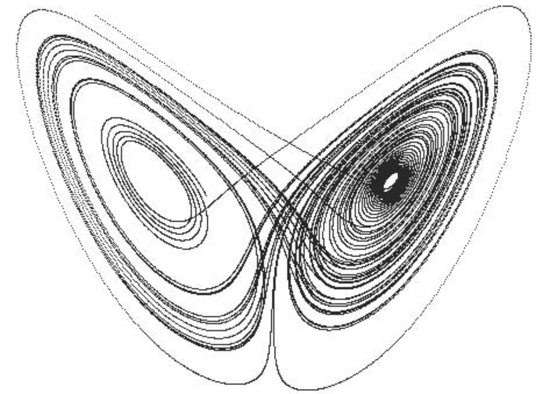
Exploring the Space of Jets: Correlation Dimension

VOLUME 50, NUMBER 5 PHYSICAL REVIEW LETTERS 31 JANUARY 1983

Characterization of Strange Attractors

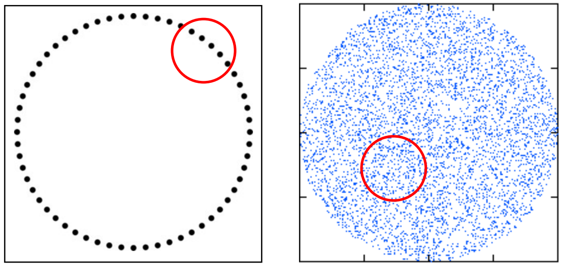
Peter Grassberger[a] and Itamar Procaccia
Chemical Physics Department, Weizmann Institute of Science, Rehovot 76100, Israel
(Received 7 September 1982)

A new measure of strange attractors is introduced which offers a practical algorithm to determine their character from the time series of a single observable. The relation of this new measure to fractal dimension and information-theoretic entropy is discussed.



Intuition:

$$N_{\text{neighboring points}}(r) \propto r^{\dim} \quad \Longrightarrow \quad \dim(r) = \mathrm{r}\frac{\partial}{\partial r}\ln N_{\text{neighbors}}(r)$$



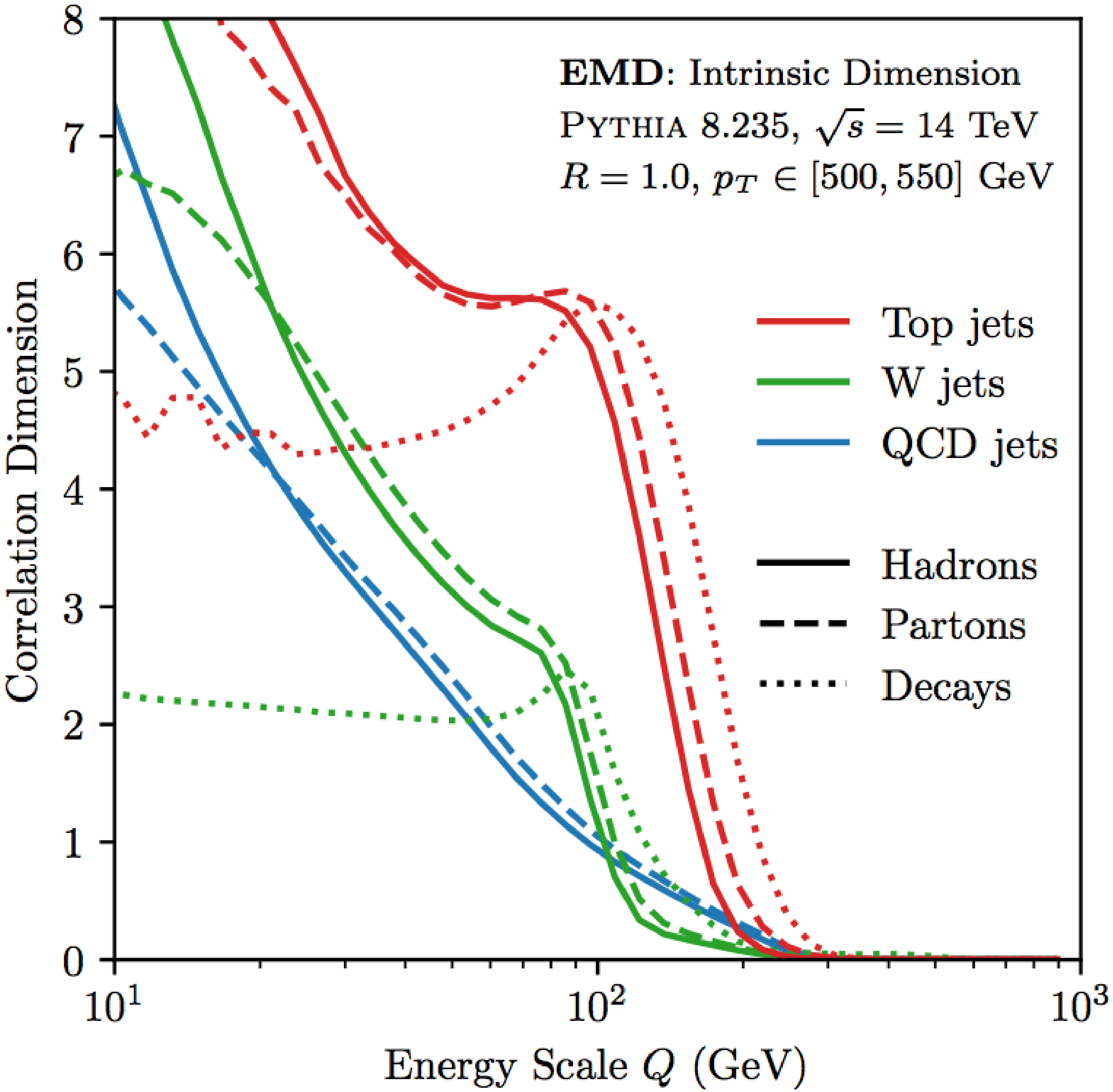
Correlation dimension:

