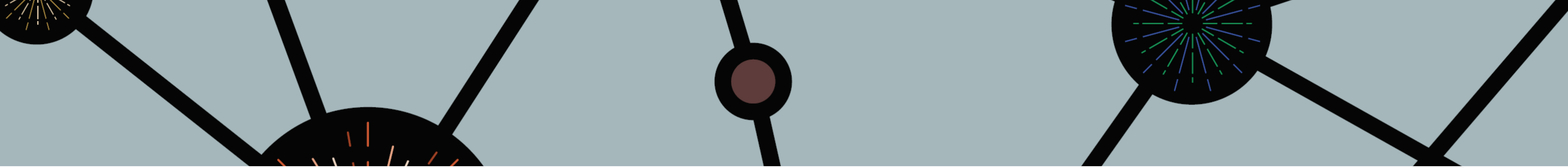


The Hidden Geometry of Particle Collisions

ATLAS Jet & Photon Physics Subgroup

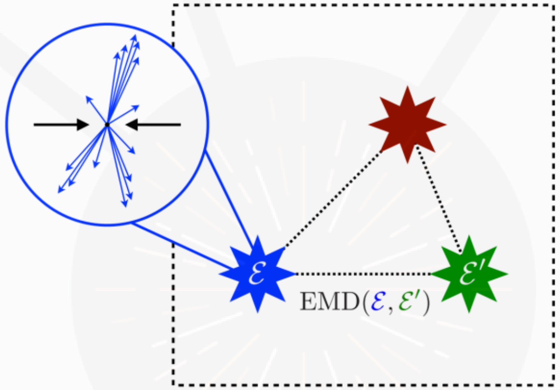
May 25, 2020

Eric M. Metodiev

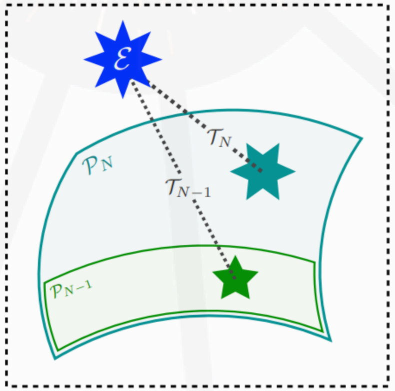
Center for Theoretical Physics, MIT

Based on work with Patrick Komiske and Jesse Thaler [2004.04159]

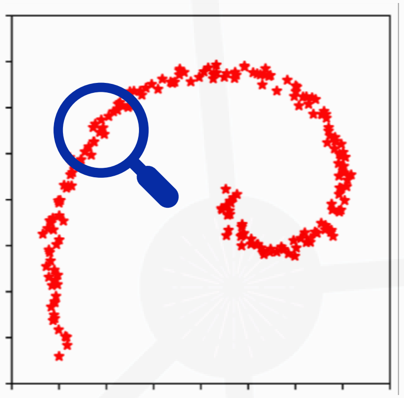
[Image: Chelsea Turner, MIT News]



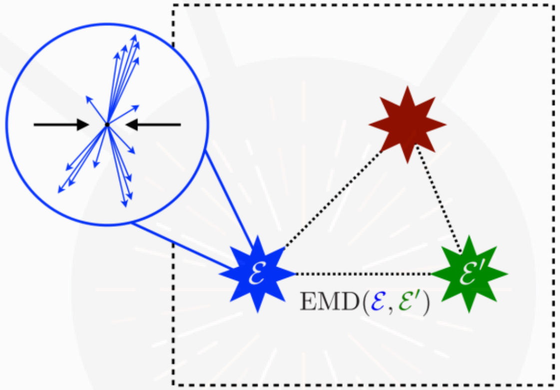
The Space of Collider Events
Building a Metric for Particle Collisions



Unifying Ideas in Collider Physics
Observables, Jets, and Pileup as Geometry



Enabling New Directions
The Fractal Dimension of QCD



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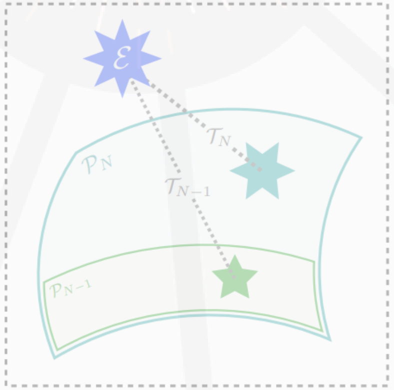

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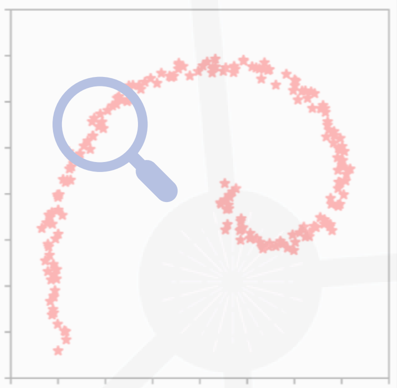
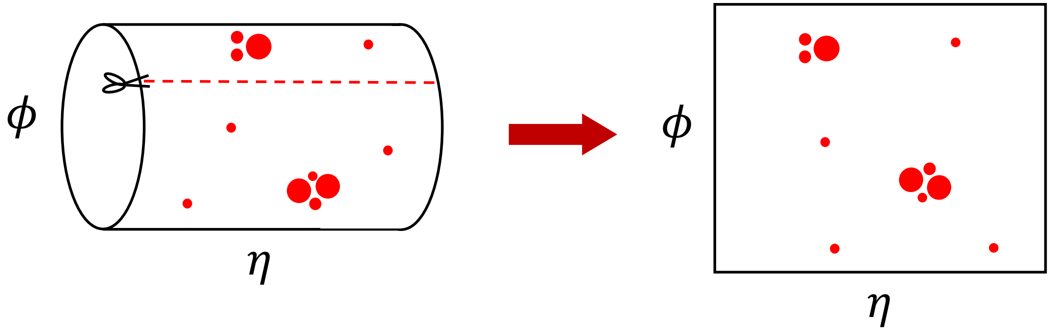


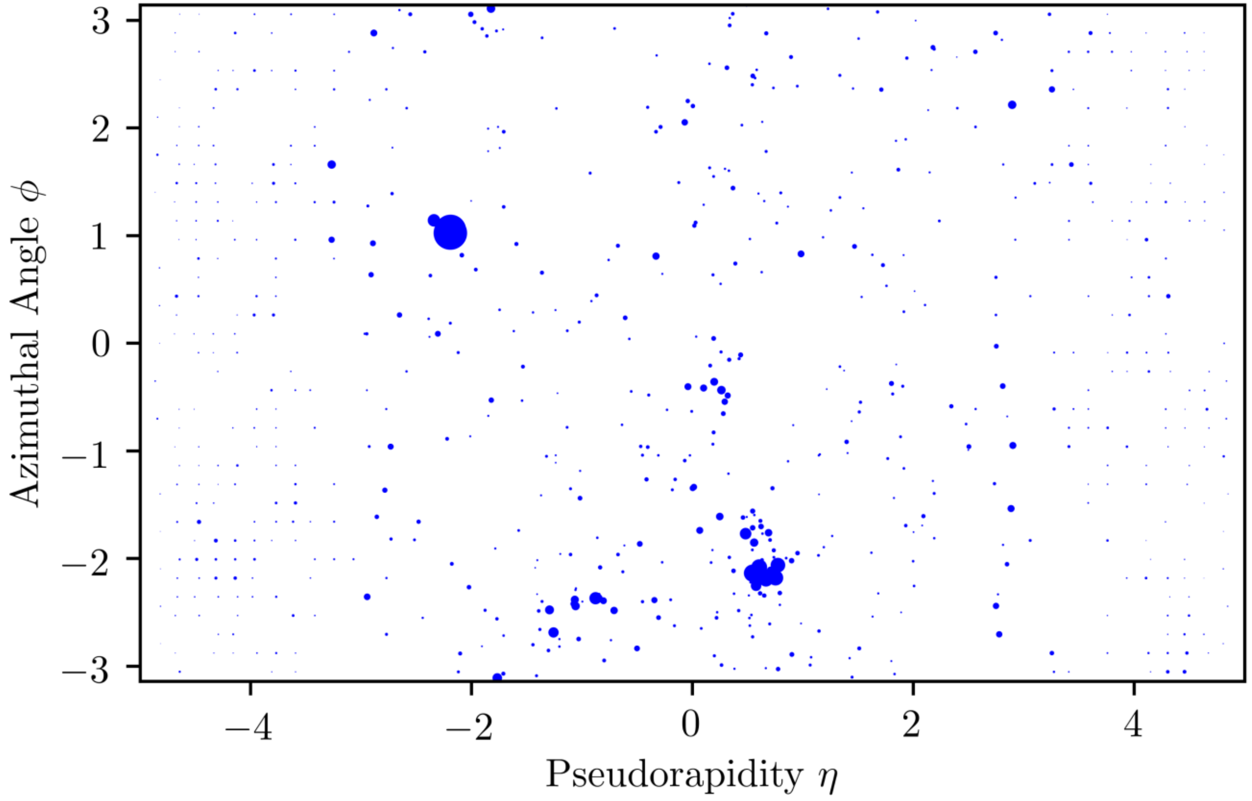
Enabling New Directions
The Fractal Dimension of QCD

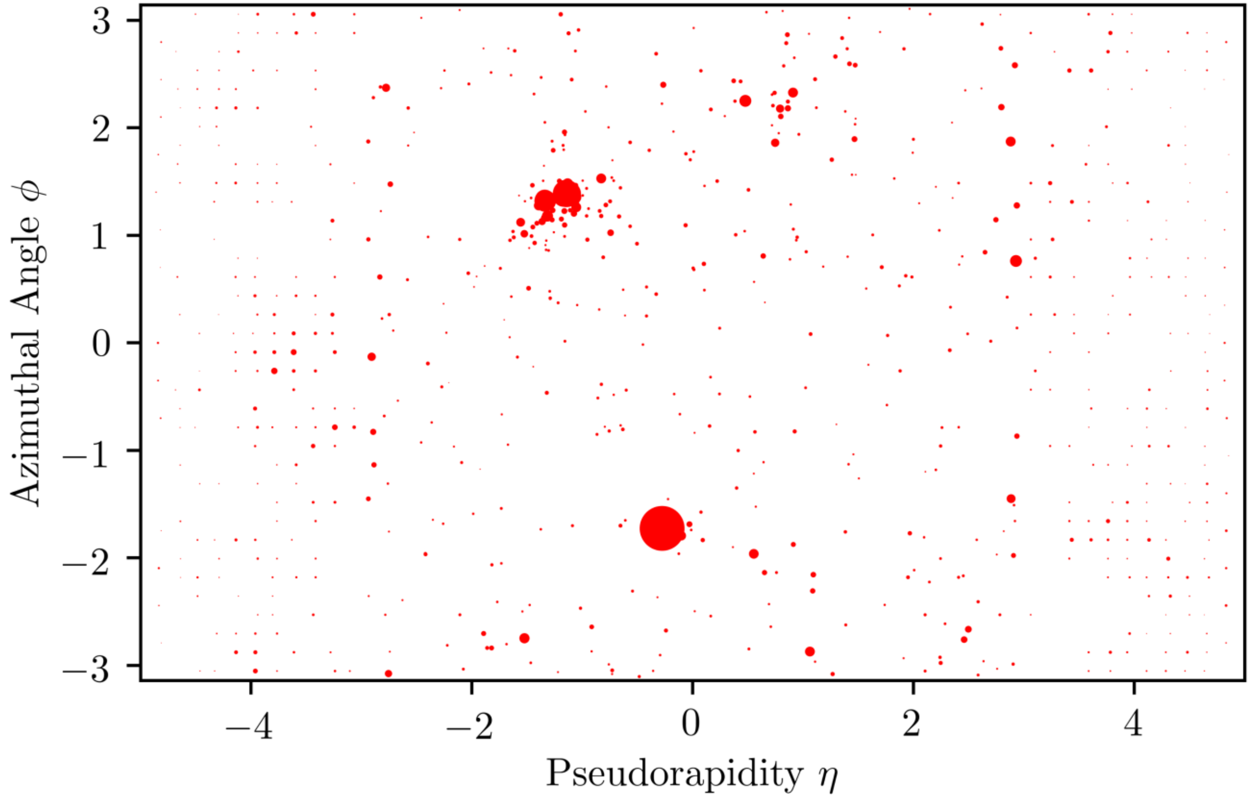
When are two collisions similar?



Infrared and Collinear Safety says distance must be invariant under:

- \+ • Addition of zero-energy particles
- Collinear splitting of one particle into two



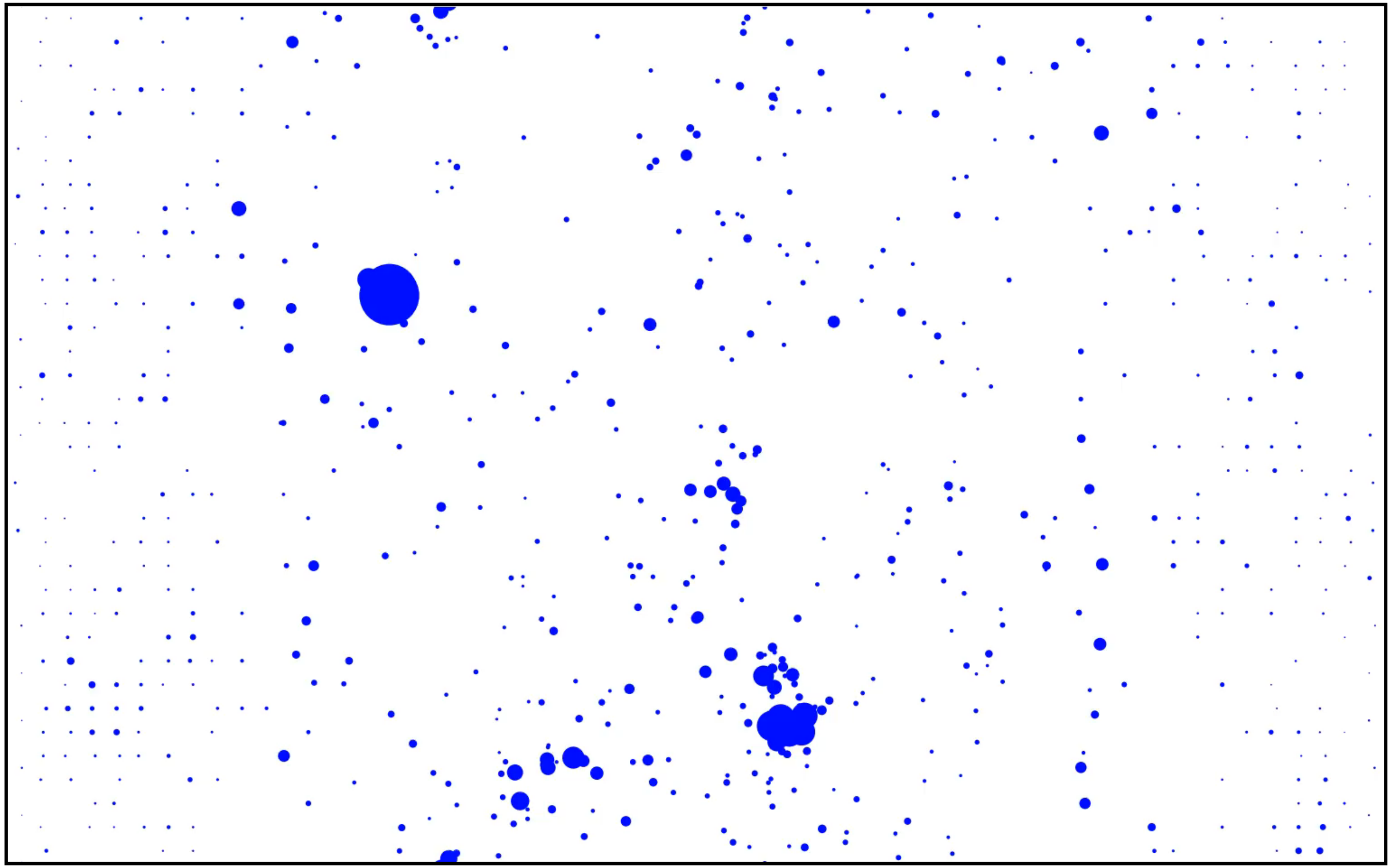


Dijet events from 2011 CMS Open Data – Particle Flow Candidates.

When are two collisions similar?

The Energy Mover's Distance (EMD)

[Komiske, **EMM**, Thaler, PRL, 1902.02346]



The "work" required to rearrange one collision event into another.

Plus a cost to create or destroy energy.

Infrared and collinear safe notion of distance!

Deeply related to the event "energy flow"

$$\mathcal{E}(\hat{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt\, \hat{n}_i T^{0i}(t, r\hat{n})$$

[Sveshnikov, Tkachov, PLB, 9512370]
[Tkachov, IJMP, 9601308]

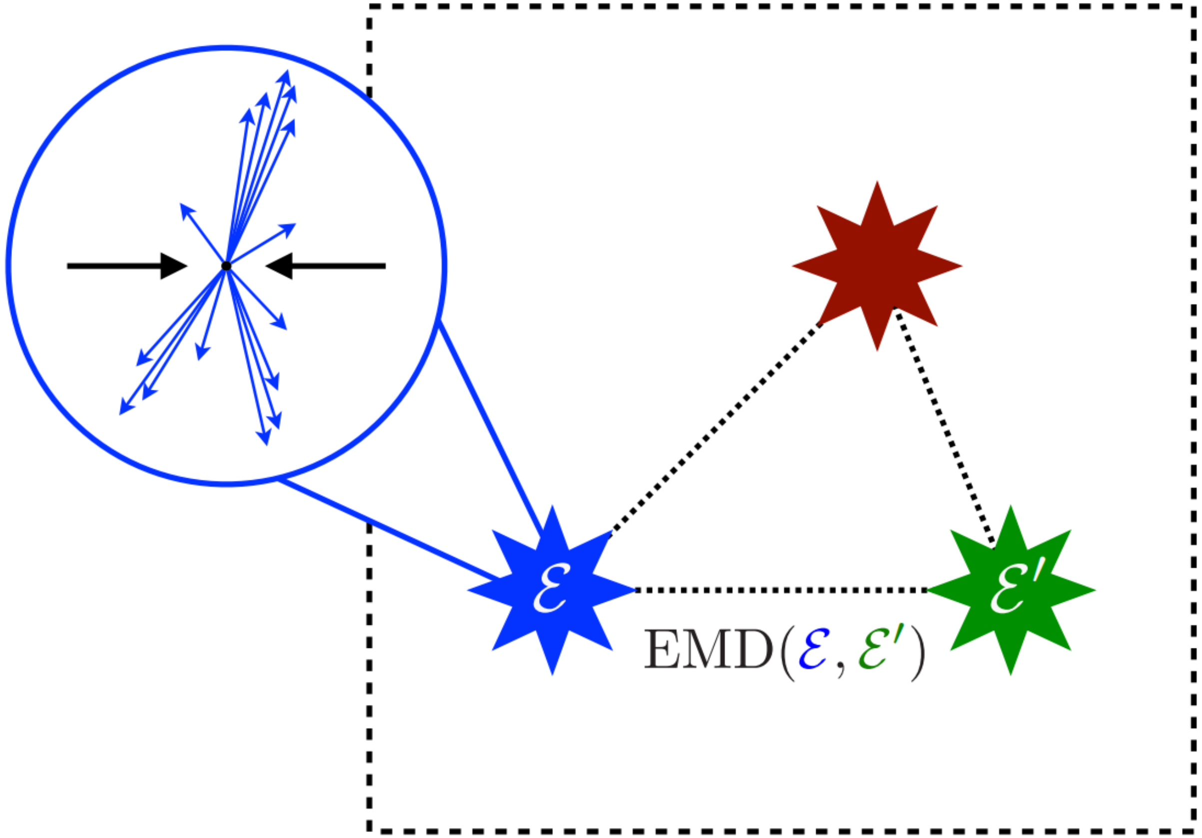
Based on the Earth Mover's or Wasserstein Distance

[Peleg, Werman, Rom, PAMI,1989]
[Rubner, Tomasi, Guibas, IJCV, 2000]

Optimal Transport Problem

python optimal transport library

The Space of Collider Events



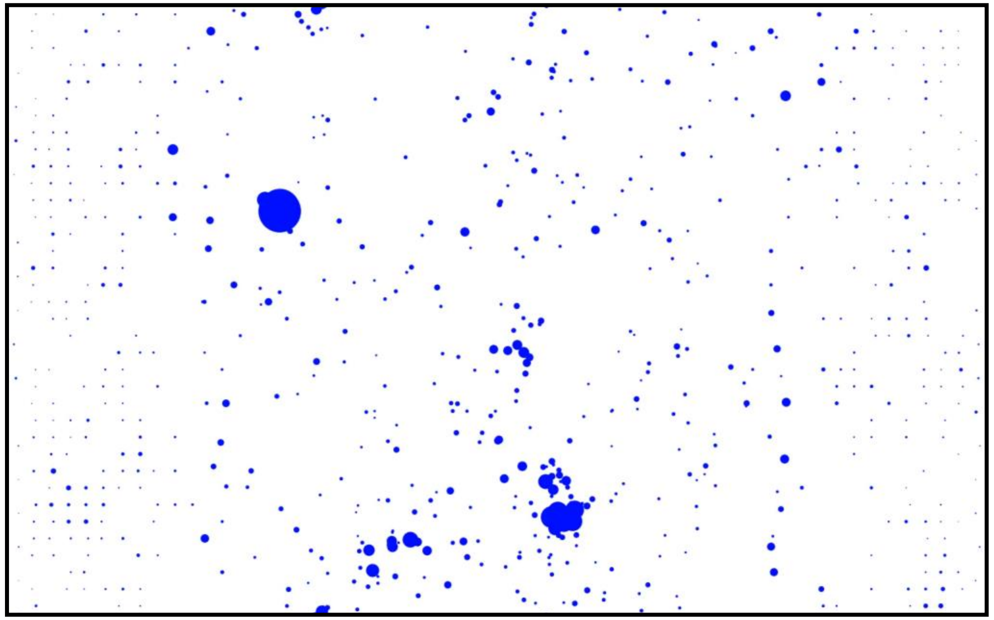
Difference in radiation pattern

Difference in total energy

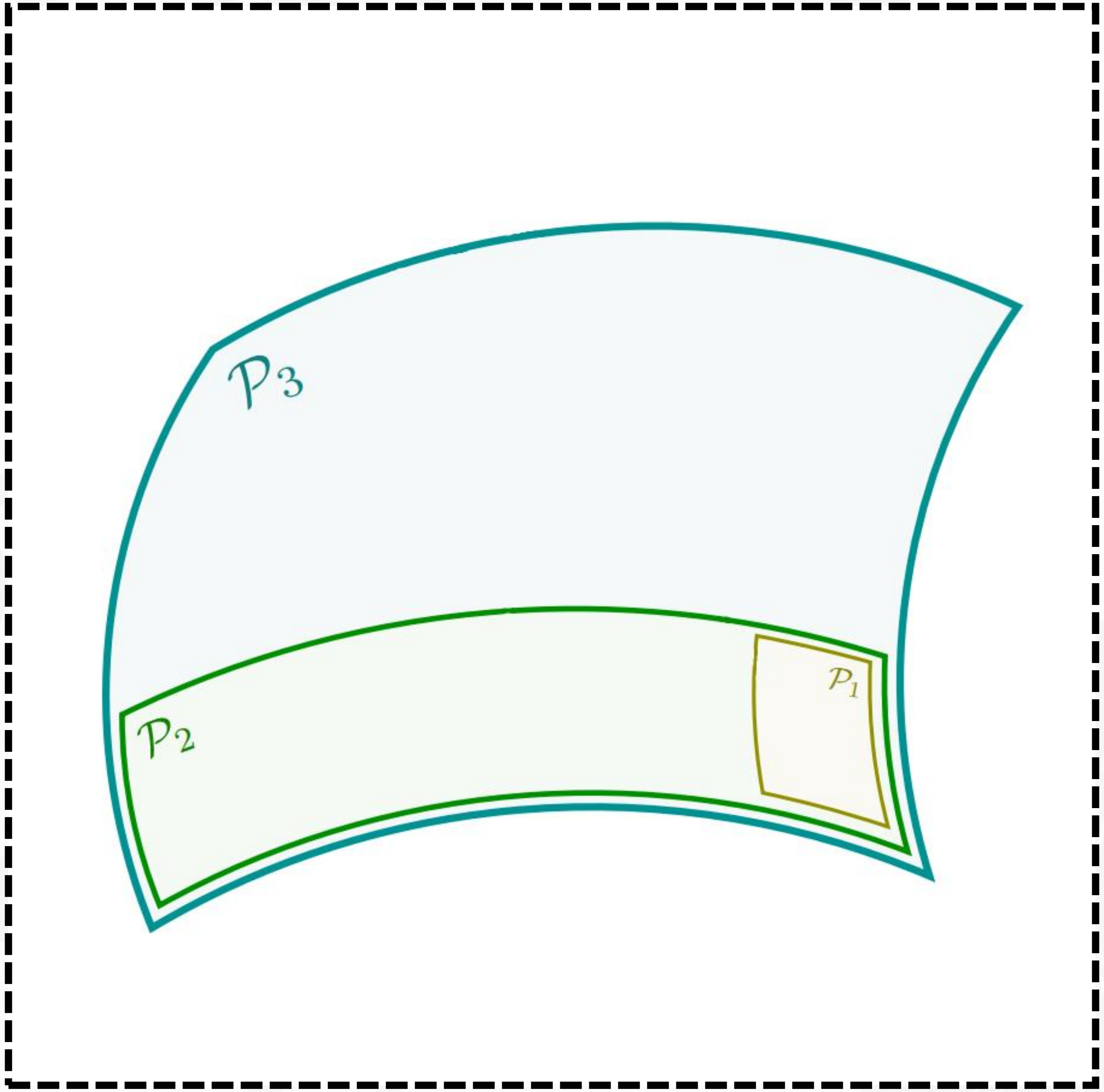
$$\mathrm{EMD}_{\beta,\mathrm{R}}(\mathcal{E},\mathcal{E}') = \min_{\{f_{ij}\}} \sum_{i=1}^{M}\sum_{j=1}^{M'} f_{ij}\frac{\theta_{ij}^{\beta}}{R^{\beta}} + \left|\sum_{i=1}^{M} E_i - \sum_{j=1}^{M'} E'_j\right|$$

β : angular weighting factor

R : tradeoff between moving energy and creating it



The Space of Collider Events



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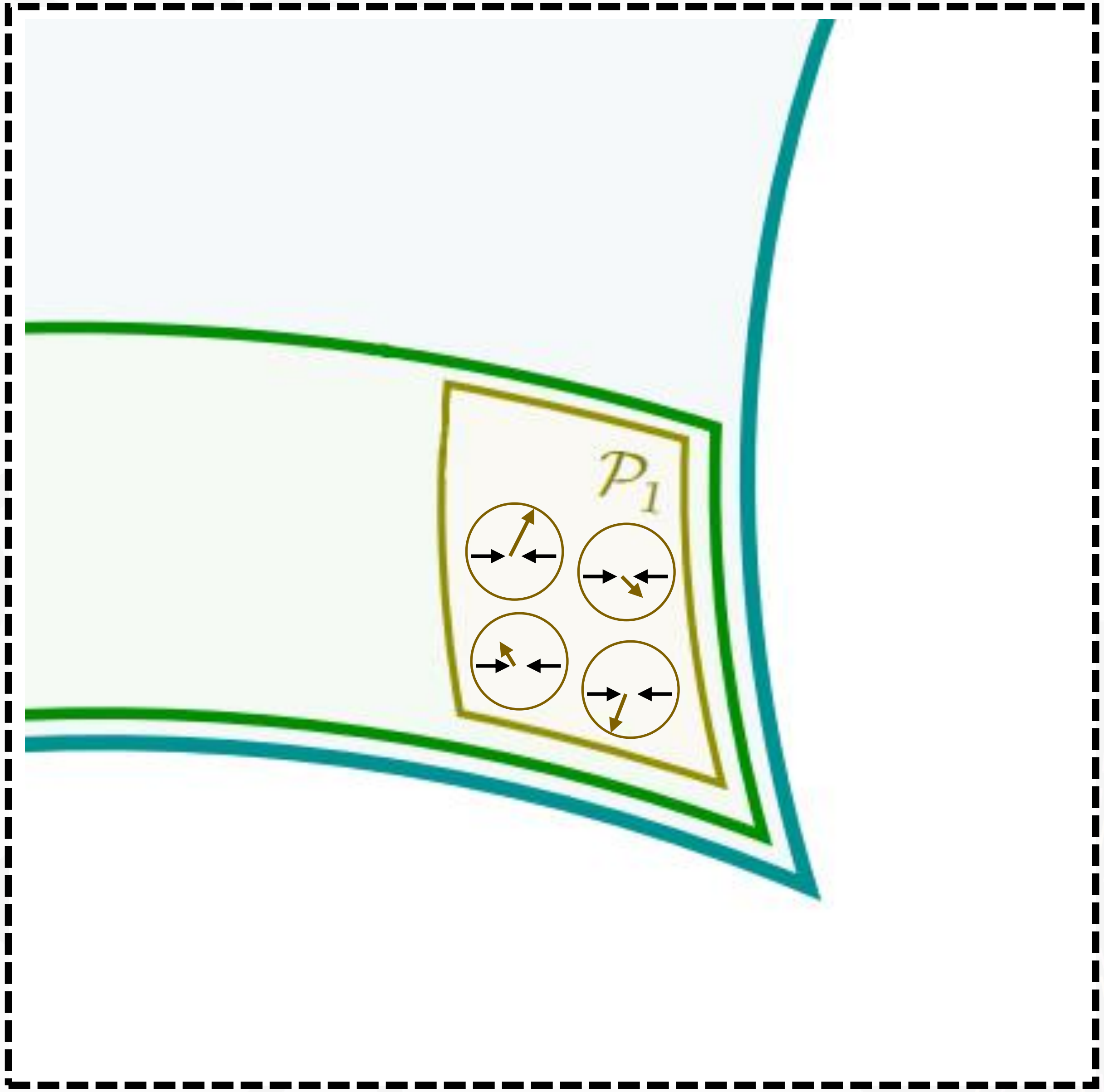
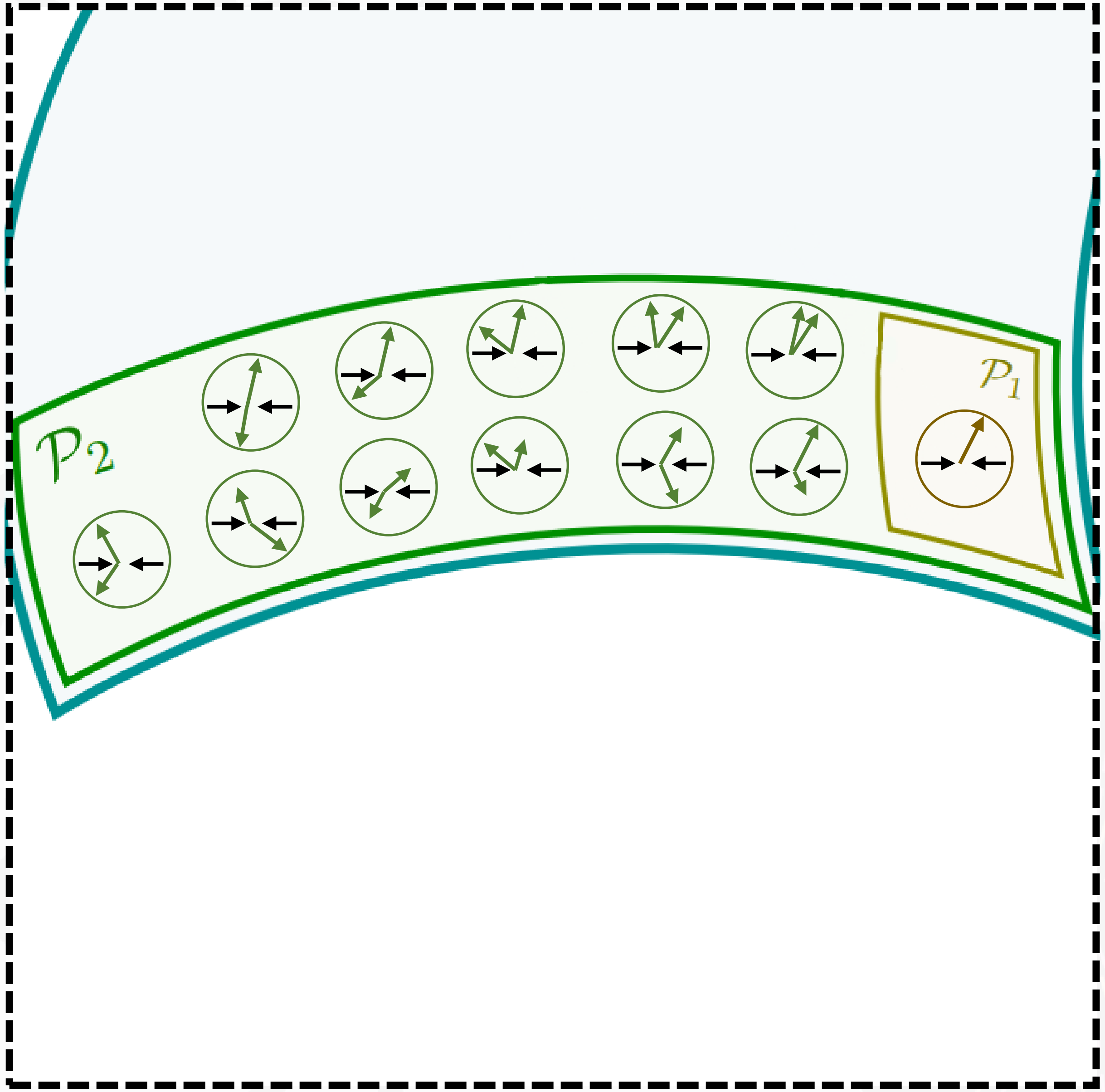


$\mathcal{P}_1$: Manifold of 1 particle events

All events consisting of a single particle

The Space of Collider Events



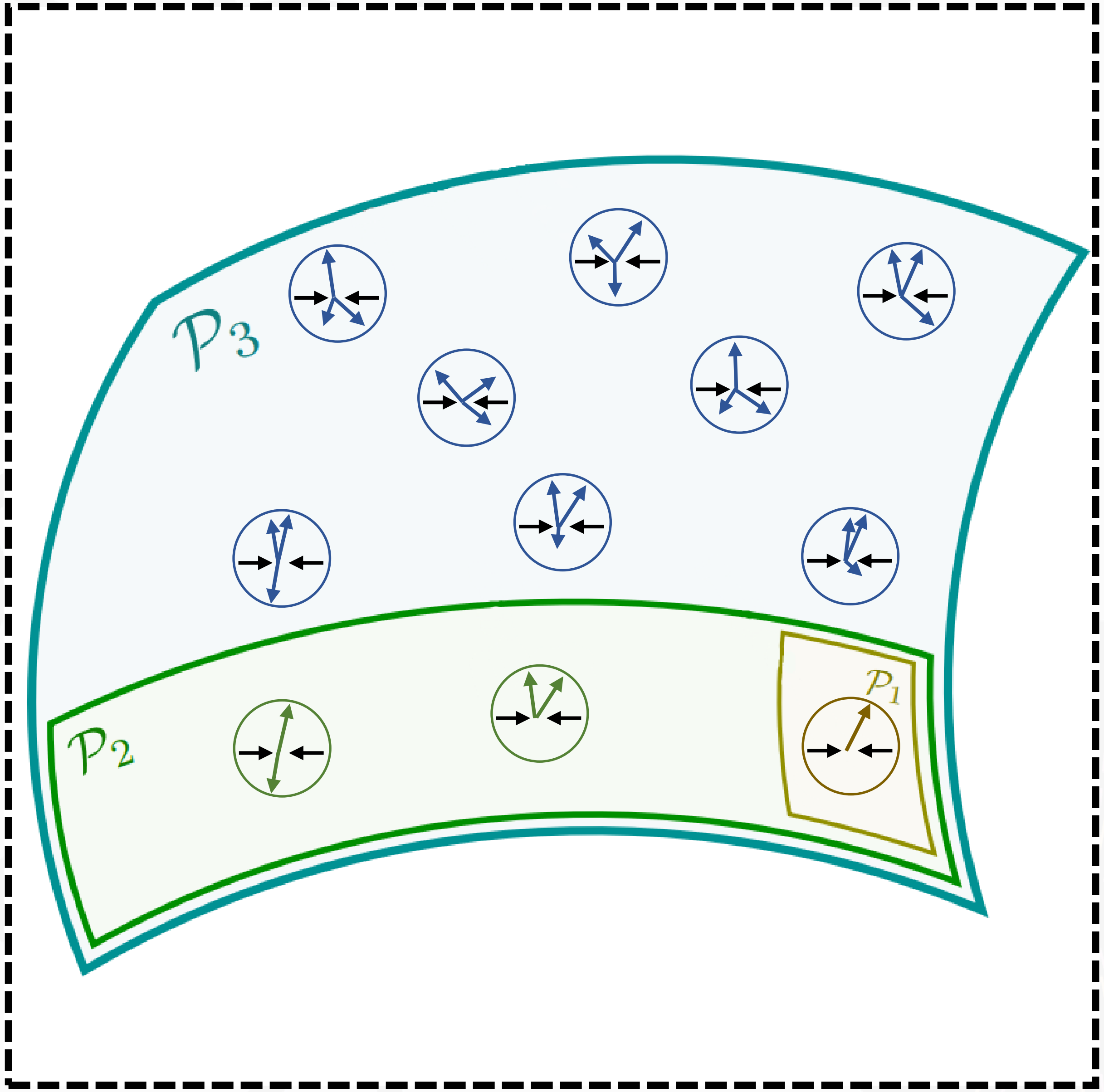
$\mathcal{P}_1$: Manifold of 1 particle events

$\mathcal{P}_2$: Manifold of 2 particle events
All events consisting of two particles

When a particle becomes **soft** or **collinear**, we recover the 1 particle manifold

$$\mathcal{P}_1 \subset \mathcal{P}_2$$

The Space of Collider Events



$\mathcal{P}_1$: Manifold of 1 particle events

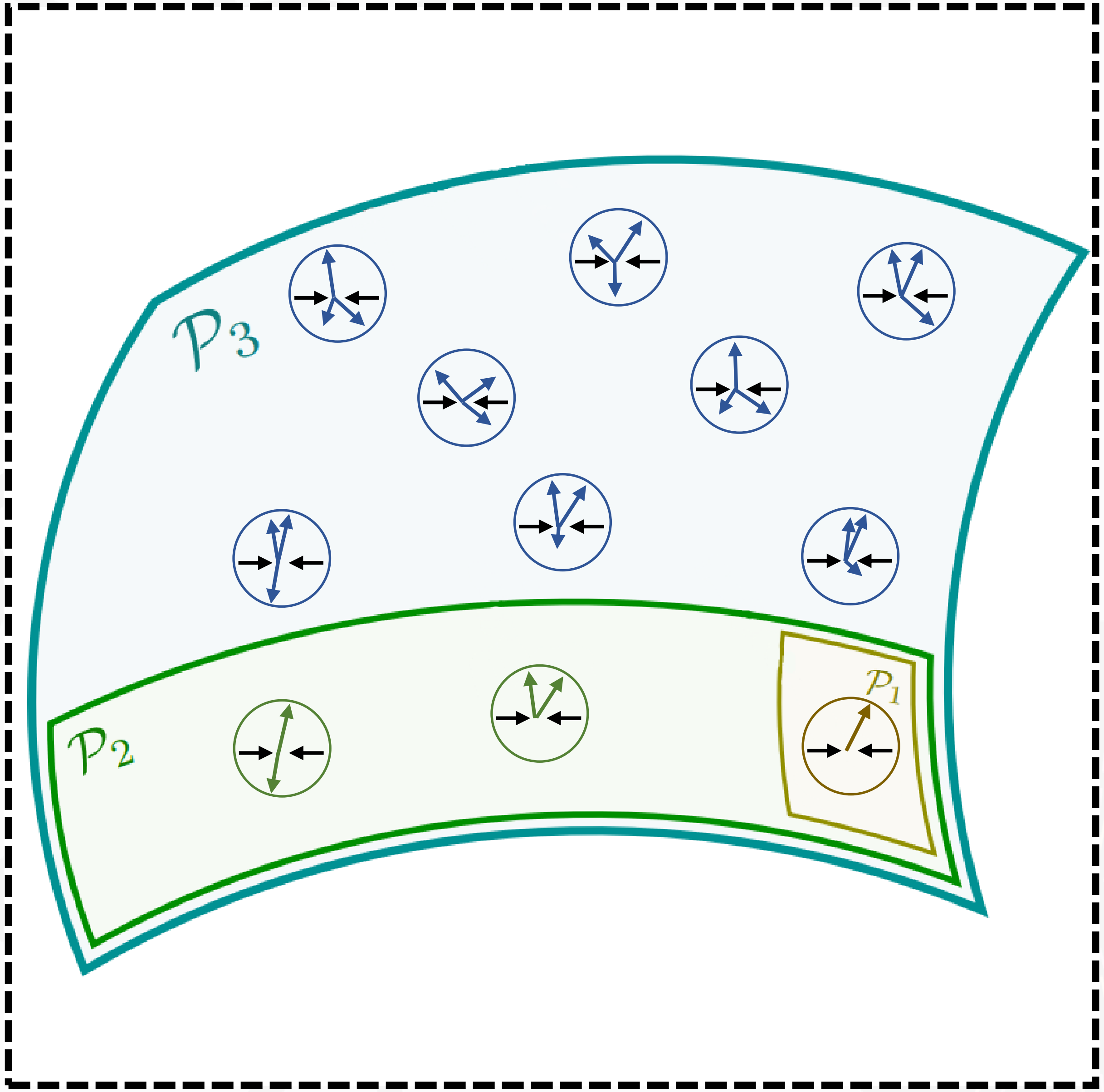
$\mathcal{P}_2$: Manifold of 2 particle events

$\mathcal{P}_3$: **Manifold of 3 particle events**
All events consisting of three particles

When a particle becomes **soft** or **collinear**, we recover the 2-particle manifold

$\mathcal{P}_1 \subset \mathcal{P}_2 \subset \mathcal{P}_3$

The Space of Collider Events



$\mathcal{P}_1$: Manifold of 1 particle events

$\mathcal{P}_2$: Manifold of 2 particle events

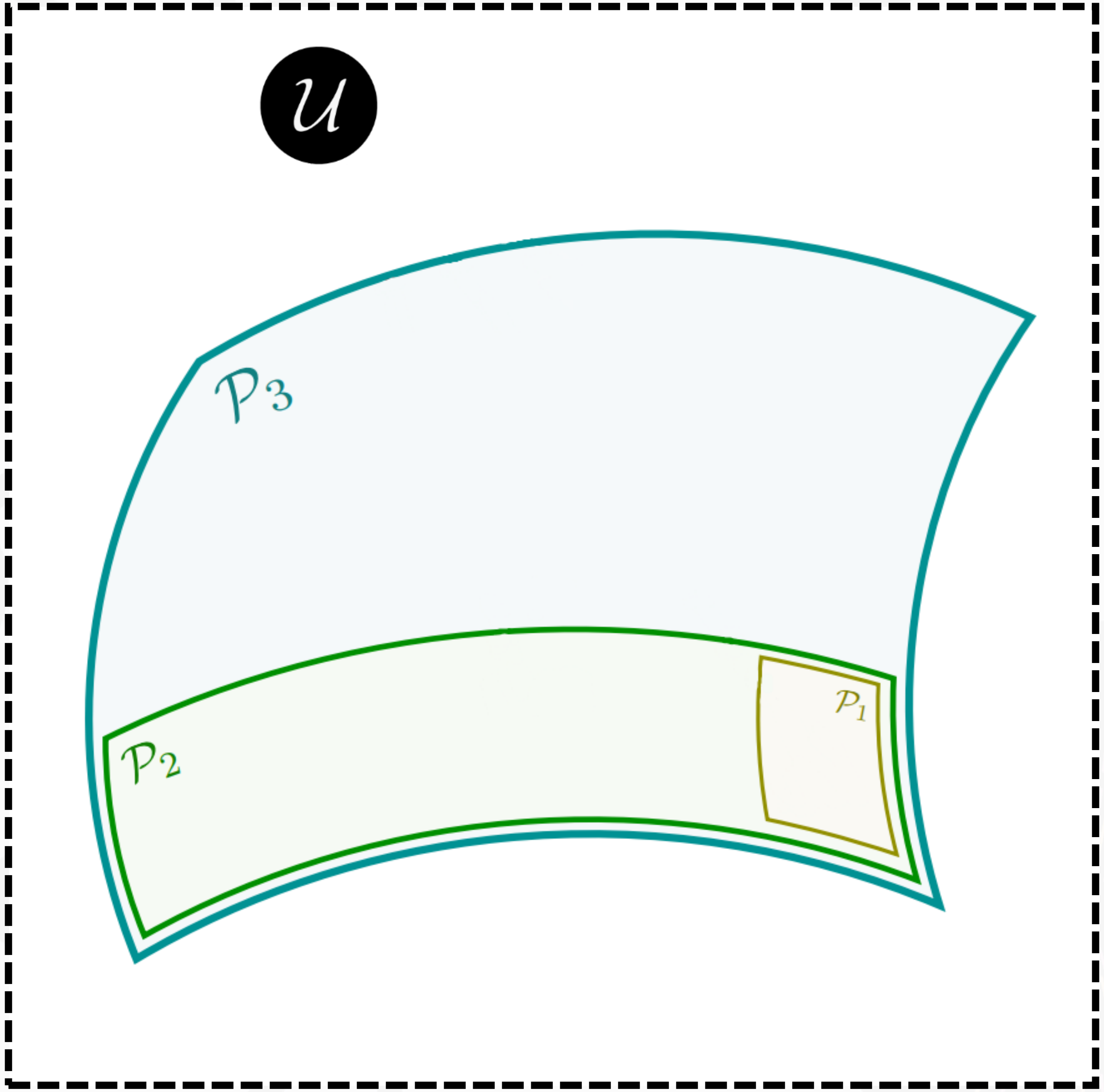
$\mathcal{P}_3$: Manifold of 3 particle events

⋮

$\mathcal{P}_N$: Manifold of N particle events
All events consisting of N particles

$$\mathcal{P}_1 \subset \mathcal{P}_2 \subset \mathcal{P}_3 \subset \cdots \subset \mathcal{P}_N$$

The Space of Collider Events



$\mathcal{P}_1$: Manifold of 1 particle events

$\mathcal{P}_2$: Manifold of 2 particle events

$\mathcal{P}_3$: Manifold of 3 particle events

⋮

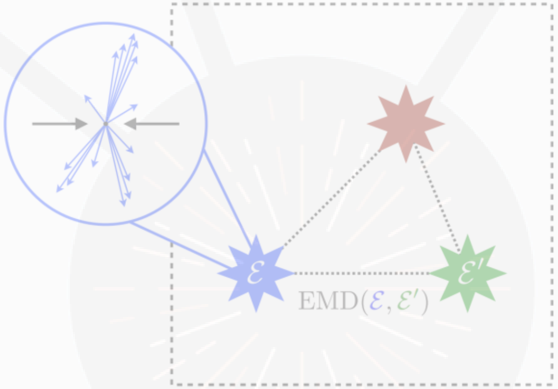
$\mathcal{P}_N$: Manifold of N particle events

$\mathcal{U}$: Uniform event

Plus many more!