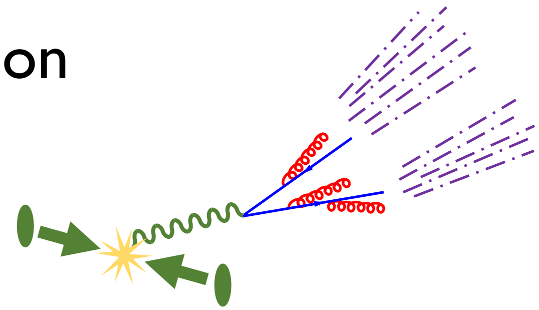
$$\dim(Q) = Q\frac{\partial}{\partial Q}\ln \sum_{i=1}^{N}\sum_{j=1}^{N}\Theta[\mathrm{EMD}(\mathcal{E}_i,\mathcal{E}_j) < Q]$$

Energy scale Q dependence

Count neighbors in ball of radius Q

Exploring the Space of Jets: Correlation Dimension





QCD jets are simplest.

W jets are more complicated.

Top jets are most complex.

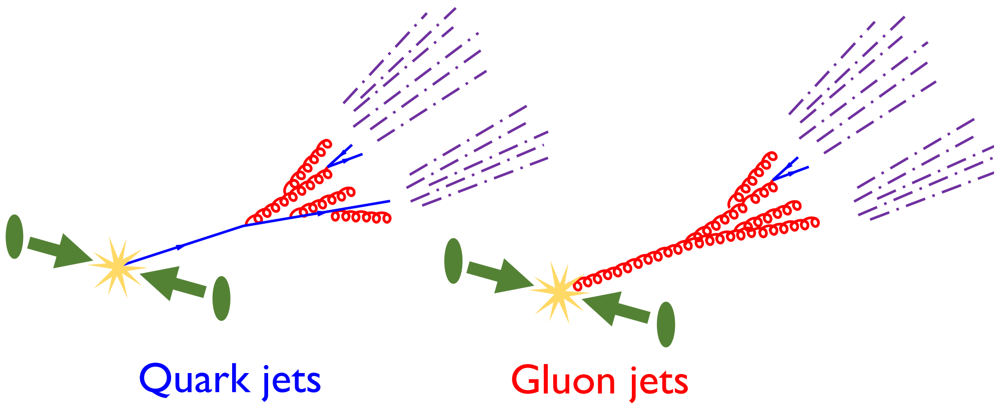
"Decays" have ~constant dimension.

Fragmentation becomes more complex at lower energy scales.

Hadronization becomes relevant at scales around 20 GeV.

Can we understand this analytically?