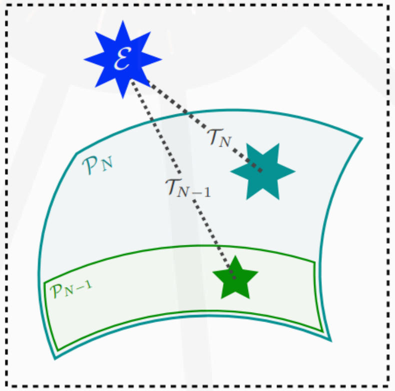
$$\mathcal{P}_1 \subset \mathcal{P}_2 \subset \mathcal{P}_3 \subset \cdots \subset \mathcal{P}_N$$

$$\mathcal{U} \not\subset \mathcal{P}_N$$



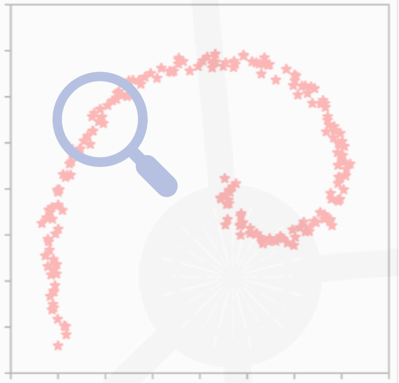
The Space of Collider Events

Building a Metric for Particle Collisions



Unifying Ideas in Collider Physics

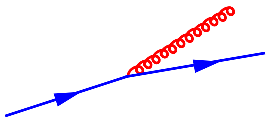
Observables, Jets, and Pileup as Geometry



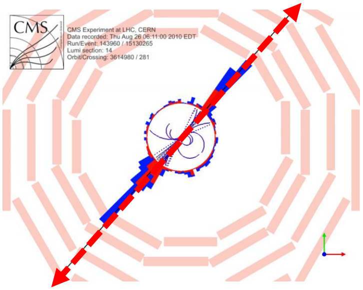
Enabling New Directions

The Fractal Dimension of QCD

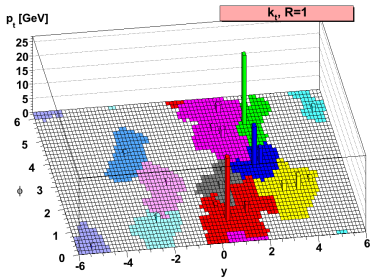
Six Decades of Collider Techniques



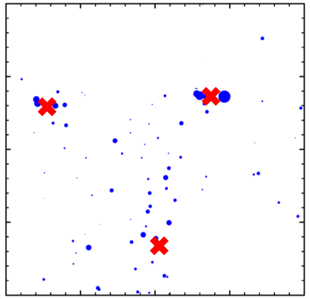
Taming infinities



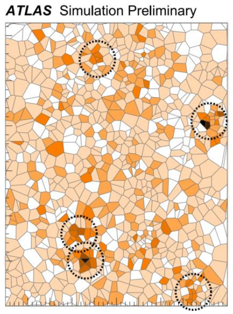
Event Shapes



Jet Algorithms



Jet Substructure



Pileup

1960

1962-1964
Infrared Safety
[Kinoshita, JMP 1962]
[Lee, Nauenberg, PR 1964]

1977
Thrust, Sphericity
[Farhi, PRL 1977]
[Georgi, Machacek, PRL 1977]

1993
k_T jet clustering
[Ellis, Soper, PRD 1993]
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1997-1998
C/A jet clustering
[Wobisch, Wengler, 1998]
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2010-2015
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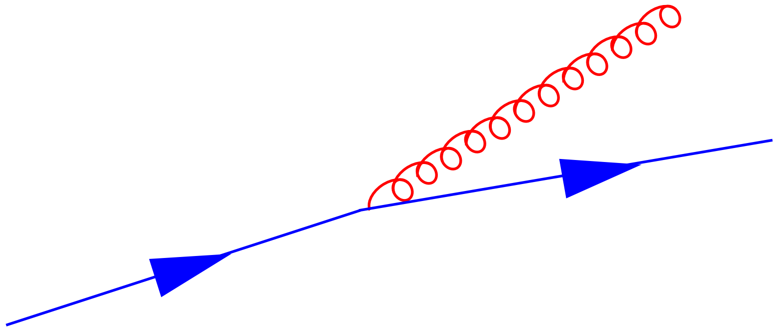
2014-2019
Constituent Subtraction
[Berta, Spousta, Miller, Leitner, JHEP 2014]
[Berta, Masetti, Miller, Spousta, JHEP 2019]

And many more!

2020

Six Decades of Collider Techniques

Infrared and Collinear Safety is robustness to low energy additions and collinear splittings.



$$\mathcal{O}(p_1^\mu, \ldots, p_M^\mu) = \lim_{\varepsilon \to 0} \mathcal{O}(\varepsilon p_0^\mu, p_1^\mu, \ldots, p_M^\mu)$$

$$\mathcal{O}(p_1^\mu, \ldots, p_M^\mu) = \lim_{p_0^\mu \to p_1^\mu} \mathcal{O}(\lambda\, p_0^\mu, (1-\lambda) p_1^\mu, \ldots, p_M^\mu)$$

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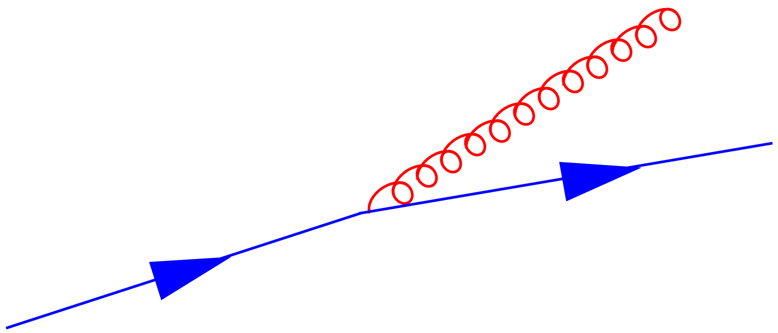
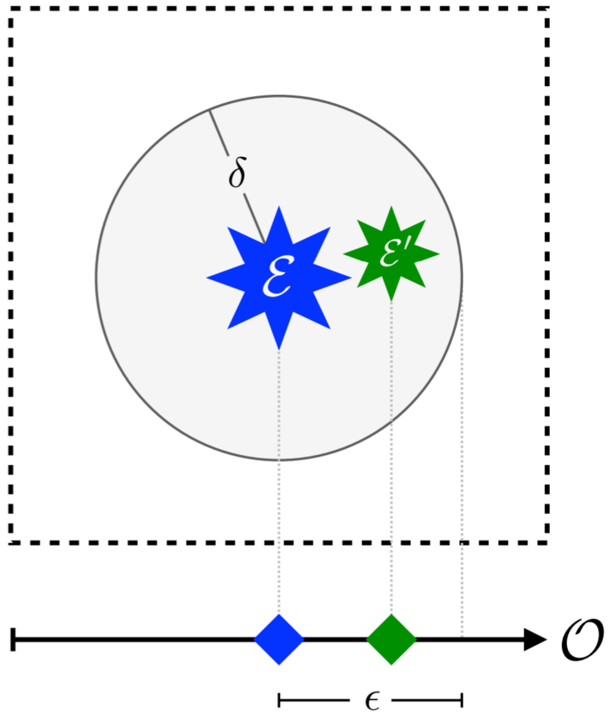
And many more!

Six Decades of Collider Techniques as Geometry!

Infrared and Collinear Safety is continuity in the space of events.





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$$\text{EMD}\,(\mathcal{E},\mathcal{E}') < \delta \Rightarrow |\mathcal{O}(\mathcal{E}) - \mathcal{O}(\mathcal{E}')| < \epsilon$$

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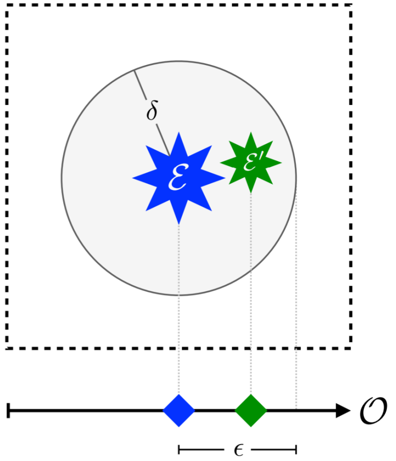
Six Decades of Collider Techniques as Geometry!

IRC Safety is smoothness in the space of events



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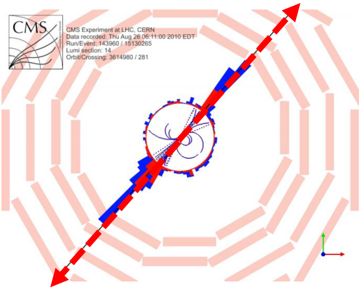

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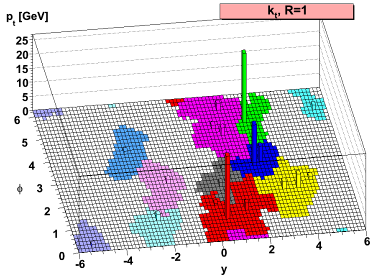

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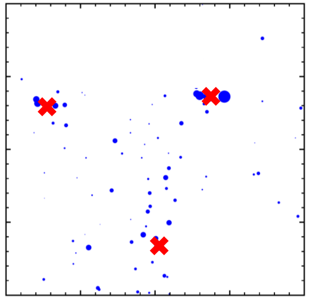

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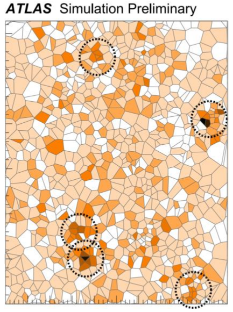

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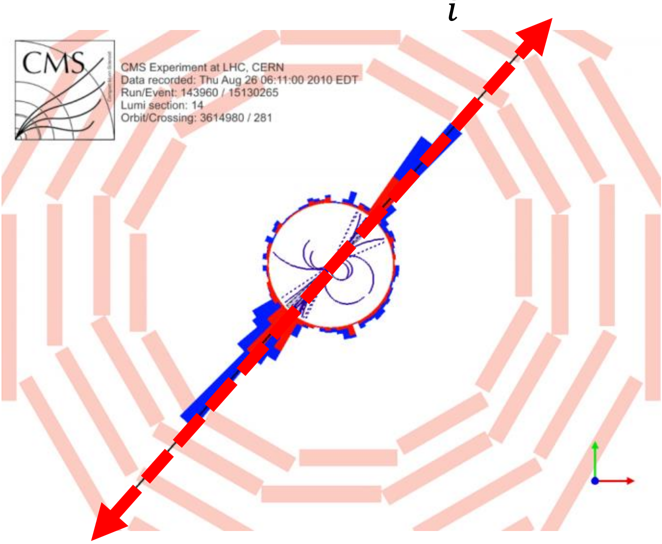
And many more!

[Farhi, PRL., 1977]

Six Decades of Collider Techniques as Geometry!

Thrust is a classic event shape that measures how pencil-like an event is.

$$t(\mathcal{E}) = E - \max_{\hat{n}} \sum_i |\vec{p}_i \cdot \hat{n}|$$



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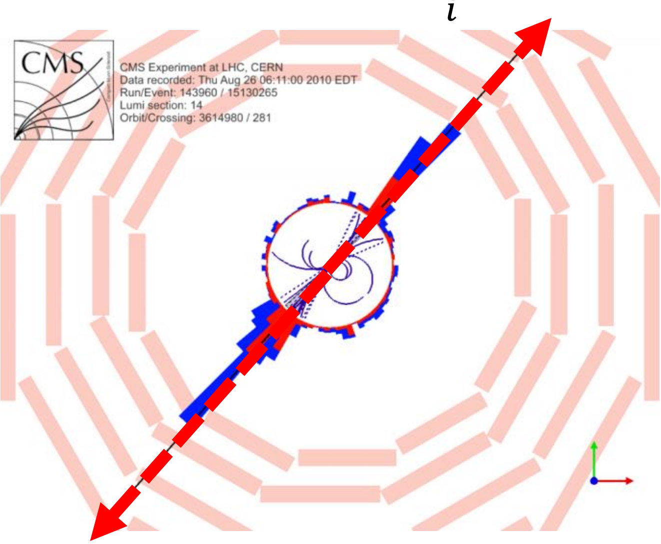
And many more!

Six Decades of Collider Techniques as Geometry!

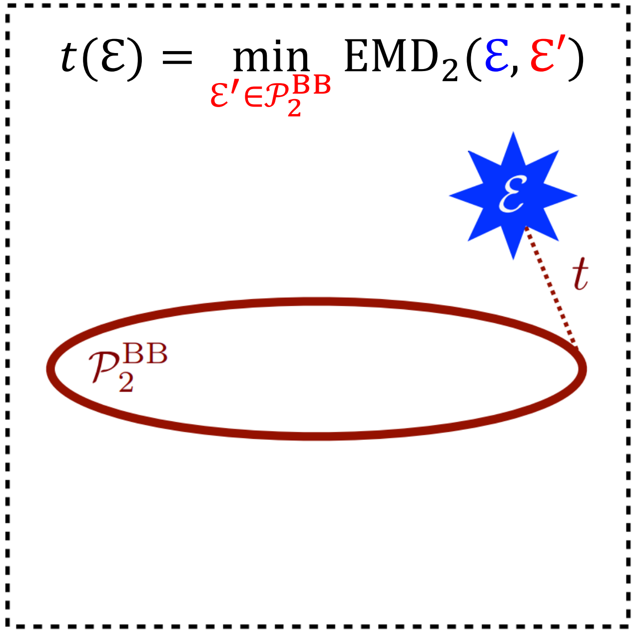
[Farhi, PRL., 1977]
[Komiske, **EMM**, Thaler, 2004.04159]

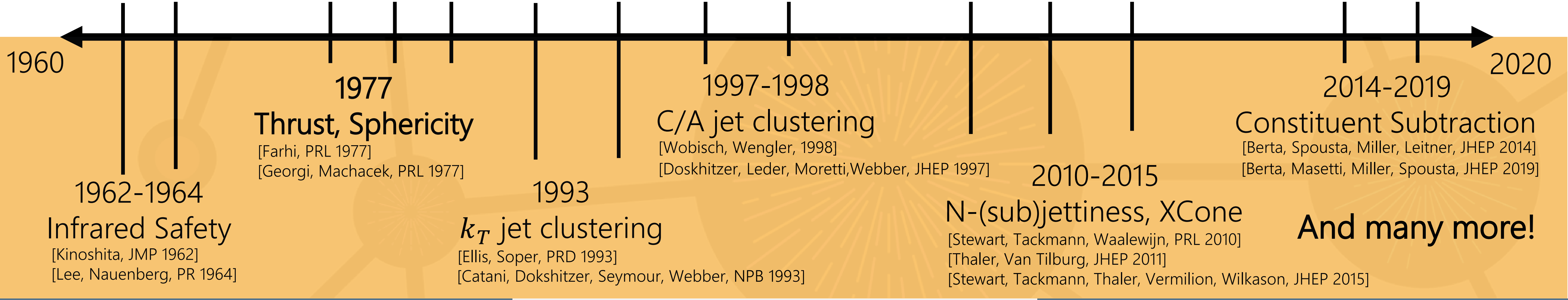
Thrust is the EMD between the event and the closest two-particle back-to-back event.

$$t(\mathcal{E}) = E - \max_{\hat{n}} \sum_i |\vec{p}_i \cdot \hat{n}|$$



$$t(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{P}_2^{\mathrm{BB}}} \mathrm{EMD}_2(\mathcal{E}, \mathcal{E}')$$





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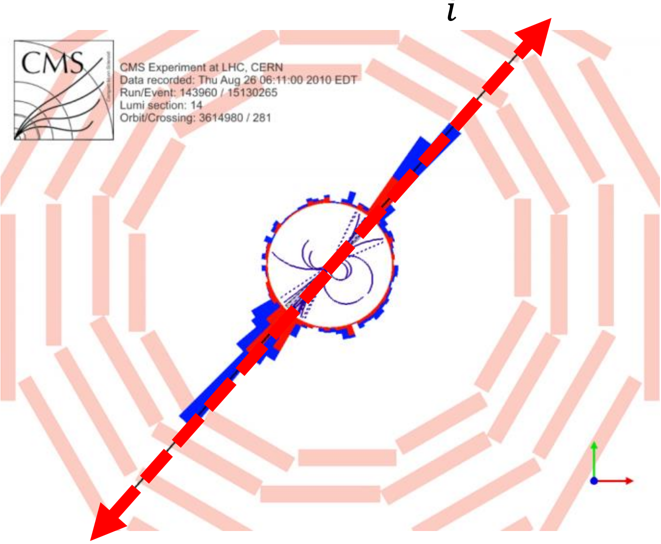

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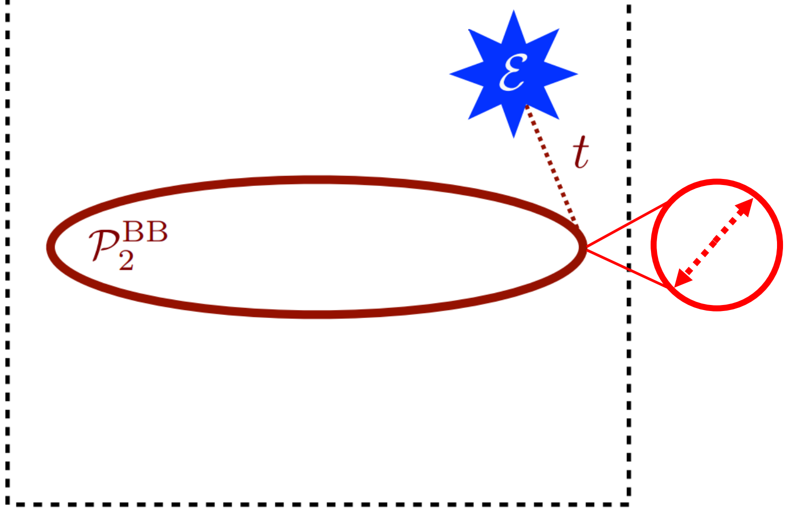

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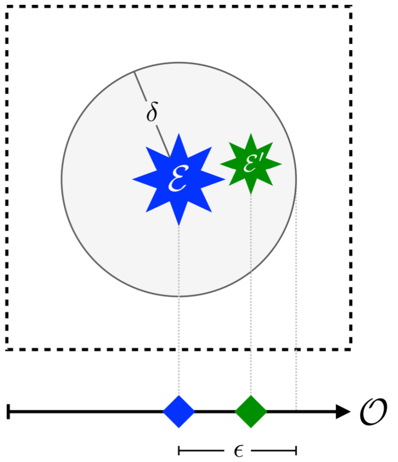
Six Decades of Collider Techniques as Geometry!

IRC Safety is smoothness in the space of events



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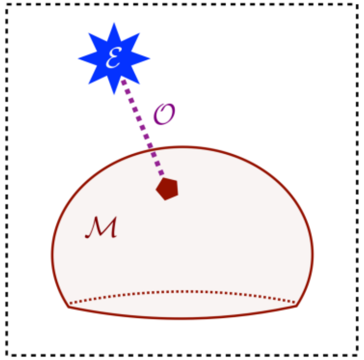

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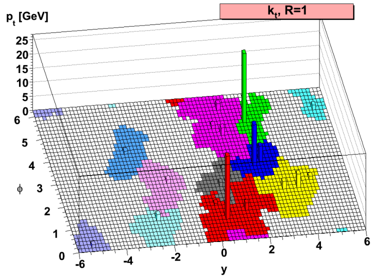

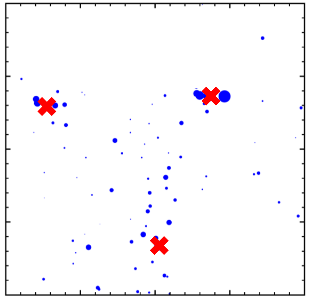

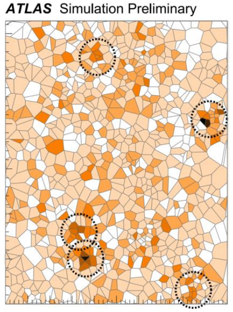

Taming infinities | Event Shapes | Jet Algorithms | Jet Substructure | Pileup

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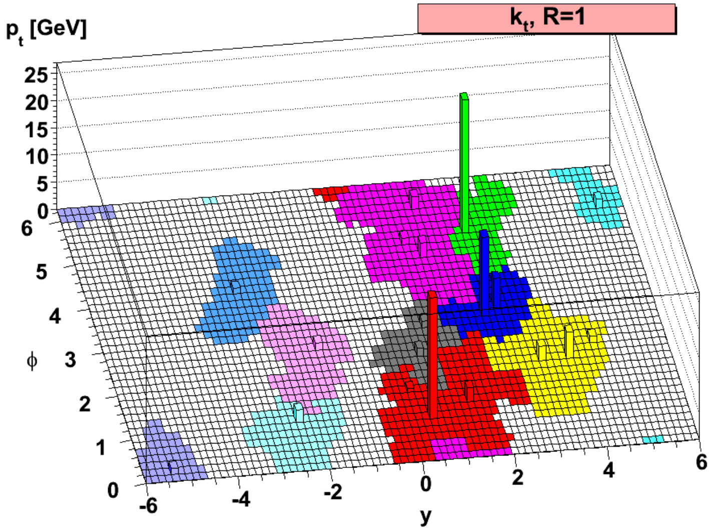
And many more!