Exploring the Space of Jets: Correlation Dimension

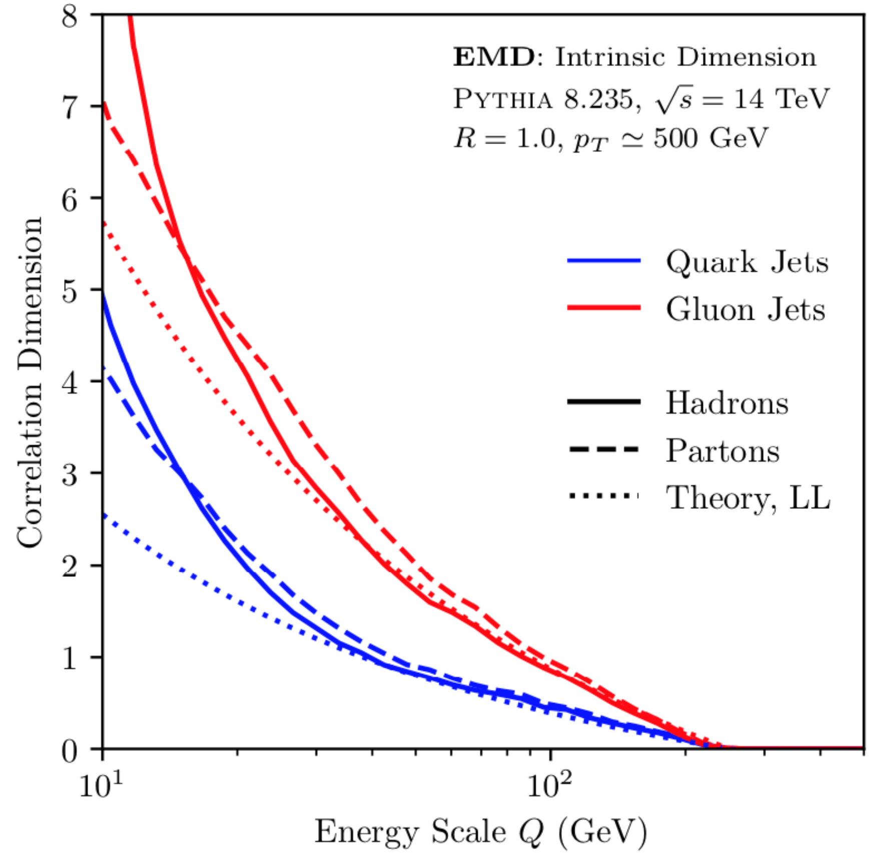


$$\text{At LL:}\quad \dim_{q/g}(Q) = -\frac{8\alpha_s C_{q/g}}{\pi} \ln \frac{Q}{p_T/2}$$

+ 1-loop running of α_s

$$C_q = C_F = \frac{4}{3}$$

$$C_g = C_A = 3$$

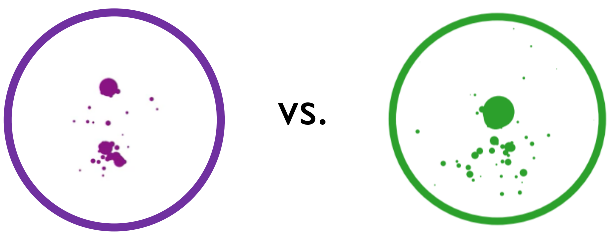


Dimension blows up at low energies.

Jets are "more than fractal"?