Six Decades of Collider Techniques as Geometry!

The $\boldsymbol{k_T}$ **algorithm** sequentially merges the closest particles to cluster the event into jets.

$$d_{ij} = \min(E_i, E_j)\ \theta_{ij}$$



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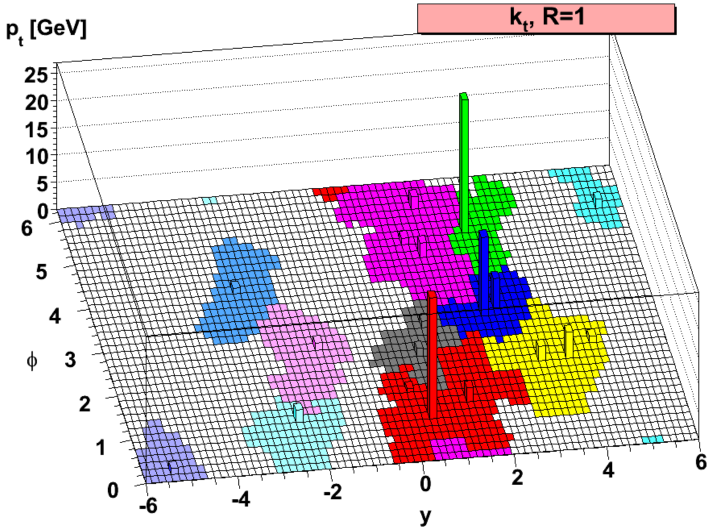
2020

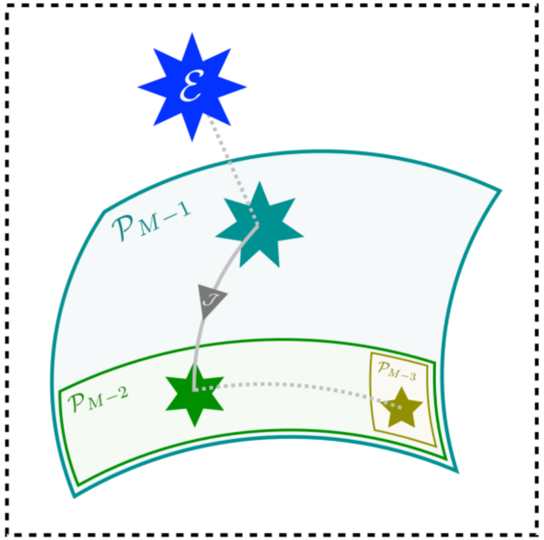
Six Decades of Collider Techniques as Geometry!

The $\boldsymbol{k_T}$ **algorithm** sequentially projects an M-particle event to the $M-1$-particle manifold.

$$d_{ij} = \min(E_i, E_j)\,\theta_{ij}$$



$$\mathcal{J} = \underset{\mathcal{E}' \in \mathcal{P}_{M-1}}{\operatorname{argmin}} \operatorname{EMD}_{\beta,R}(\mathcal{E}, \mathcal{E}')$$



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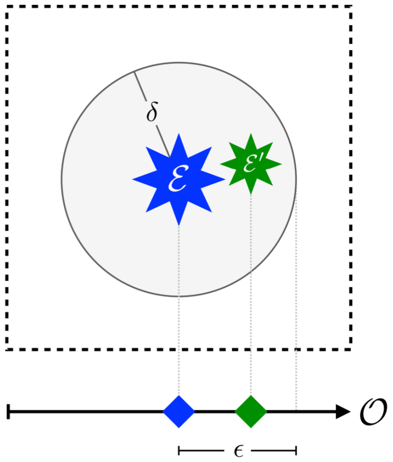

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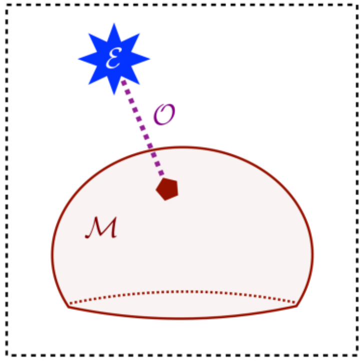
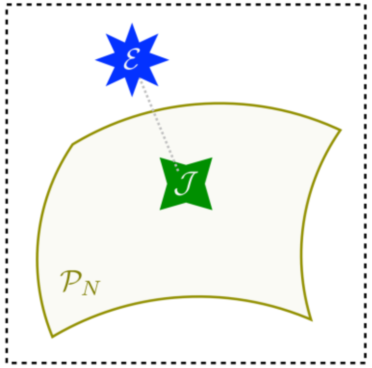
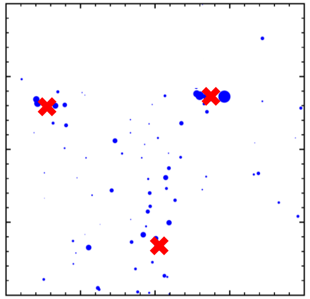


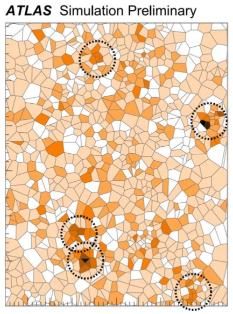
$$\mathcal{O}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{M}} \text{EMD}_{\beta,R}(\mathcal{E}, \mathcal{E}')$$

Jets are projections to few-particle manifolds.



$$\mathcal{J} = \underset{\mathcal{E}' \in \mathcal{P}_N}{\text{argmin}}\, \text{EMD}_{\beta,R}(\mathcal{E}, \mathcal{E}')$$





Taming infinities | Event Shapes | Jet Algorithms | Jet Substructure | Pileup

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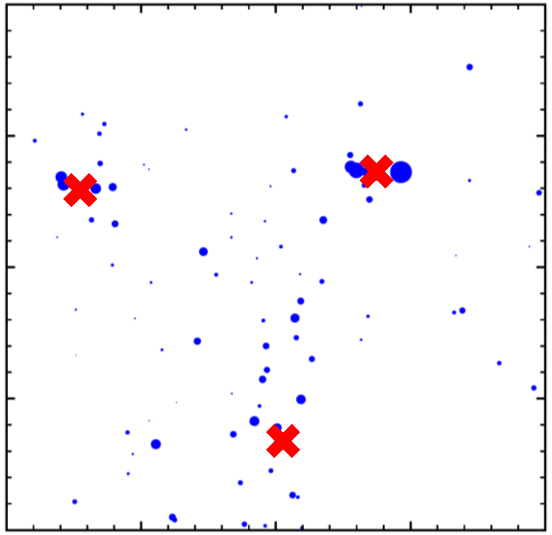
And many more!

Six Decades of Collider Techniques as Geometry!

***N*-subjettiness** probes how aligned a jet is along N subjet directions.

$$\tau_N(\mathcal{J}) = \min_{N \text{ axes}} \sum_{i=1}^{M} E_i \min\{\theta_{1,i}^{\beta}, \theta_{2,i}^{\beta}, \ldots, \theta_{N,i}^{\beta}\}$$



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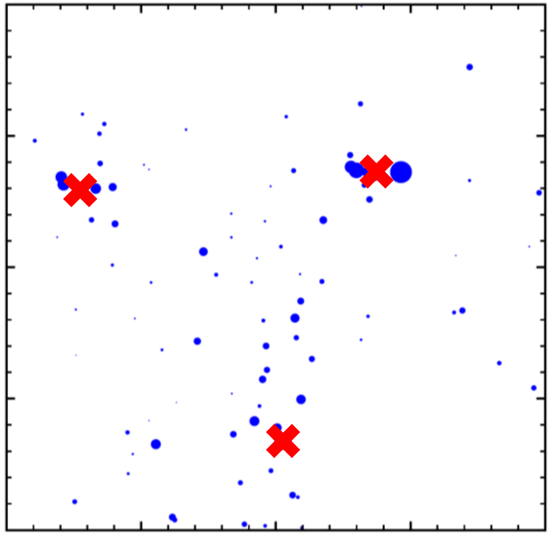
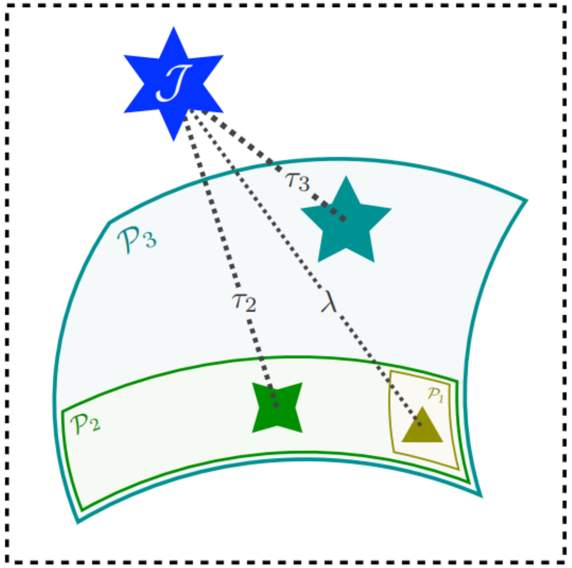
Six Decades of Collider Techniques as Geometry!

***N*-subjettiness** is the EMD from the event to the *N*-particle manifold.

$$\tau_N(\mathcal{J}) = \min_{N \text{ axes}} \sum_{i=1}^{M} E_i \min\{\theta_{1,i}^{\beta}, \theta_{2,i}^{\beta}, \dots, \theta_{N,i}^{\beta}\}$$



$$\tau_N(\mathcal{J}) = \min_{\mathcal{E}' \in \mathcal{P}_N} \text{EMD}_\beta(\mathcal{J}, \mathcal{E}').$$



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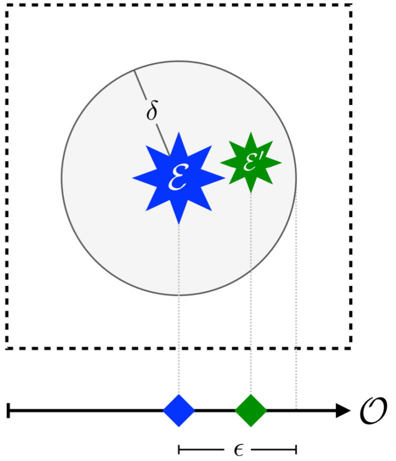
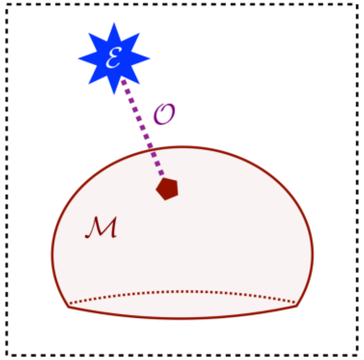
Six Decades of Collider Techniques as Geometry!

IRC Safety is smoothness in the space of events



Taming infinities

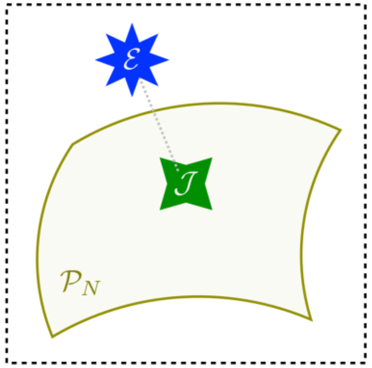
Event shapes are distances from events to manifolds.



$$\mathcal{O}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{M}} \text{EMD}_{\beta,R}(\mathcal{E}, \mathcal{E}')$$

Event Shapes

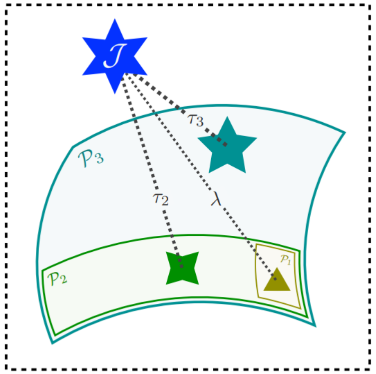
Jets are projections to few-particle manifolds.



$$\mathcal{J} = \underset{\mathcal{E}' \in \mathcal{P}_N}{\text{argmin}}\, \text{EMD}_{\beta,R}(\mathcal{E}, \mathcal{E}')$$

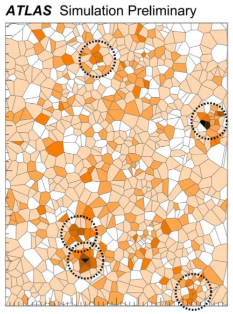
Jet Algorithms

Substructure resolves emissions within the jet.



$$\tau(\mathcal{J}) = \min_{\mathcal{E}' \in \mathcal{P}_N} \text{EMD}_{\beta}(\mathcal{J}, \mathcal{E}').$$

Jet Substructure



Pileup

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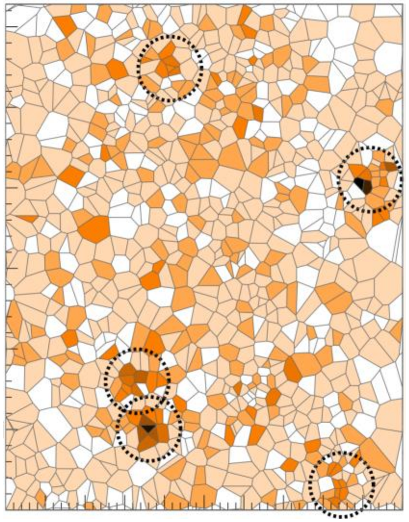
And many more!

Six Decades of Collider Techniques as Geometry!

Constituent subtraction adds in "negative" uniform radiation and clusters it with the event.



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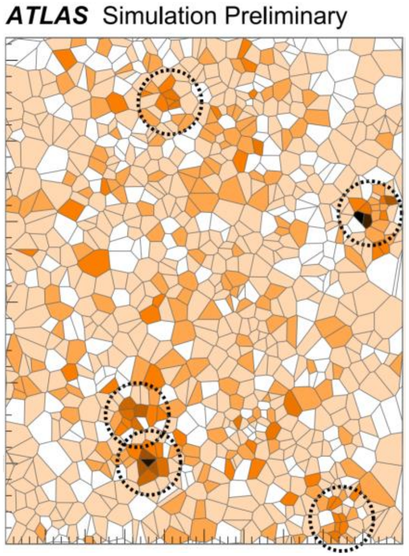
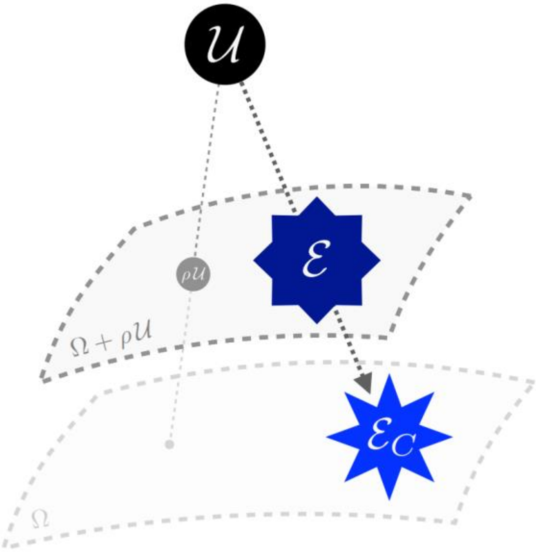
Constituent subtraction finds the closest event consistent with **uniform contamination**.



$$\mathcal{E}_C = \arg\min_{\mathcal{E}'} \text{EMD}_1(\mathcal{E}, \mathcal{E}' + \rho\,\mathcal{U}).$$



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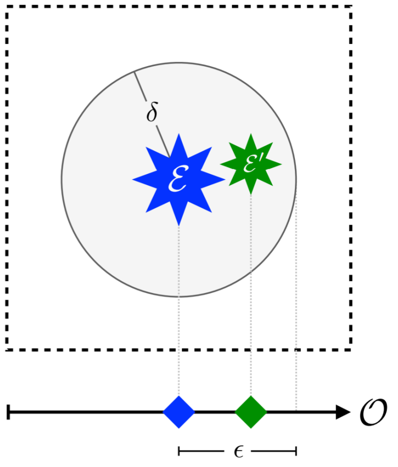

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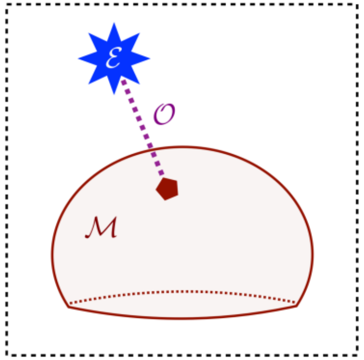

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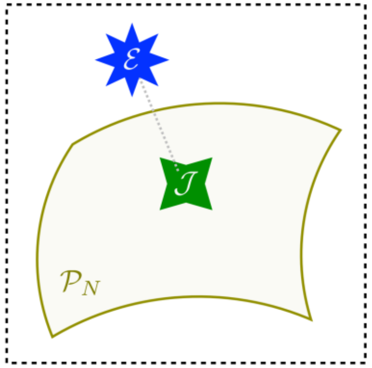

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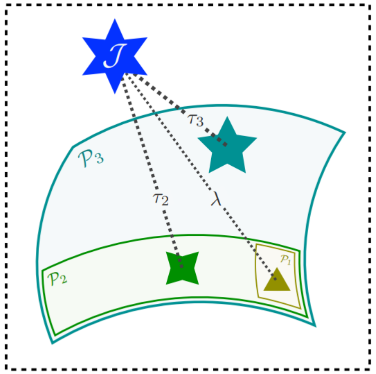
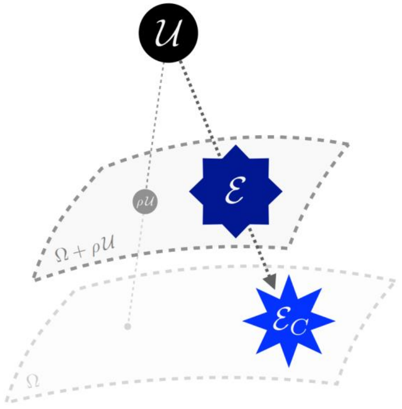


$\tau(\mathcal{J}) = \min_{\mathcal{E}' \in \mathcal{P}_N} \text{EMD}_\beta(\mathcal{J}, \mathcal{E}').$

Pileup mitigation moves away from uniform radiation.



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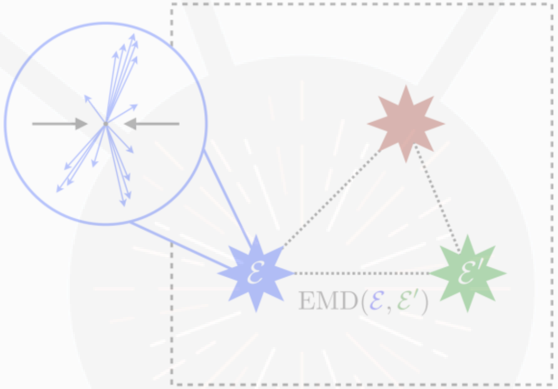
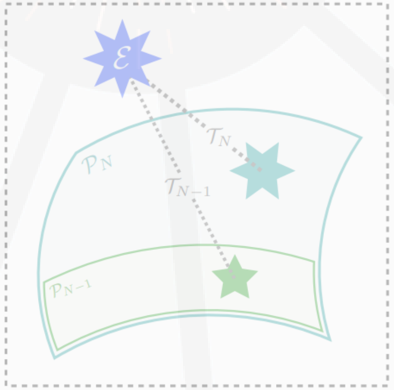


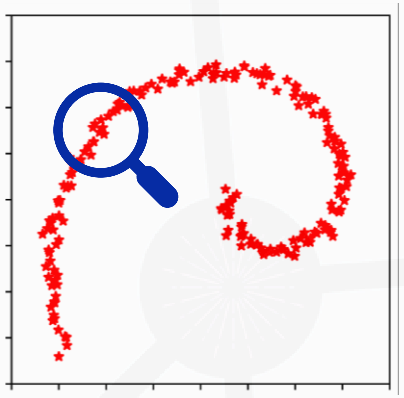
The Space of Collider Events

Building a Metric for Particle Collisions



Unifying Ideas in Collider Physics

Observables, Jets, and Pileup as Geometry



Enabling New Directions

The Fractal Dimension of QCD

Enabling New Directions: Event Isotropy

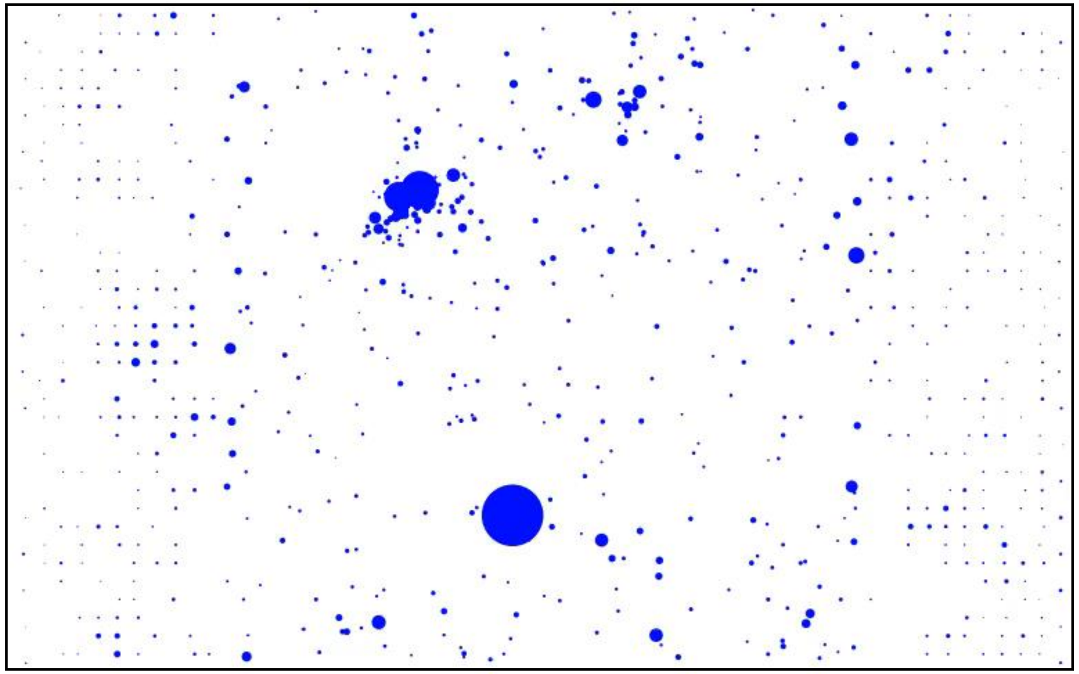
Event Isotropy is a new observable to probe how "uniform" an event is.
[Cesarotti, Thaler, 2004.06125]

It is sensitive to very different new physics scenarios, compared to existing event shapes.