Exploring the Space of Events: Jet Classification

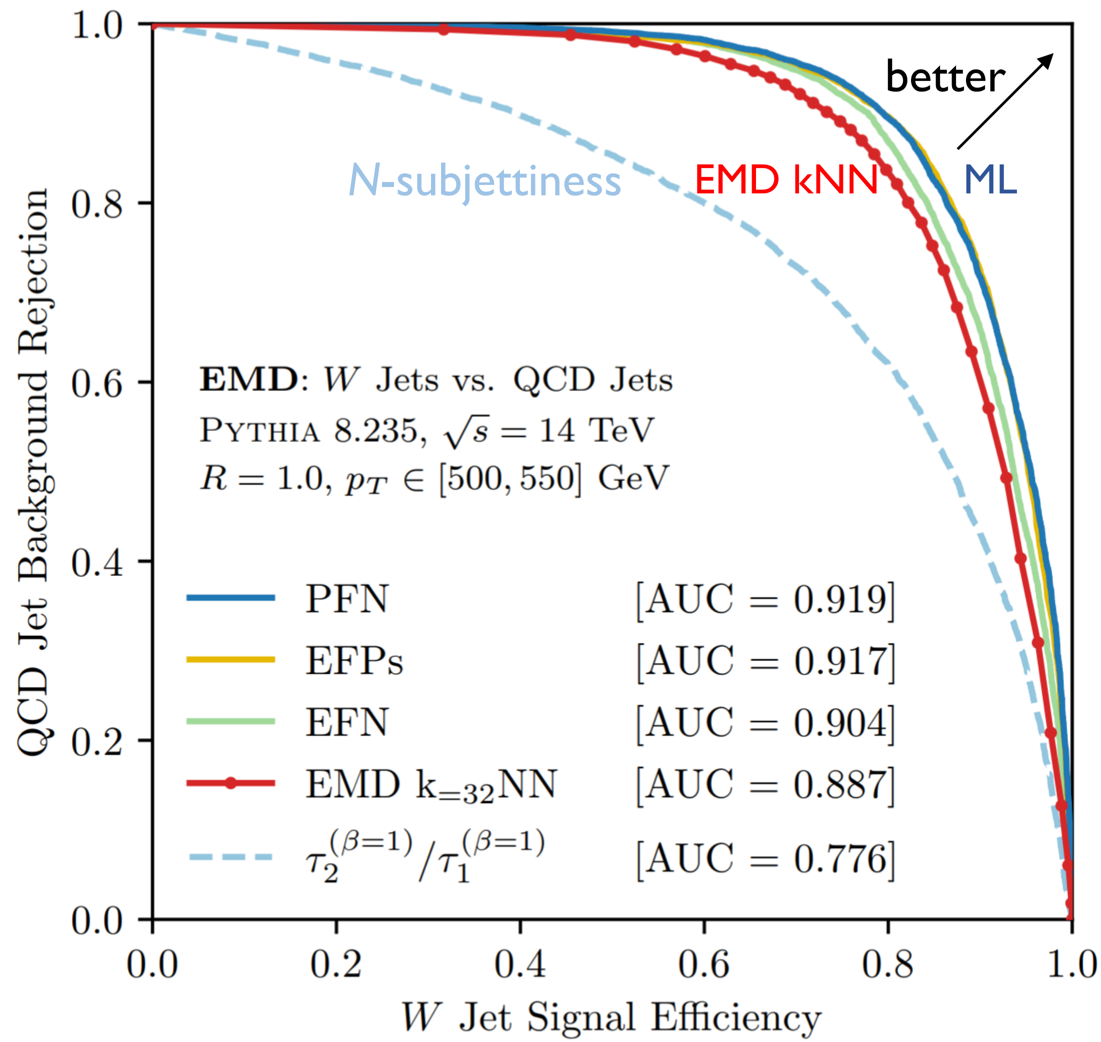
Classify *W* jets vs. QCD jets



Look at a jet's nearest neighbors (kNN) to predict its class.

Optimal IRC-safe classifier with enough data.

Nearing performance of ML.



Exploring the Space of Events

Clustering events

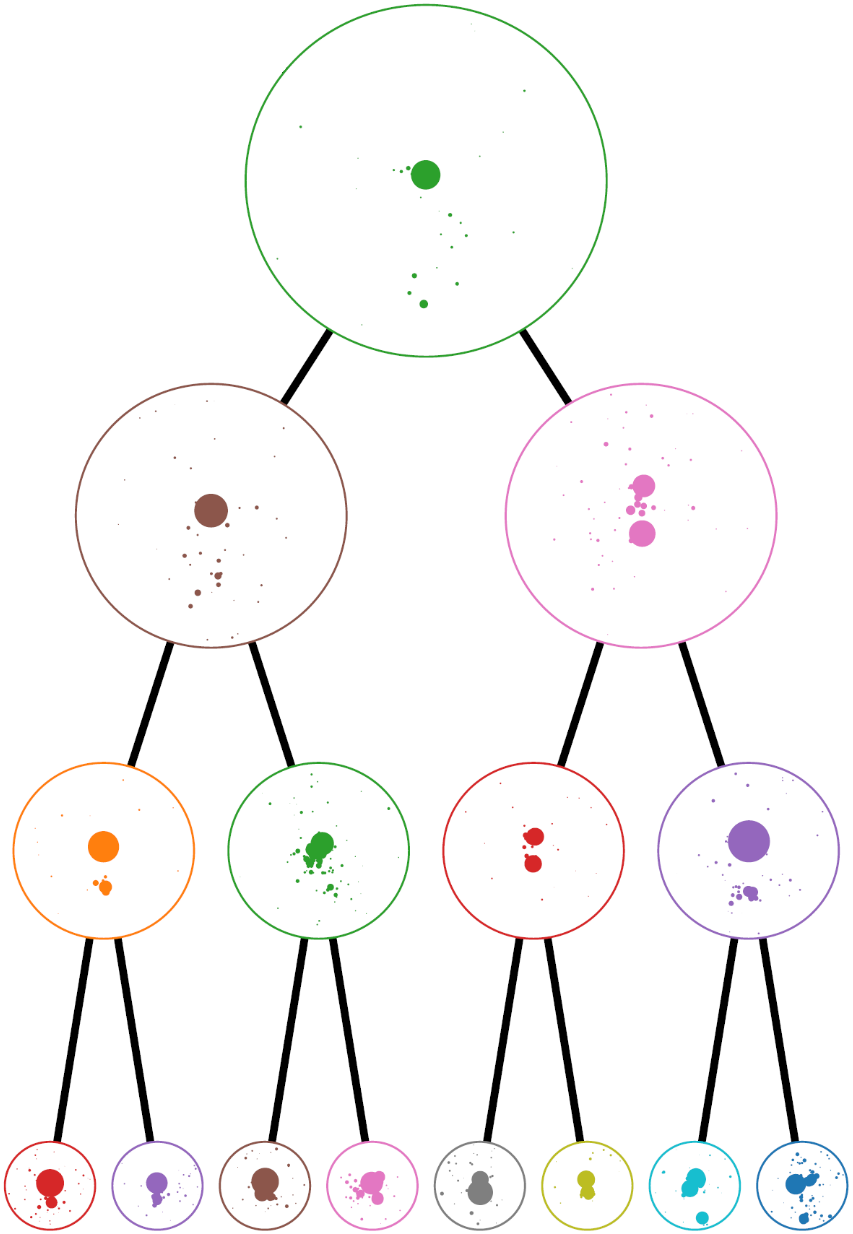
Use EMD as a measure of event similarity