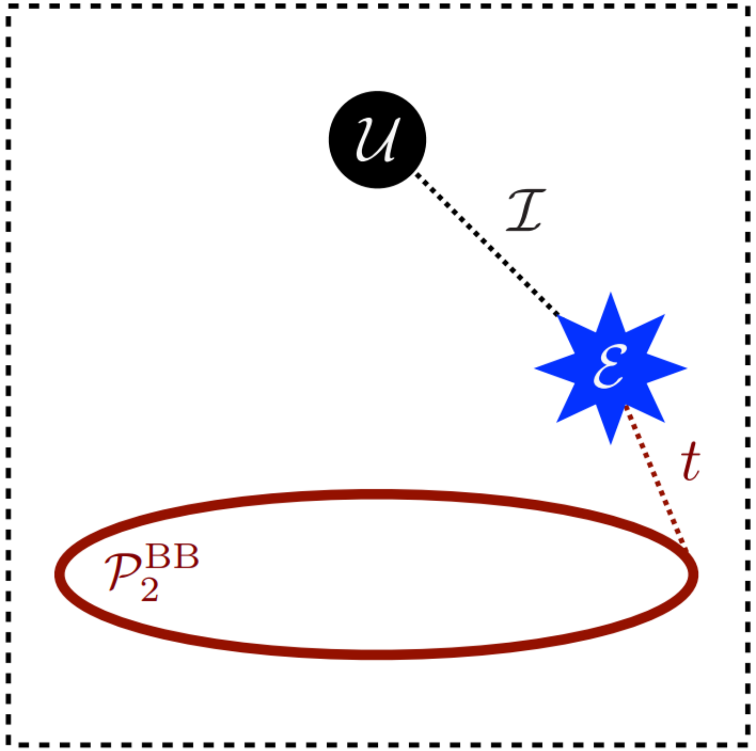
e.g. uniform radiation from micro black holes

$$\mathcal{I}(\mathcal{E}) = \mathrm{EMD}(\mathcal{E}, \mathcal{U})$$

where $\mathcal{U}$ is a fully isotropic event



Dijet event from 2011 CMS Open Data – Particle Flow Candidates.



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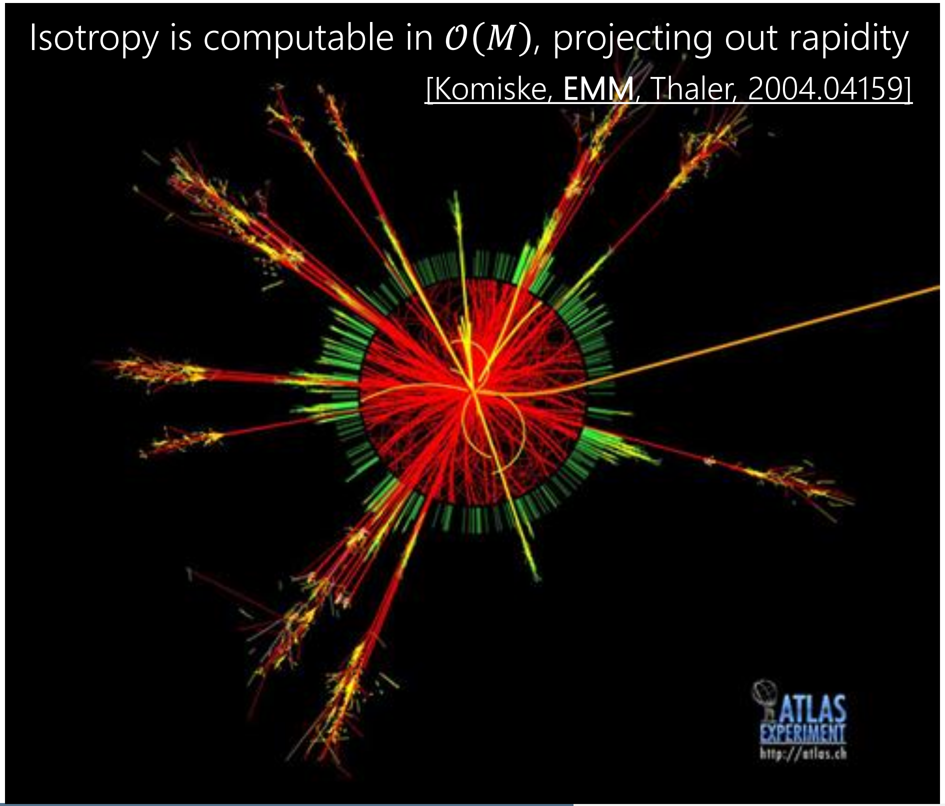
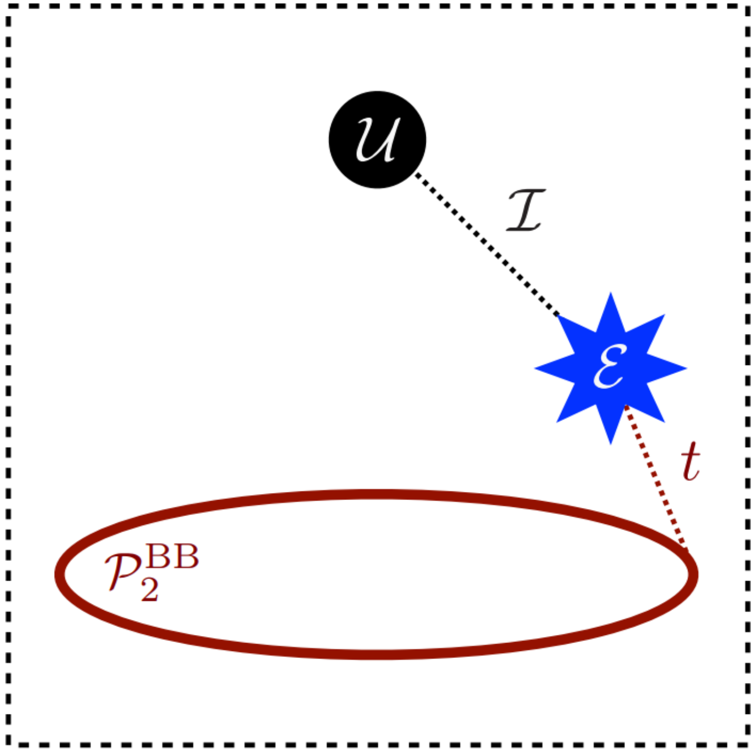
e.g. uniform radiation from micro black holes

$$\mathcal{I}(\mathcal{E}) = \mathrm{EMD}(\mathcal{E}, \mathcal{U})$$ where $\mathcal{U}$ is a fully isotropic event





Enabling New Directions: The Fractal Dimension of QCD

A new probe of the fractal nature of QCD.

Goes beyond an observable, $\mathcal{O}(\mathcal{E})$

"How much information is in a jet?"

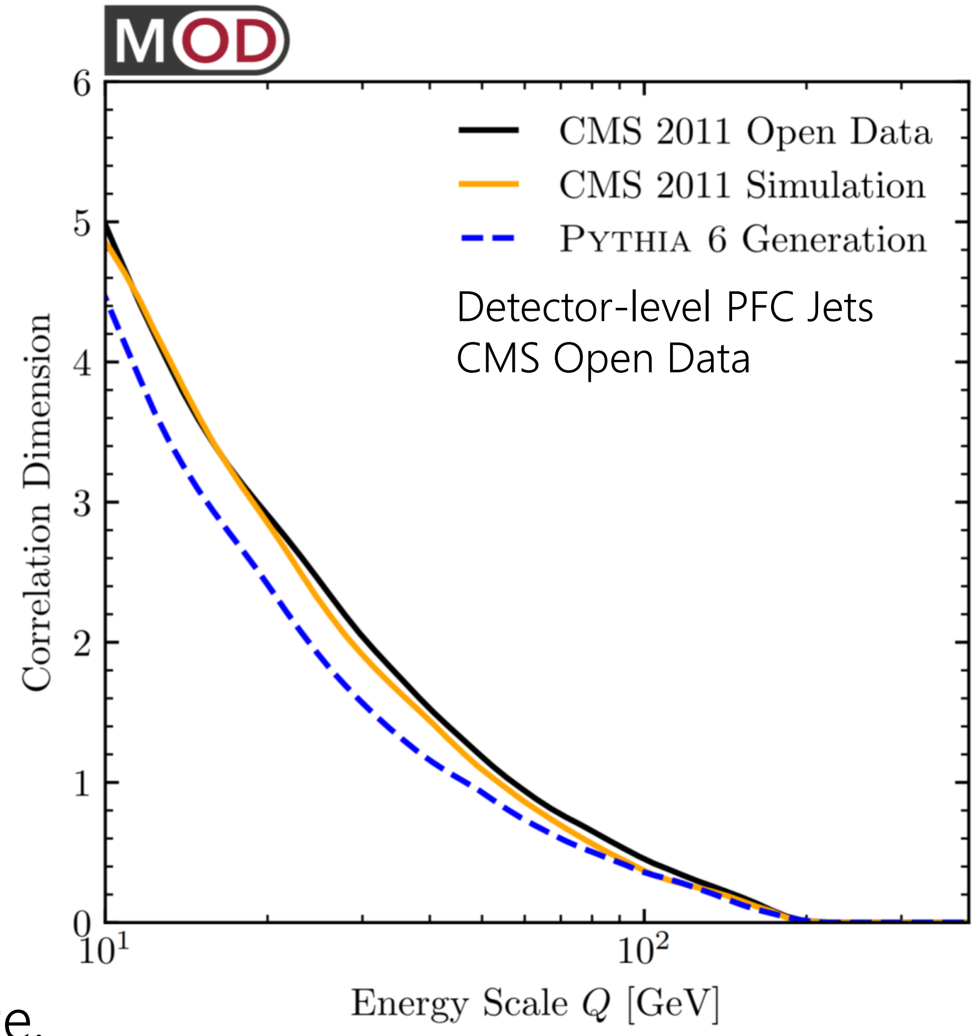
[Datta, Larkoski, JHEP, 1704.08249]

"How many particles do I resolve at this energy scale?"

[Larkoski, **EMM**, JHEP, 1906.01639]

P.S. Related to the event-event correlators of Theory Space.

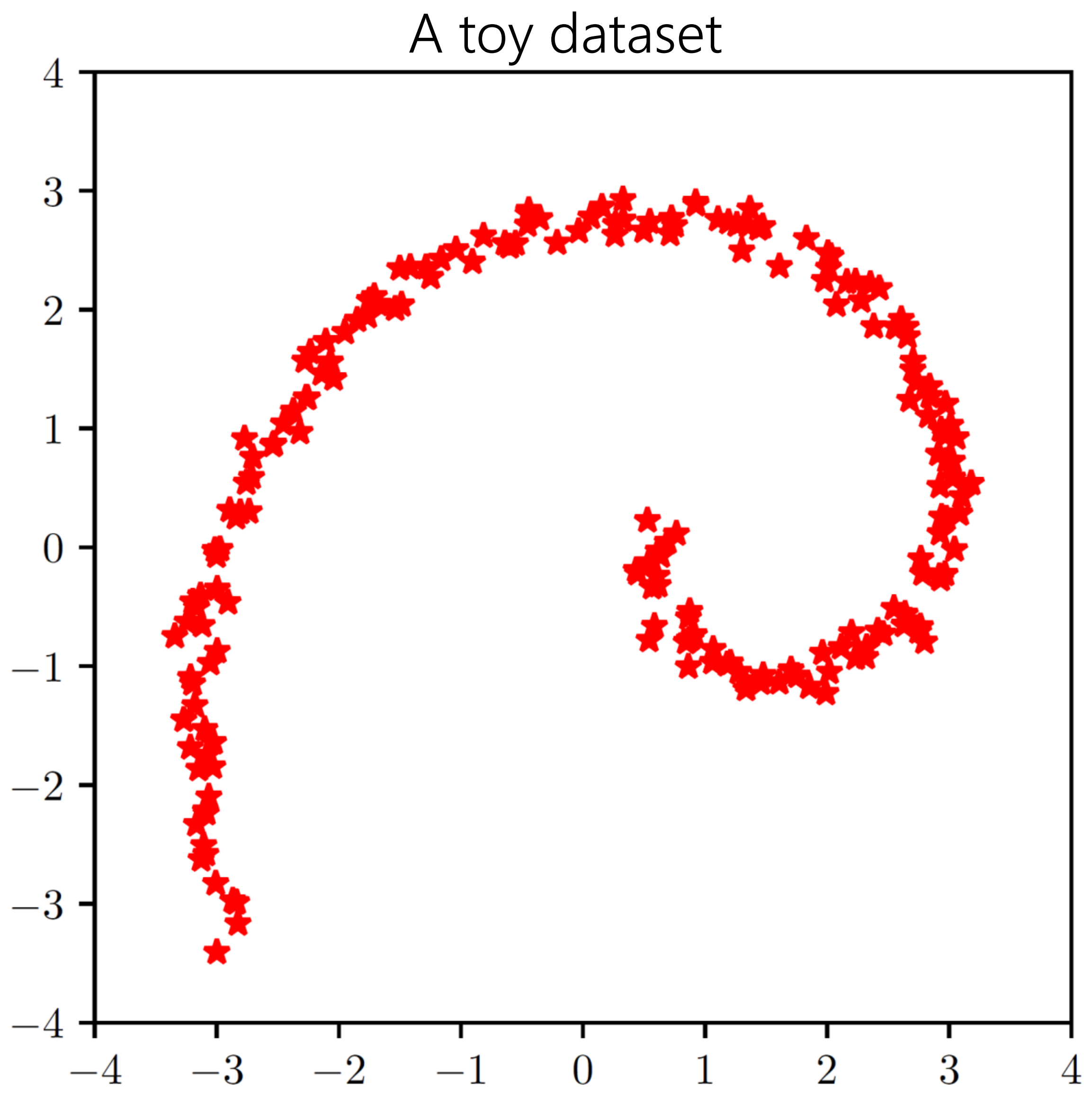
[Komiske, **EMM**, Thaler, 2004.04159]



[Komiske, **EMM**, Thaler, PRL, 1902.02346]

[Komiske, Mastandrea, **EMM**, Naik, Thaler, PRD, 1908.08542]

Enabling New Directions: The Fractal Dimension of QCD



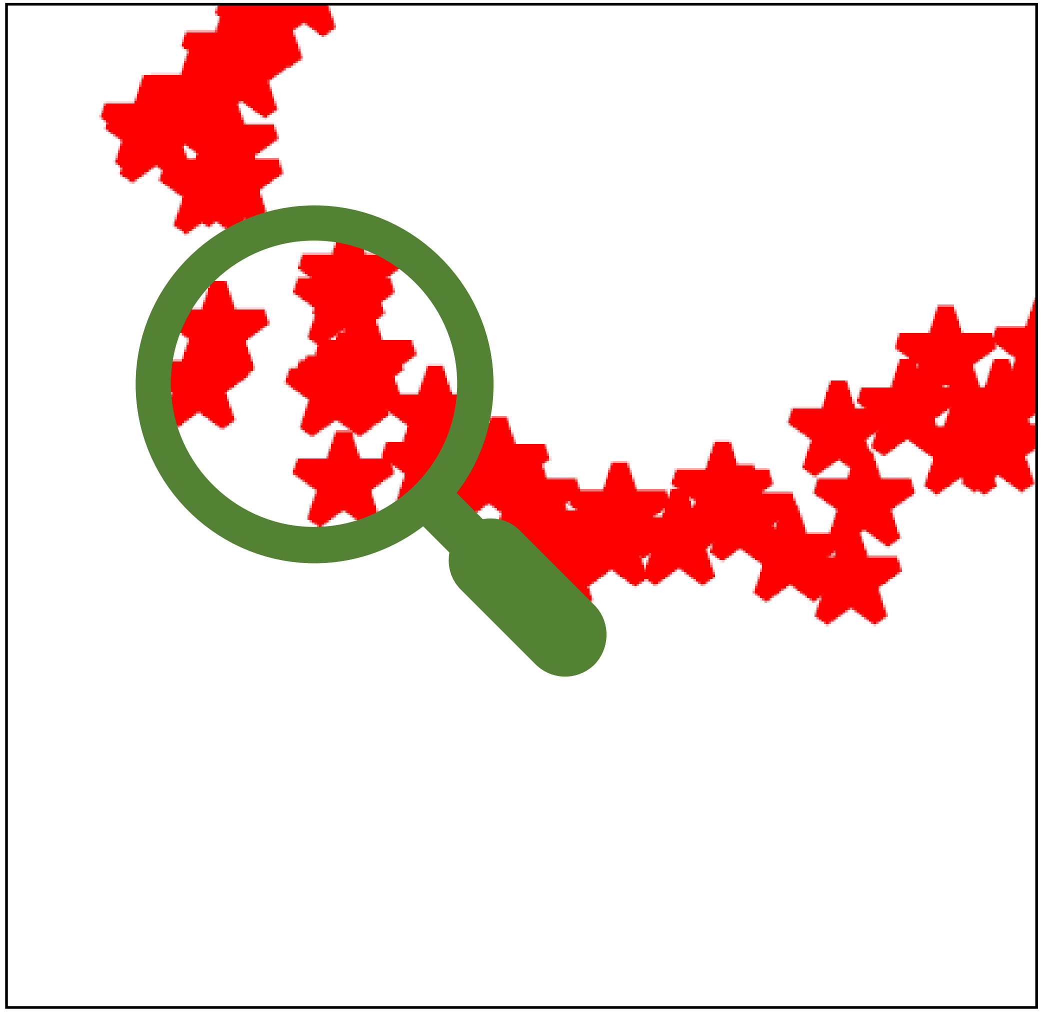
Questions

What are the scales in the system?

What is its dimensionality or complexity?

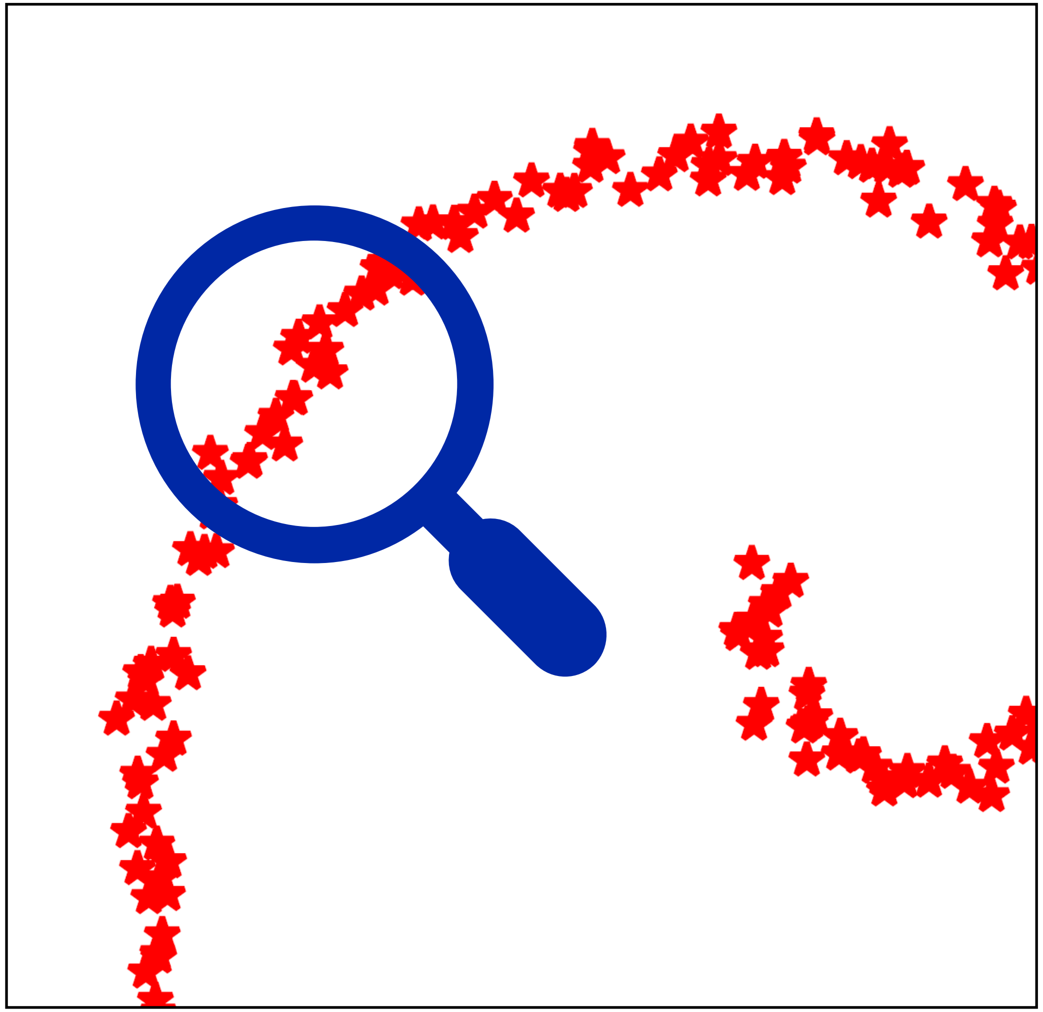
How do I characterize it?