Unsupervised clustering algorithms can be used to cluster events

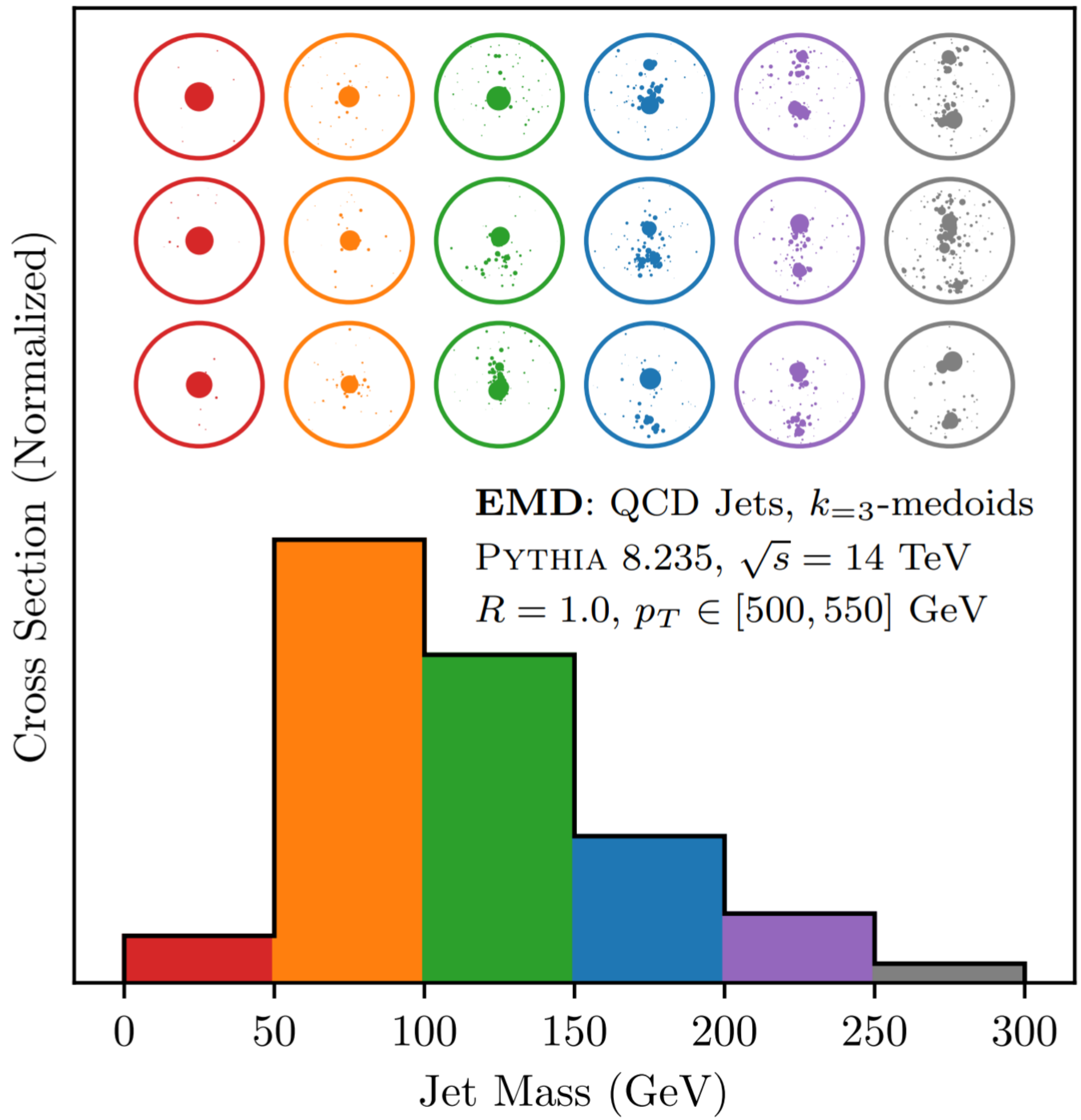
Jets are clusters of particles
???? are clusters of jets

VP Tree: O(log(N)) neighbor query time

Much more to explore.