Enabling New Directions: The Fractal Dimension of QCD



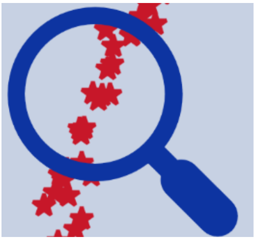
Small scales: Two-dimensional plane

Enabling New Directions: The Fractal Dimension of QCD



Small scales: Two-dimensional plane

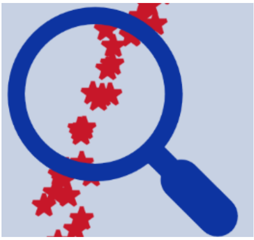
Medium scales: One-dimensional line



Enabling New Directions: The Fractal Dimension of QCD

Small scales: Two-dimensional plane

Medium scales: One-dimensional line

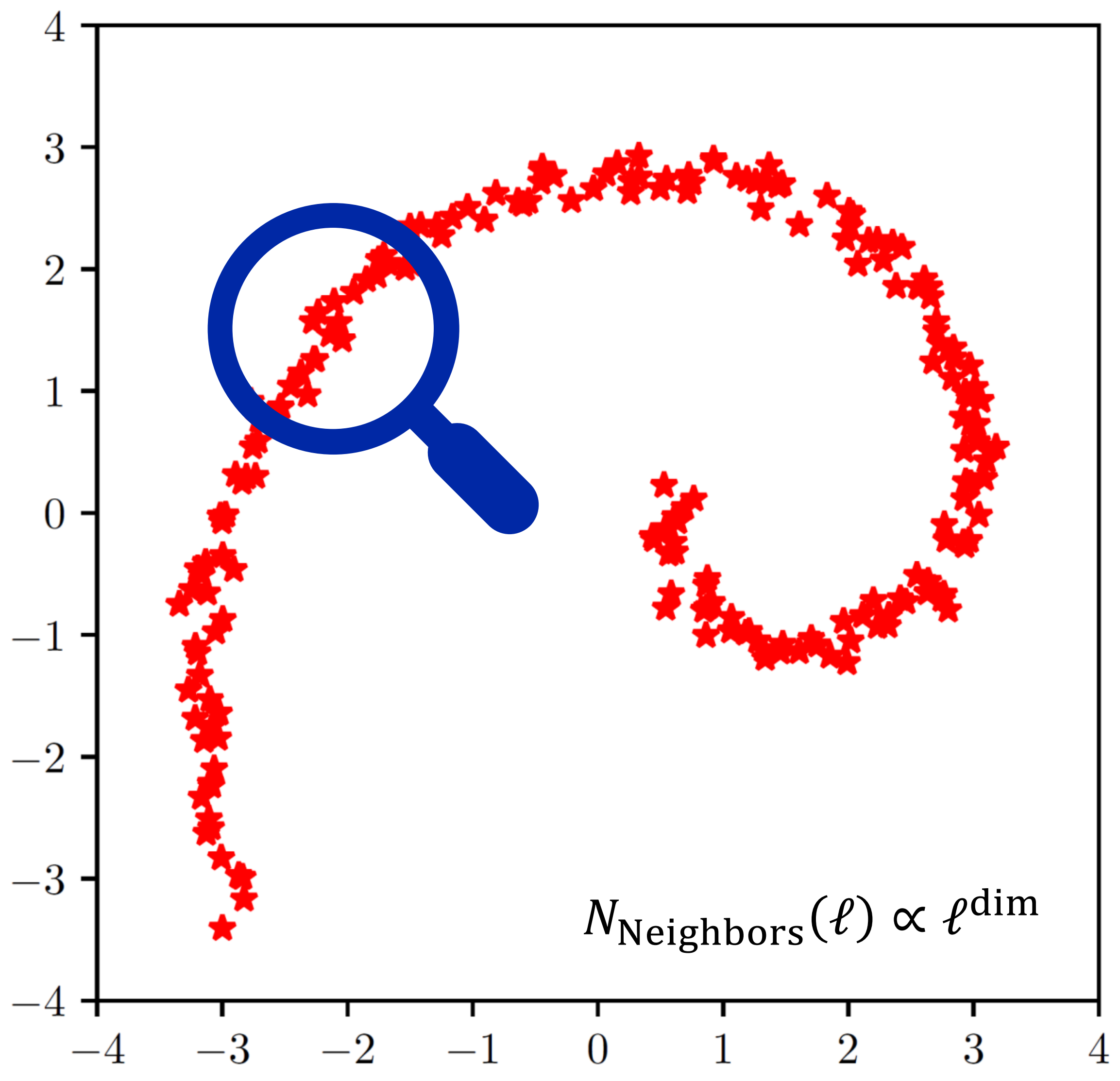


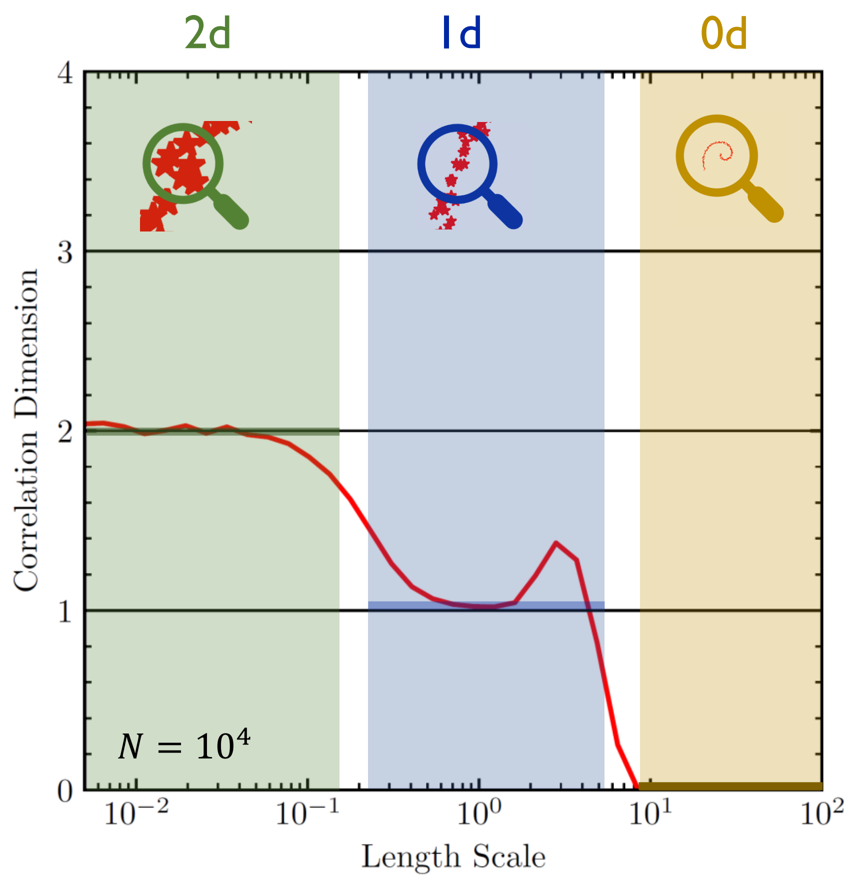
Large scales: Zero-dimensional point

Enabling New Directions

$$\dim(\ell) = \ell \frac{\partial}{\partial \ell} \ln \sum_{i=1}^{N} \sum_{j=1}^{N} \Theta[d(x_i, x_j) < \ell]$$

[Grassberger, Procaccia, PRL, 1983] [Kegl, NeurIPS, 2002]





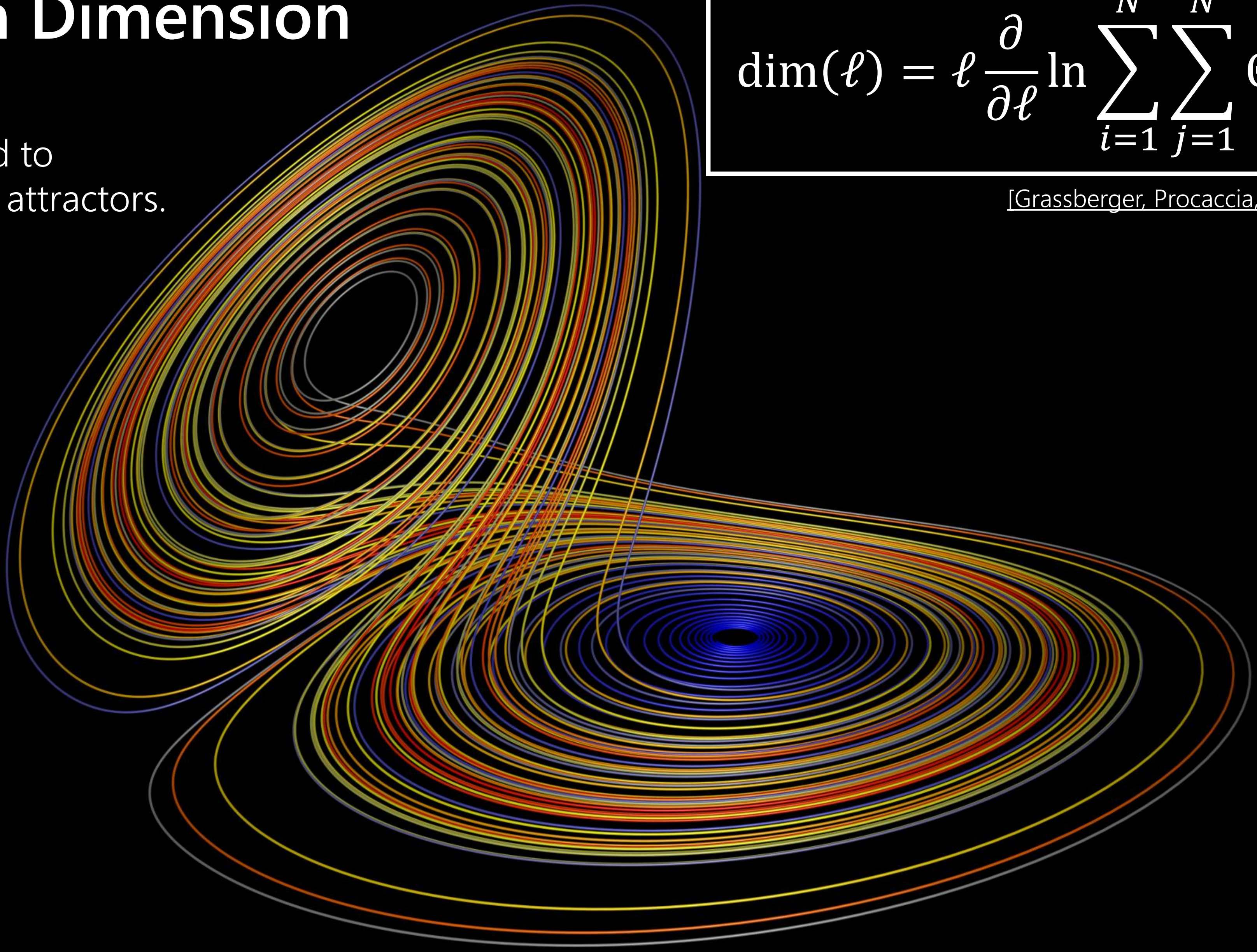
A spectrum of the dataset at a glance.

Correlation Dimension

$$\dim(\ell) = \ell \frac{\partial}{\partial \ell} \ln \sum_{i=1}^{N} \sum_{j=1}^{N} \Theta[\mathrm{d}(x_i, x_j) < \ell]$$

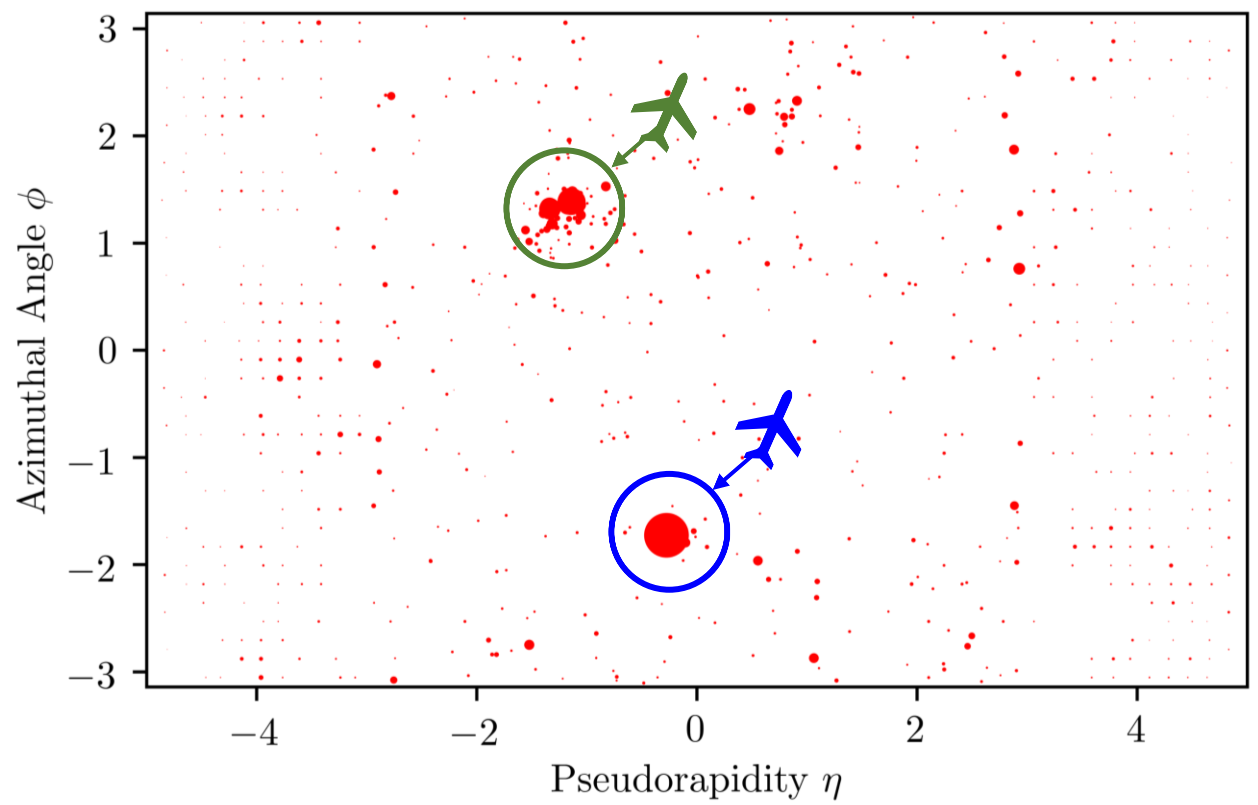
[Grassberger, Procaccia, PRL, 1983] [Kegl, NeurIPS, 2002]

Originally introduced to characterize strange attractors.



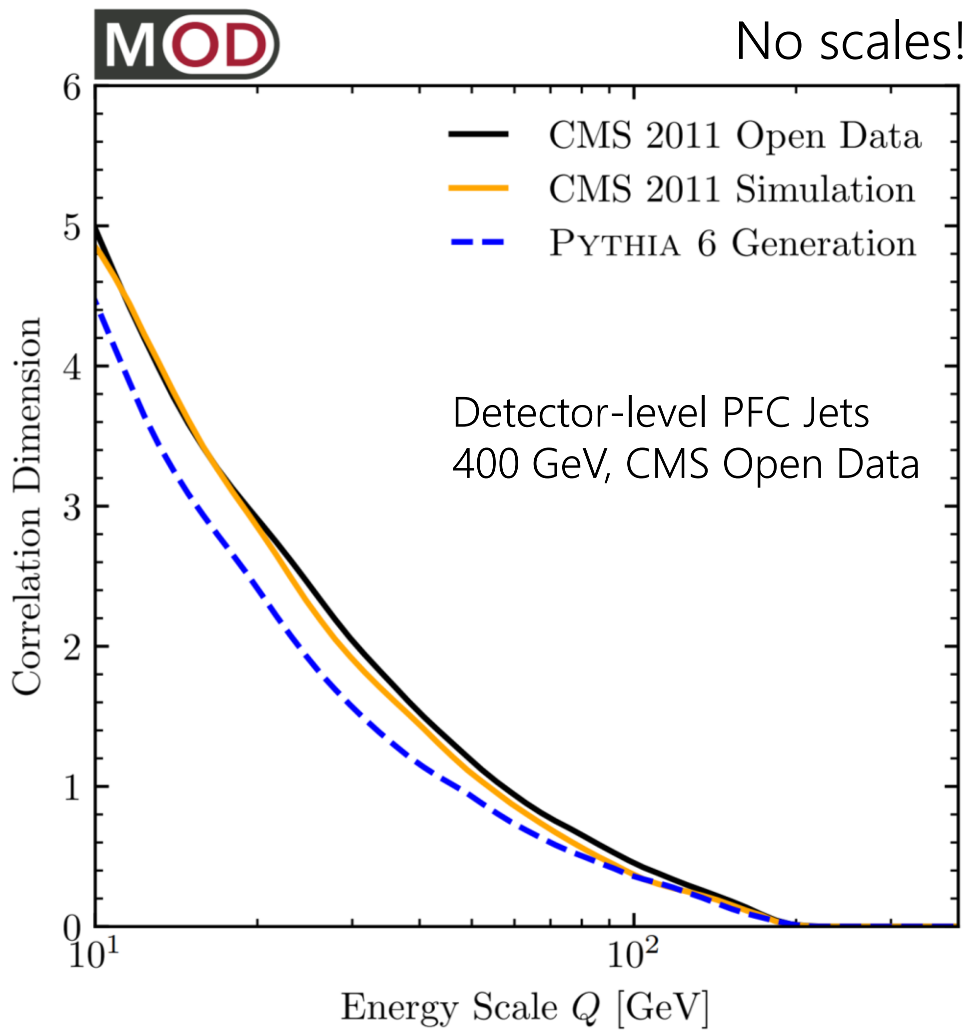
Enabling New Directions: The Fractal Dimension of QCD

$$\dim(Q) = Q\frac{\partial}{\partial Q}\ln\sum_{i=1}^{N}\sum_{j=1}^{N}\Theta[\mathrm{EMD}(\mathcal{E}_i,\mathcal{E}_j) < Q]$$



+ centering and rotation

No scales!



[Komiske, Mastandrea, **EMM**, Naik, Thaler, PRD, 1908.08542]

Enabling New Directions: The Fractal Dimension of QCD

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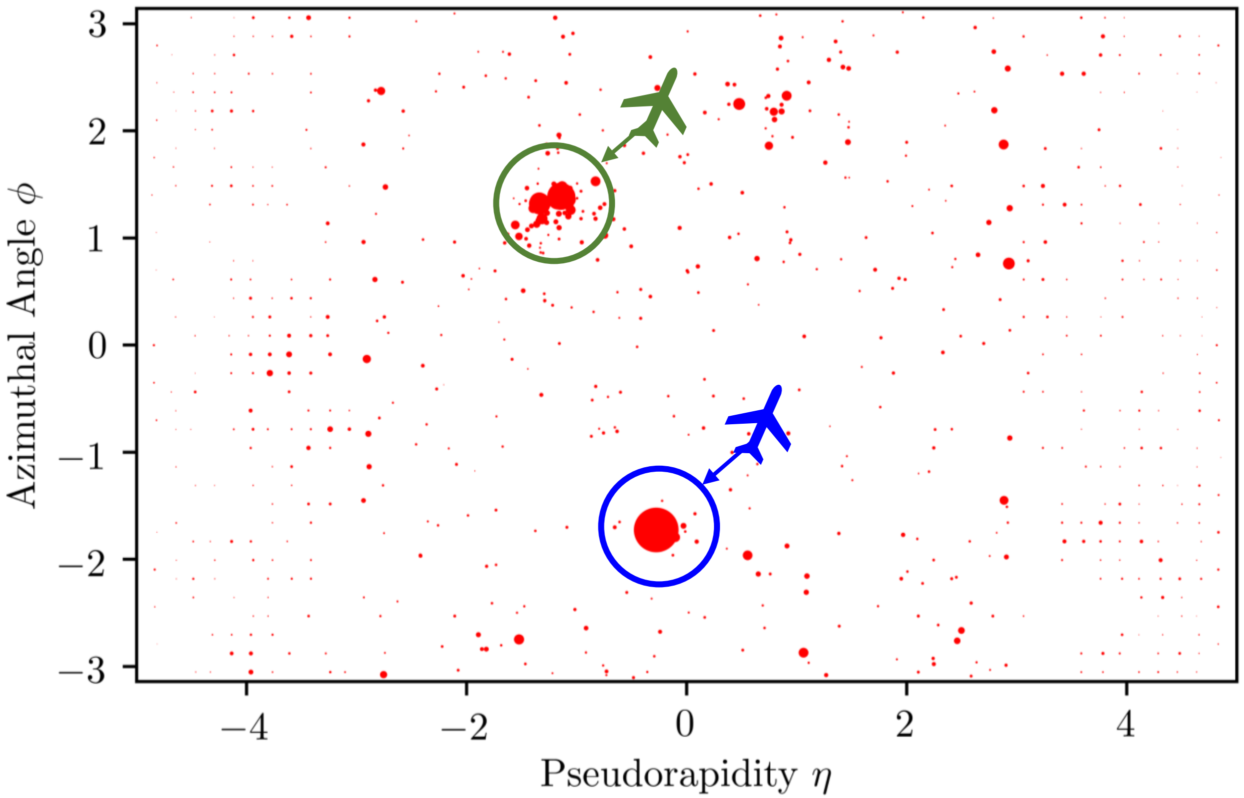
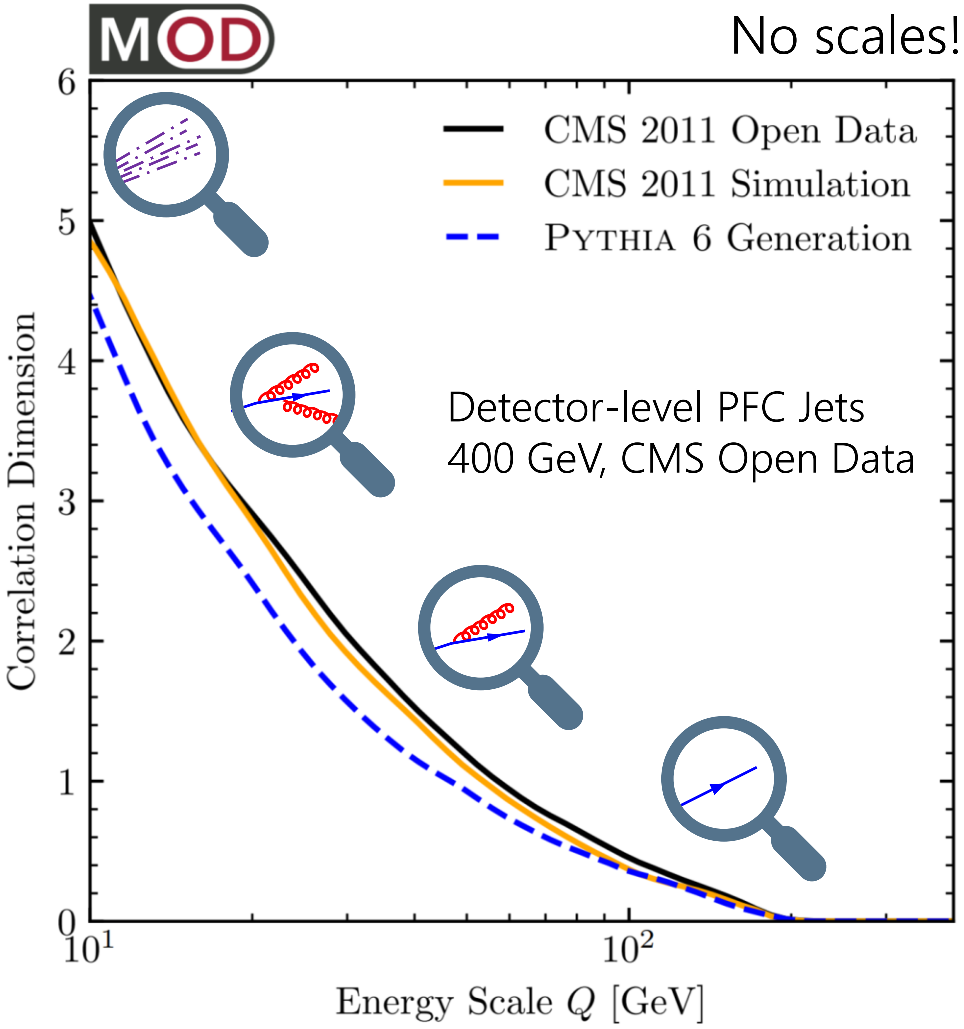
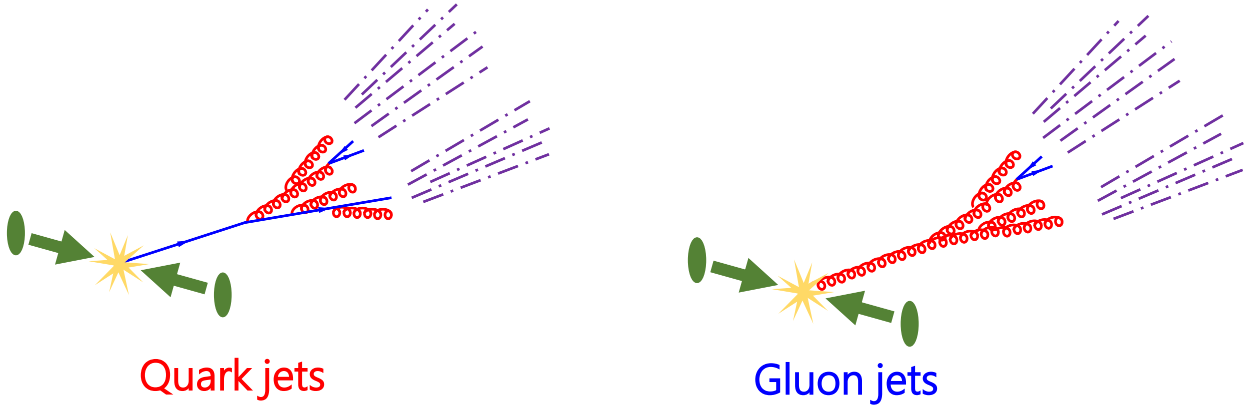


+ centering and rotation



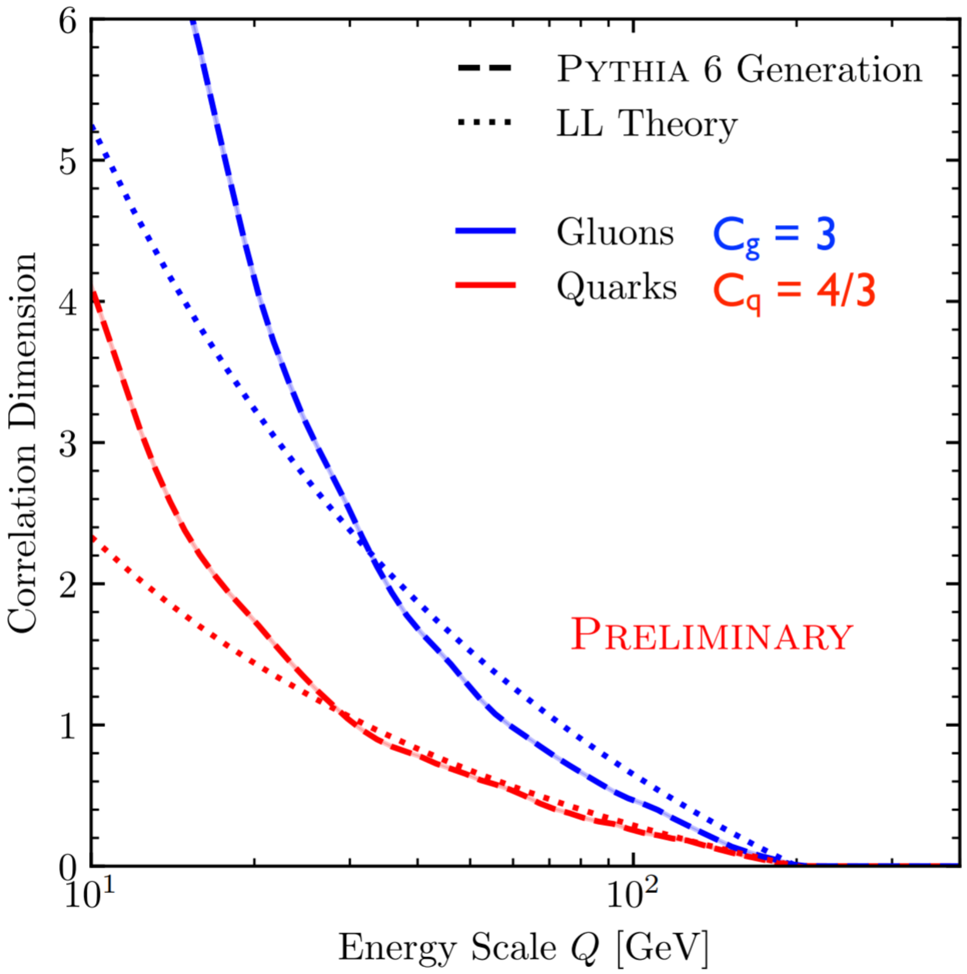
Enabling New Directions: The Fractal Dimension of QCD

$$\dim(Q) = Q\frac{\partial}{\partial Q}\ln\sum_{i=1}^{N}\sum_{j=1}^{N}\Theta[\mathrm{EMD}(\mathcal{E}_i,\mathcal{E}_j) < Q]$$

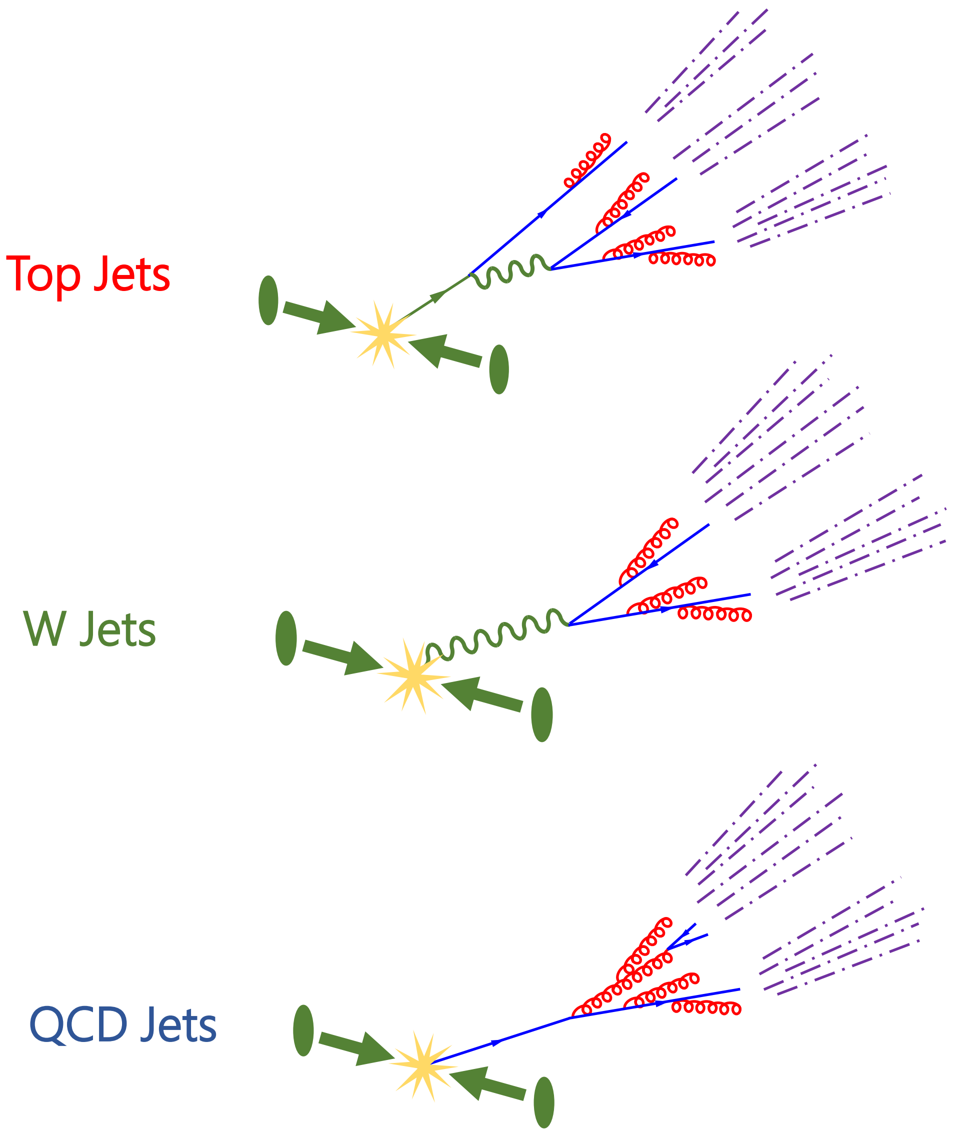


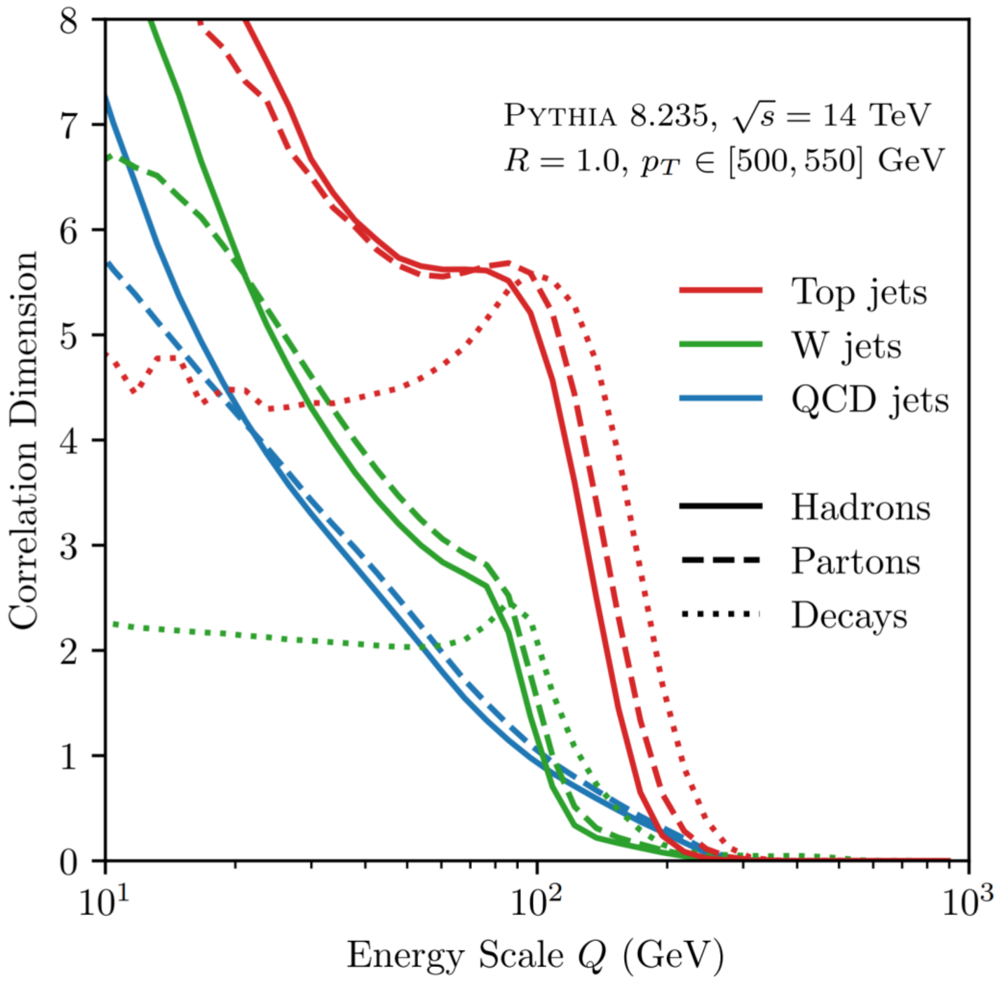
$$\text{At LL: } \dim_{q/g}(Q) = -\frac{8\alpha_s C_{q/g}}{\pi}\ln\frac{Q}{p_T/2}$$

+ 1-loop running of α_s



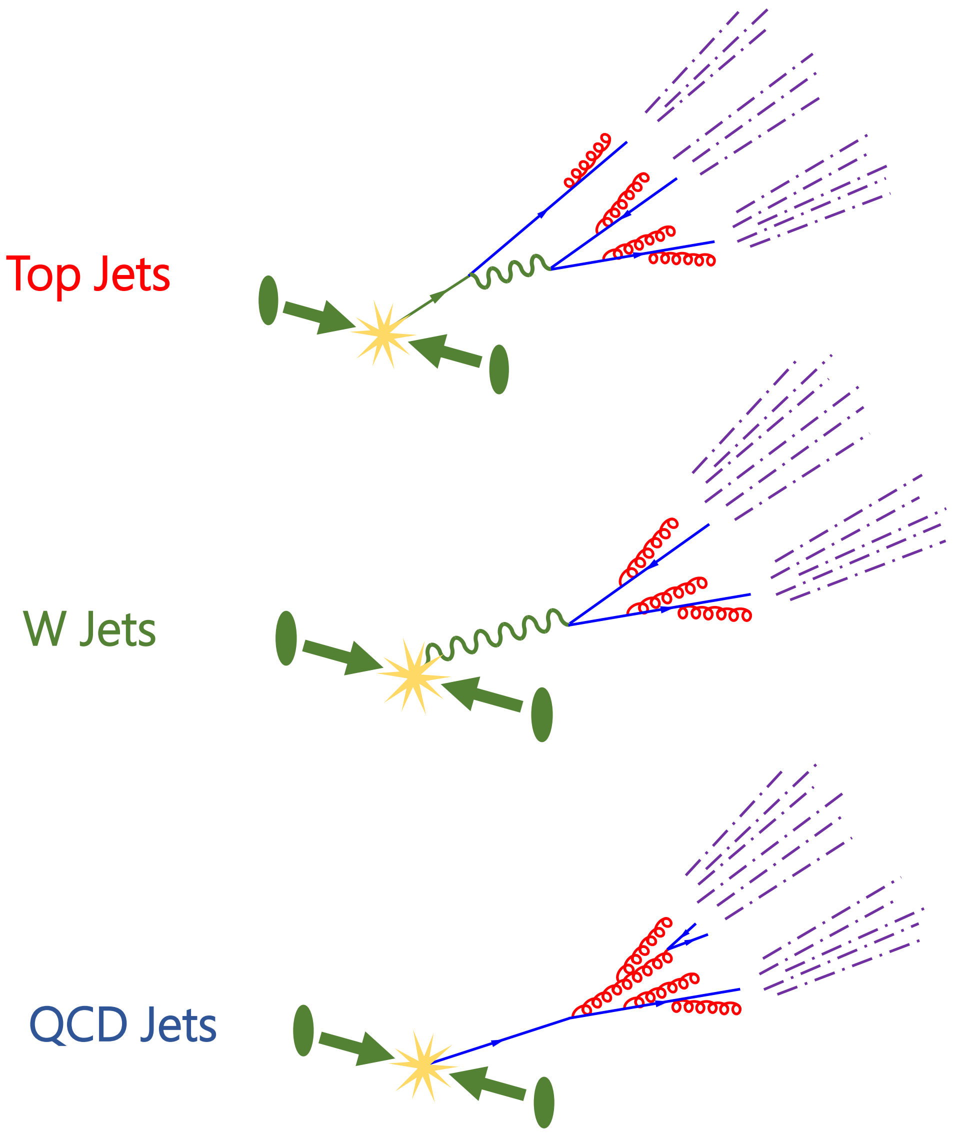
Enabling New Directions: The Fractal Dimension of the SM

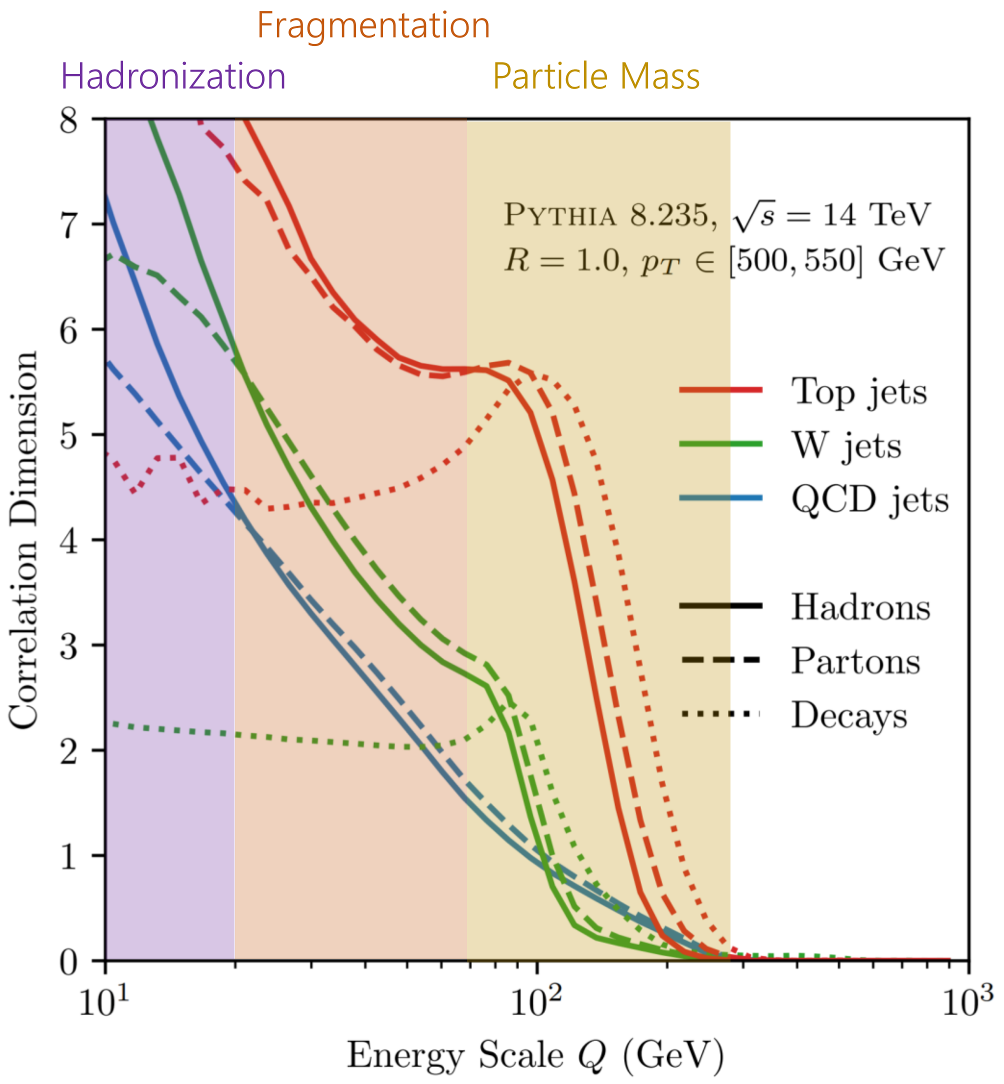




[Komiske, **EMM**, Thaler, PRL, 1902.02346]

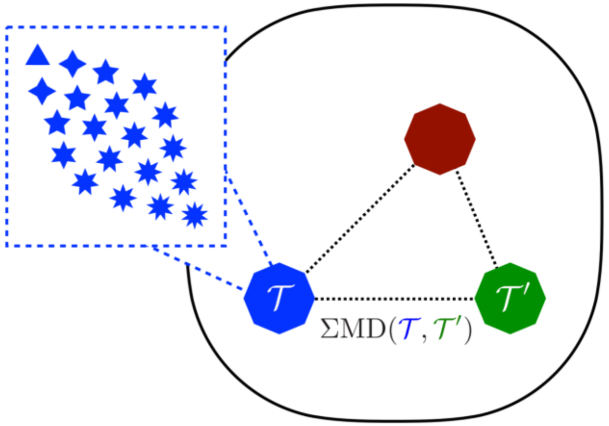
Enabling New Directions: The Fractal Dimension of the SM





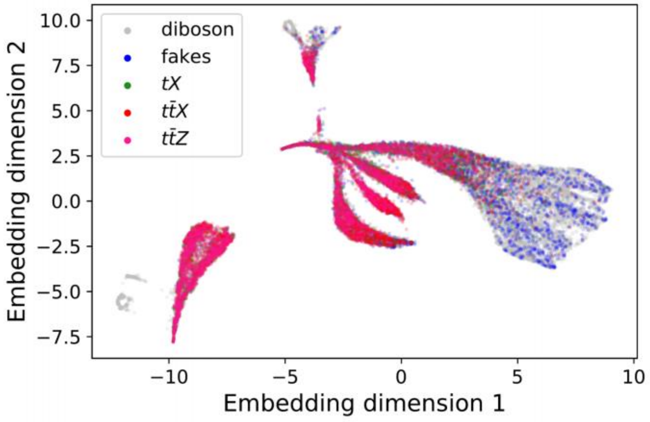
[Komiske, **EMM**, Thaler, PRL, 1902.02346]

Enabling New Directions: Beyond this talk



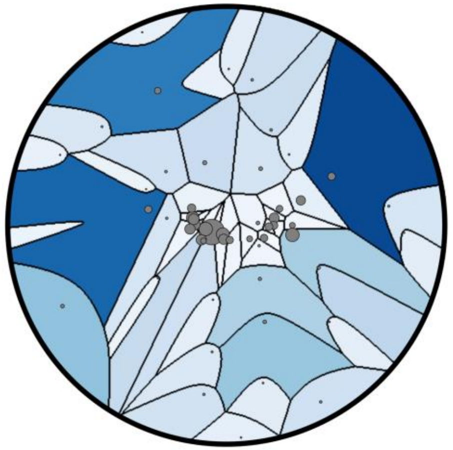
"Theory Space"

[Komiske, **EMM**, Thaler, 2004.04159]
[Thaler, CERN Theory Colloquium]



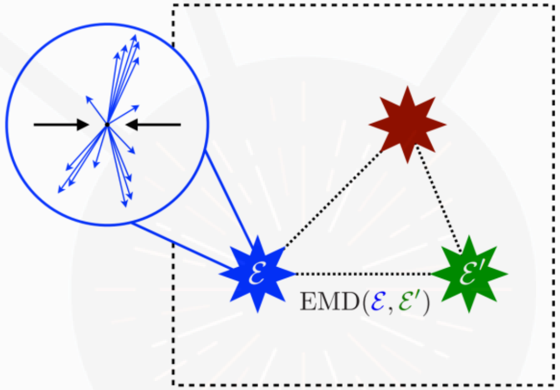
Flavor-dependence in the metric

[Romao, Castro, Milhano, Pedro, Vale, 2004.09360]



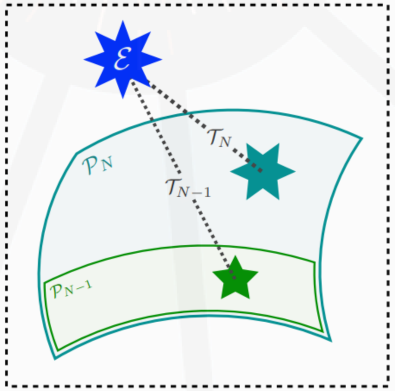
New grooming or pileup mitigation techniques?

New jet clustering algorithms?



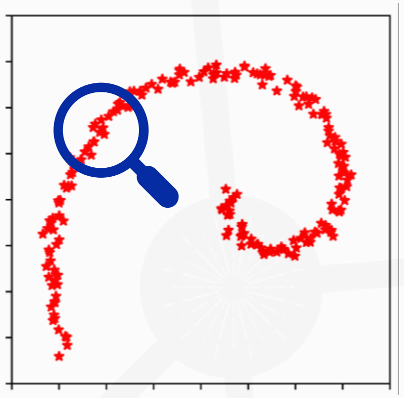
The Space of Collider Events

Building a Metric for Particle Collisions



Unifying Ideas in Collider Physics

Observables, Jets, and Pileup as Geometry



Enabling New Directions

The Fractal Dimension of QCD

The End

Thank you!