Exploring the Space of Events: *k*-medoids



Summary

Part I
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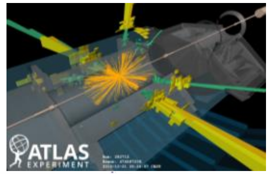

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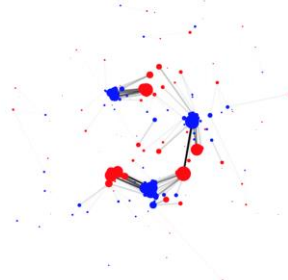

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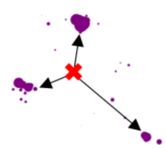

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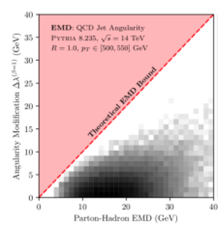
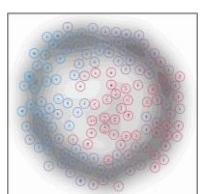


Quantifying event modifications

Hadronization, pileup, detector effects

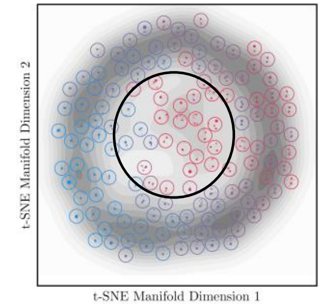


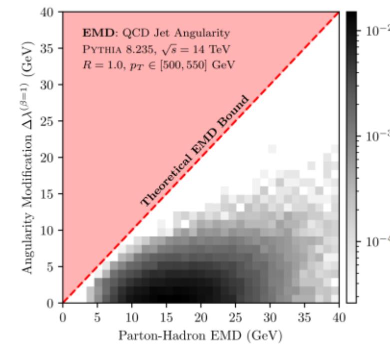
Exploring the Space of Events

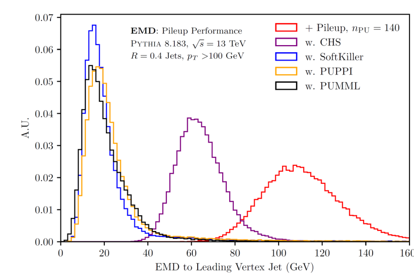
Unlock new ideas and techniques with EMD

Going Beyond

- Model (in)dependent anomaly detection?
- Sharpen the parton-hadron duality of energy flow?
- Train ML models to optimize EMD directly?
- Include flavor information?

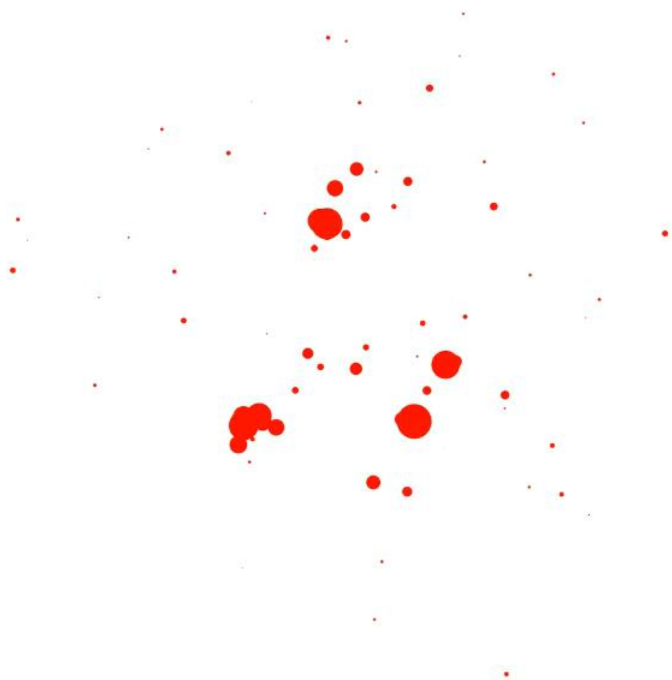




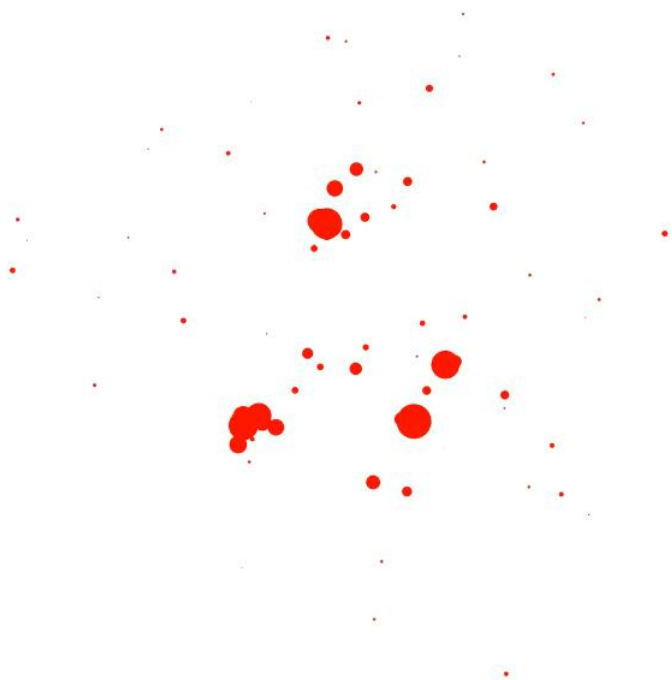


The End

Thank you!



Extra Slides



Exploring the Space of Jets: Correlation Dimension

Sketch of leading log (one emission) calculation:

$$\dim_{q/g}(Q) = Q\frac{\partial}{\partial Q}\ln\sum_{i=1}^{N}\sum_{j=1}^{N}\Theta[\mathrm{EMD}(\mathcal{E}_i,\mathcal{E}_j) < Q]$$

$$= Q\frac{\partial}{\partial Q}\ln \Pr[\mathrm{EMD} < Q]$$

$$= Q\frac{\partial}{\partial Q}\ln \Pr[\lambda^{(\beta=1)} < Q; C_{q/g} \to 2\,C_{q/g}]$$

$$= Q\frac{\partial}{\partial Q}\ln \exp\left(-\frac{4\alpha_s C_{q/g}}{\pi}\ln^2\frac{Q}{p_T/2}\right)$$

$$= -\frac{8\alpha_s C_{q/g}}{\pi}\ln\frac{Q}{p_T/2}$$

+ 1-loop running of α_s

$$C_q = C_F = \frac{4}{3}$$

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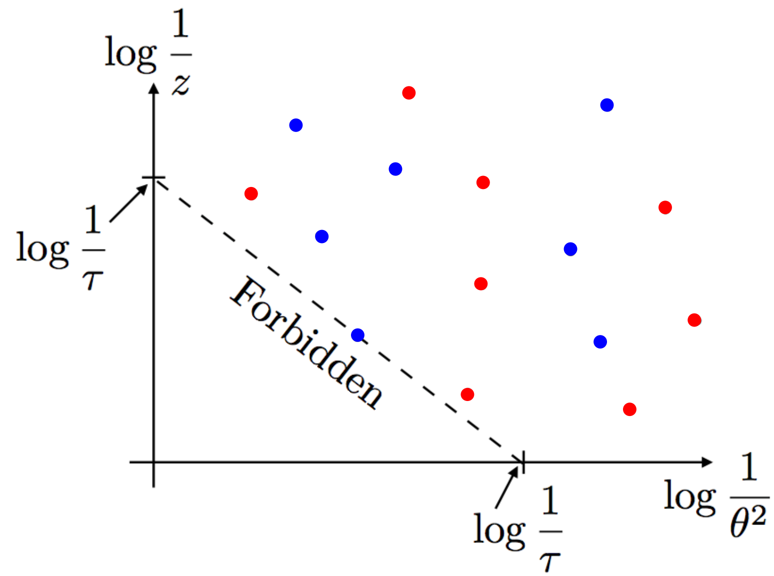
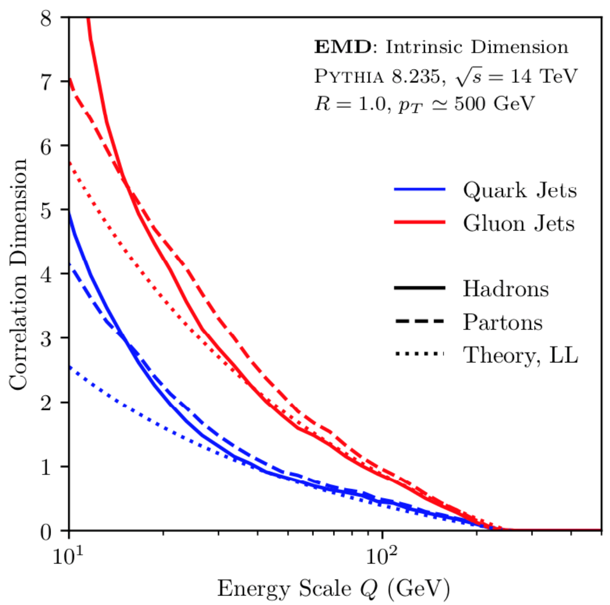
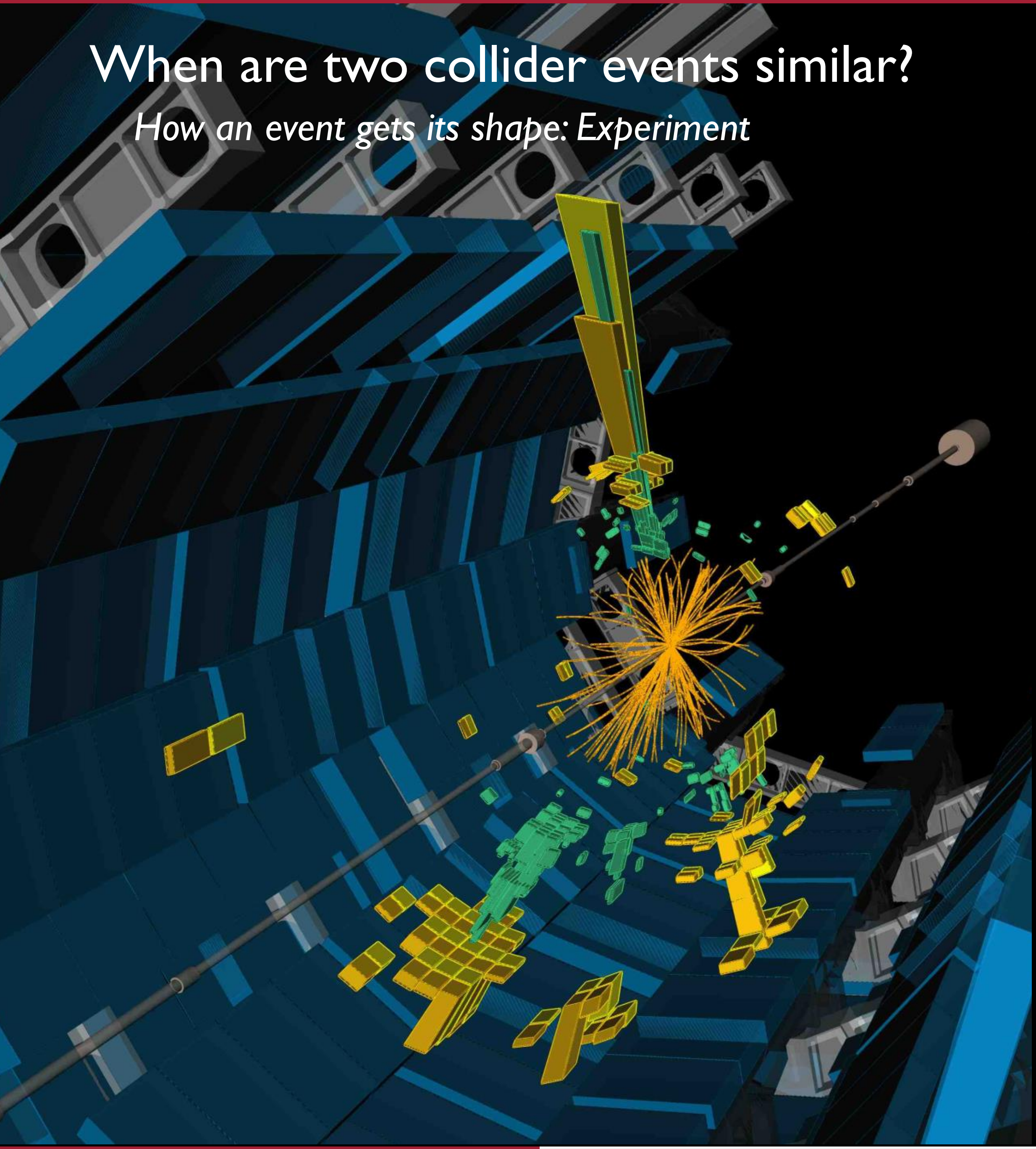


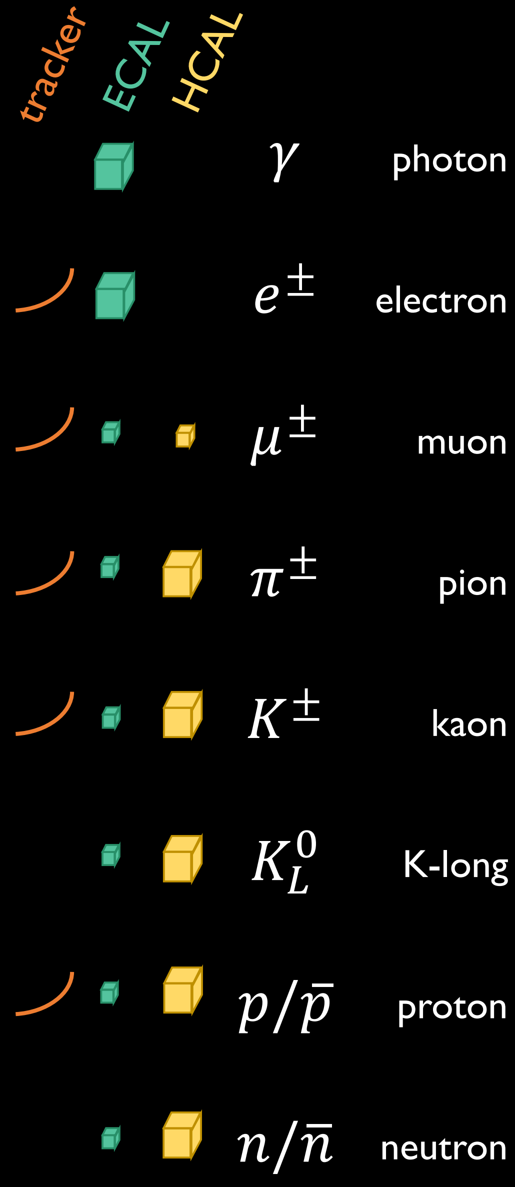
[A. Larkoski, 1709.06195]



When are two collider events similar?

How an event gets its shape: Experiment





Pileup Mitigation with PUMML