[Image: Chelsea Turner, MIT News]

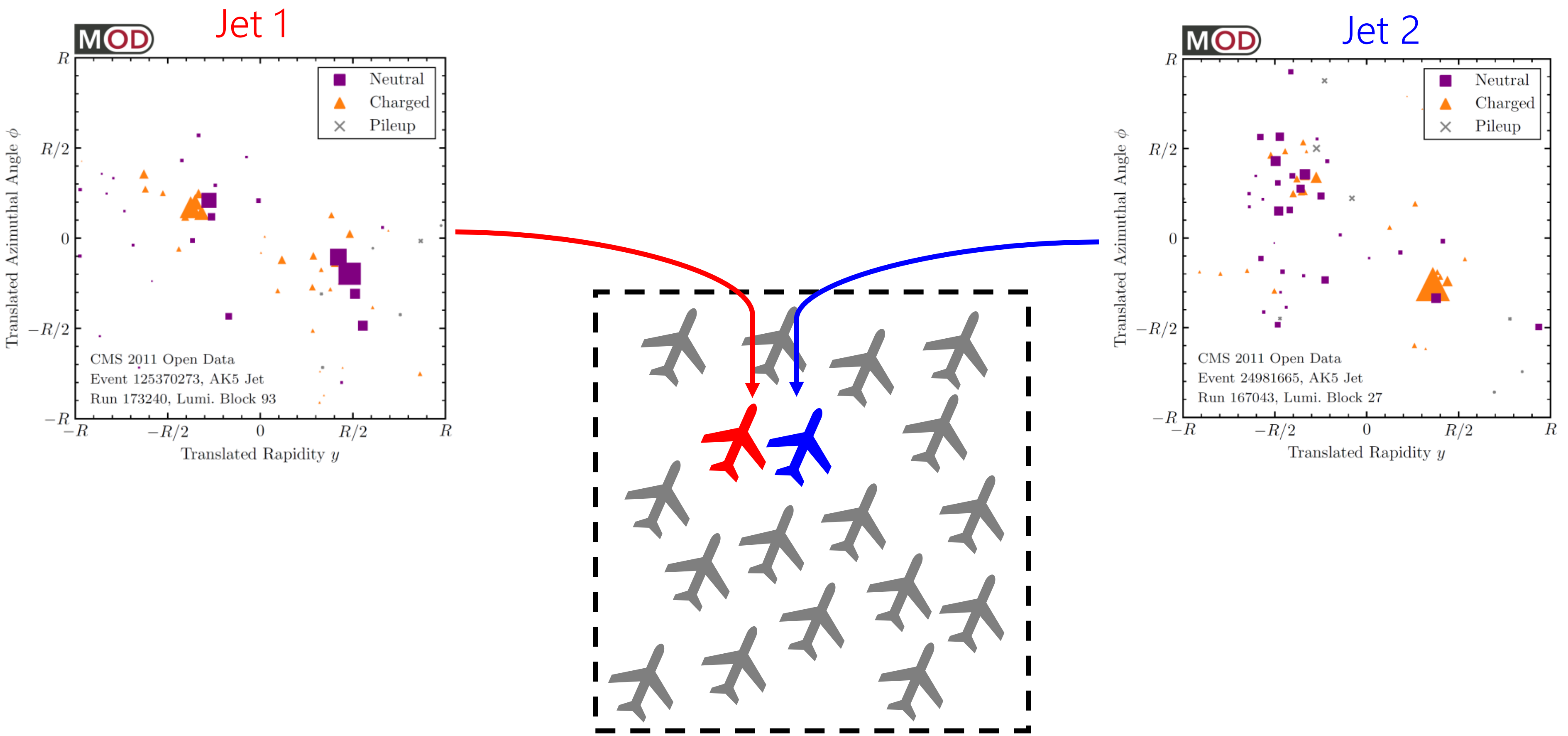
Extra Slides

[Image: Chelsea Turner, MIT News]

When are two events similar?

These two jets “look” similar, but have different numbers of particles, flavors, and locations.

How do we quantify this?



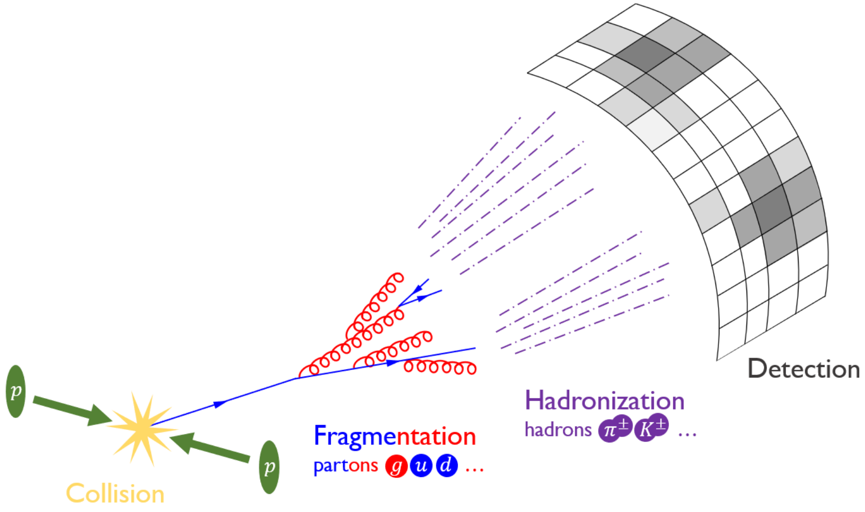
“Space of Jets”

400 GeV $R = 0.5$ anti- k_T Jets from CMS Open Data

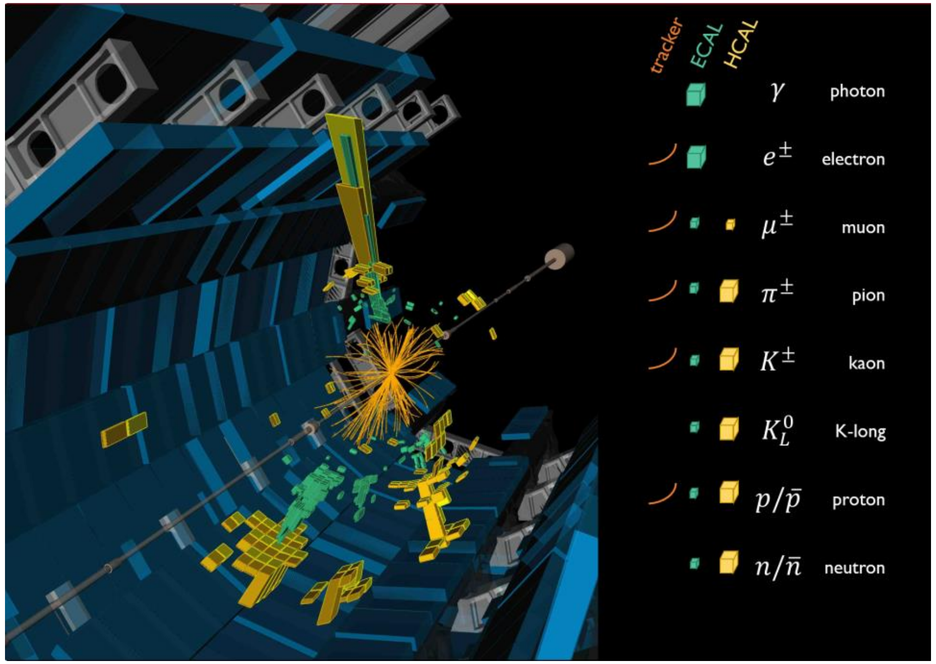
When are two events similar?

An event is…

Theoretically: very complicated



Experimentally: very complicated



However:

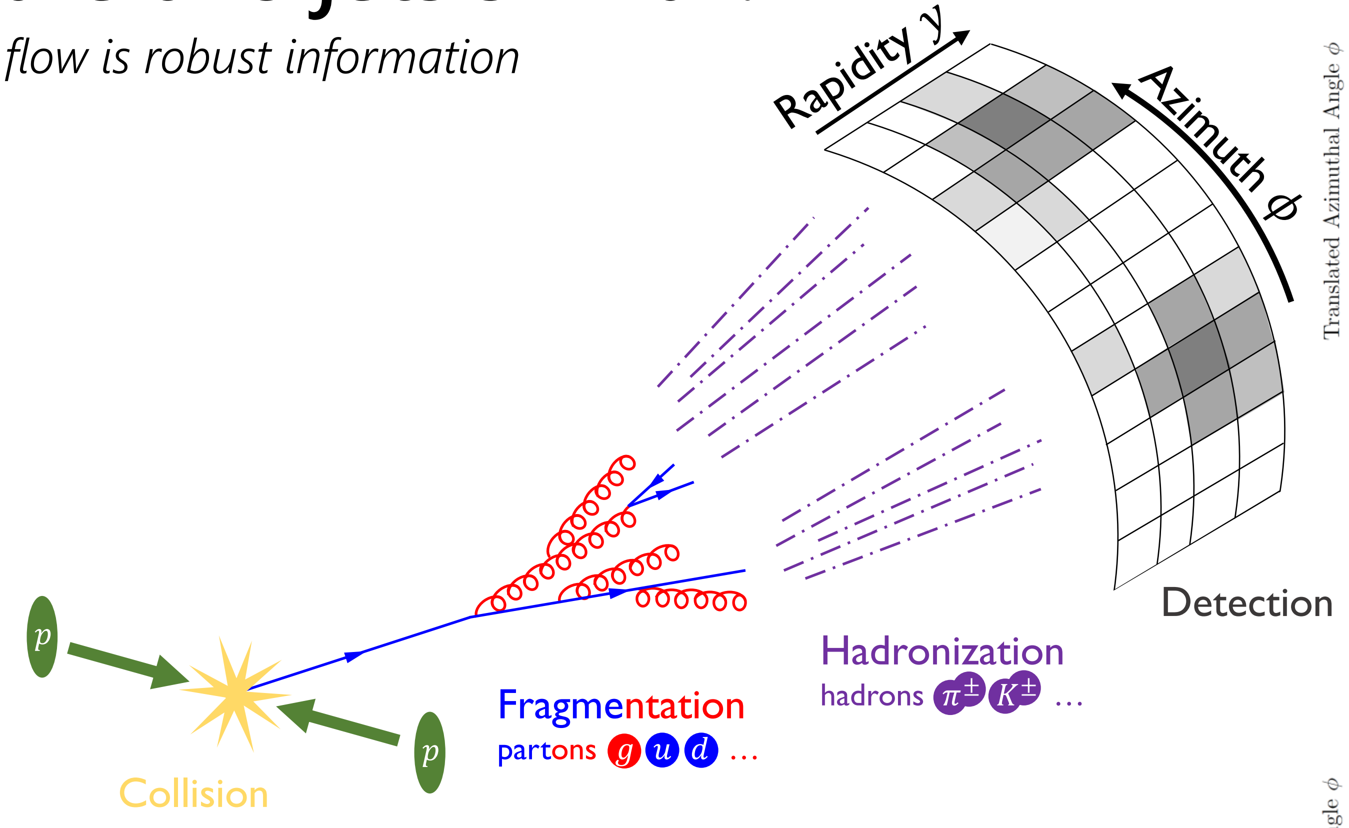
The *energy flow* (distribution of energy) is the information that is robust to: fragmentation, hadronization, detector effects, …

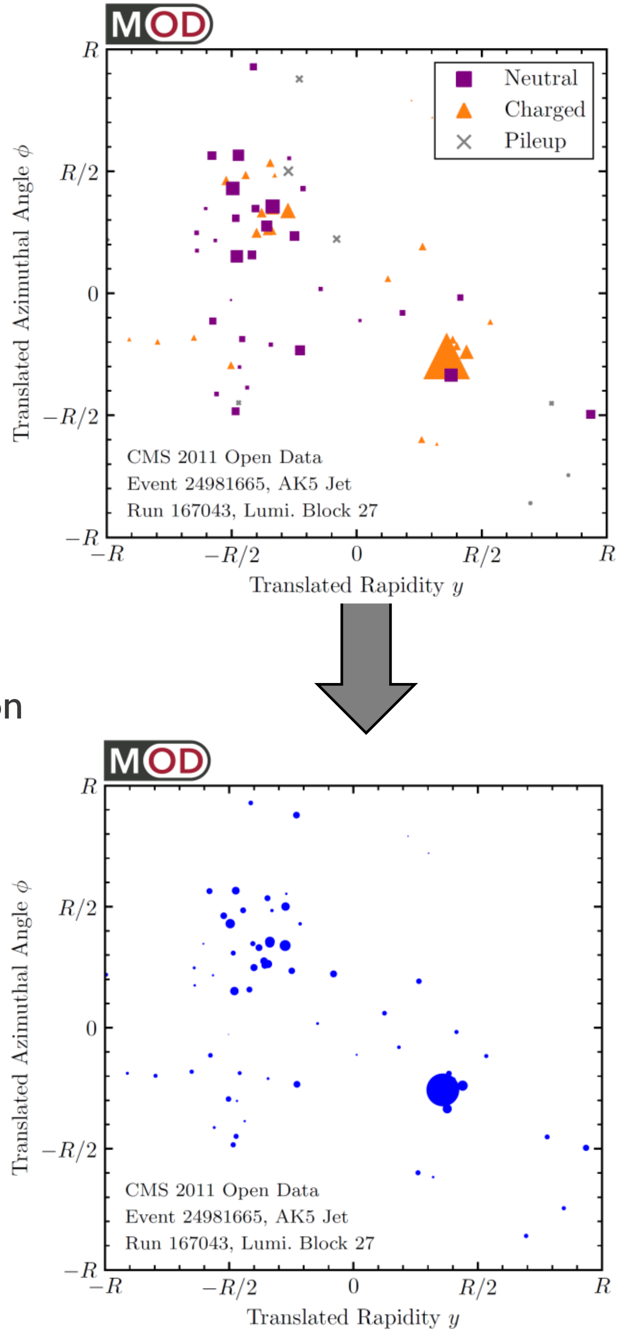
[N.A. Sveshnikov, F.V. Tkachov, 9512370]
[F.V. Tkachov, 9601308]
[P.S. Cherzor, N.A. Sveshnikov, 9710349]

Energy flow ⇔ Infrared and Collinear (IRC) Safe information

When are two jets similar?

Energy flow is robust information





Treat events as distributions of energy:

$$\mathcal{E}(\hat{n}) = \sum_{i=1}^{M} E_i \, \delta(\hat{n} - \hat{n}_i)$$

energy ↑ E_i direction ↑ $\hat{n}_i$

Ignoring particle flavor, charge…

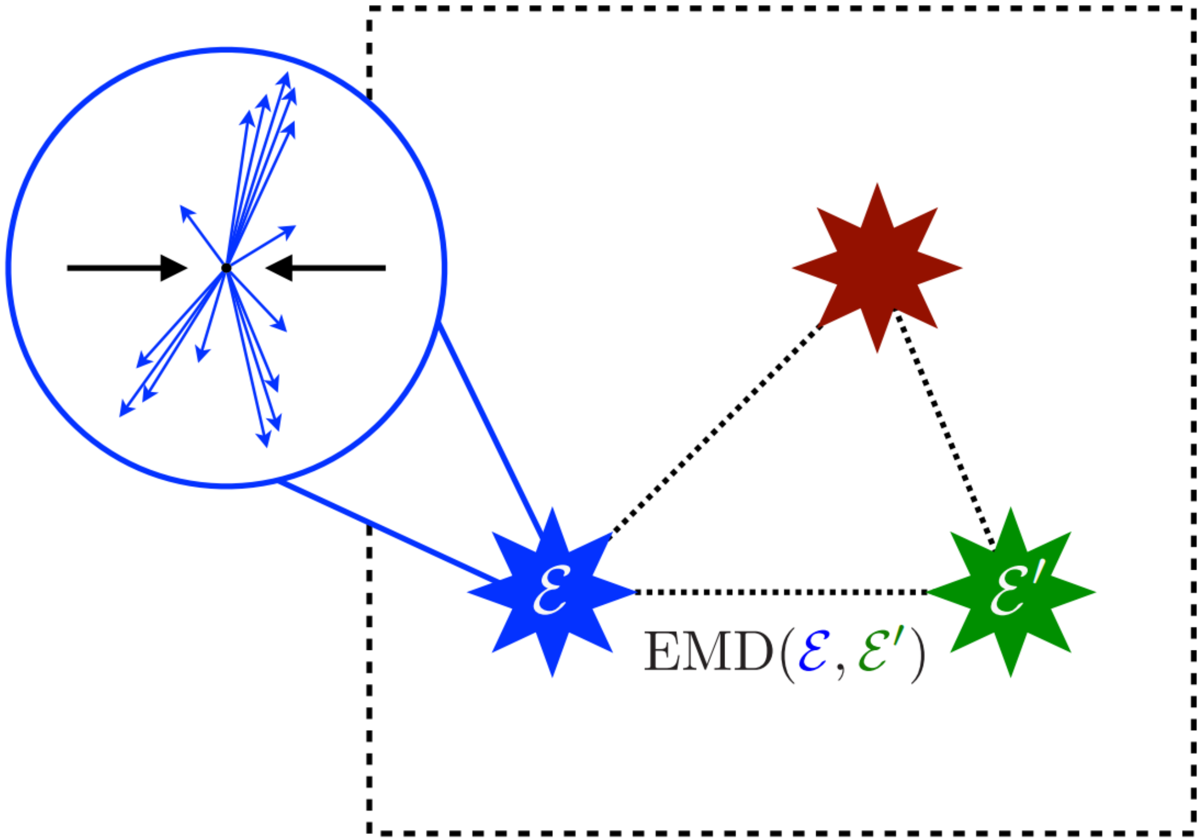
The Space of Collider Events

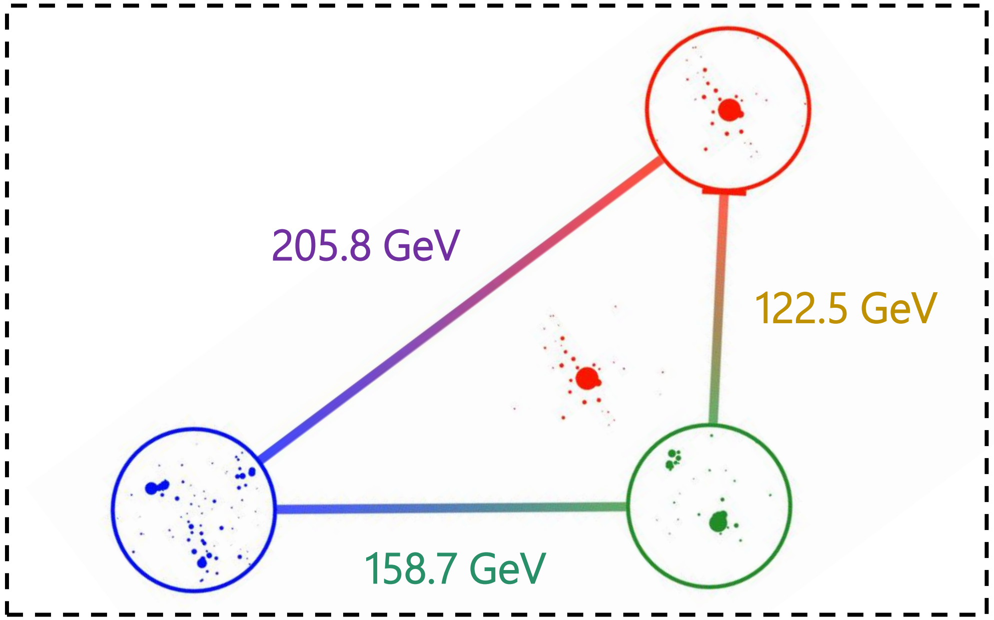
Difference in radiation pattern — Difference in total energy

$$\mathrm{EMD}_{\beta,\mathrm{R}}(\mathcal{E},\mathcal{E}') = \min_{\{f_{ij}\}} \sum_{i=1}^{M}\sum_{j=1}^{M'} f_{ij}\frac{\theta_{ij}^{\beta}}{R^{\beta}} + \left|\sum_{i=1}^{M} E_i - \sum_{j=1}^{M'} E'_j\right|$$

β : angular weighting factor

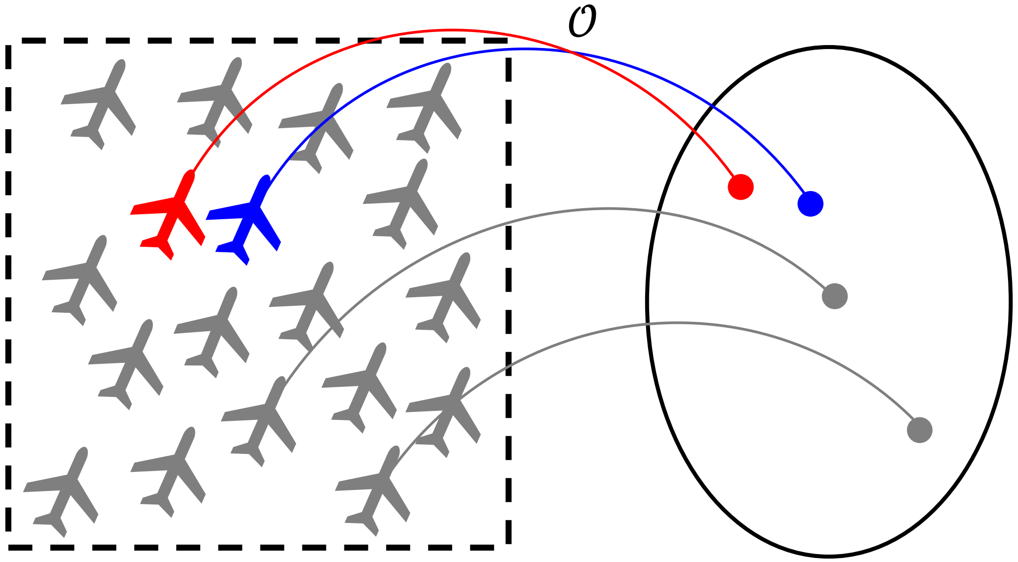
R : tradeoff between moving energy and creating it





A Geometric Language for Observables

Events close in EMD are close in any infrared and collinear safe observable!



Additive IRC-safe observables: $\mathcal{O}(\mathcal{E}) = \sum_{i=1}^{M} E_i \, \Phi(\hat{n}_i)$

Energy Mover's Distance

$$\text{EMD}(\mathcal{E}, \mathcal{E}') \geq \frac{1}{RL} \, |\mathcal{O}(\mathcal{E}) - \mathcal{O}(\mathcal{E}')|$$

Difference in observable values

"Lipschitz constant" of Φ
i.e. bound on its derivative

A Geometric Language for Observables

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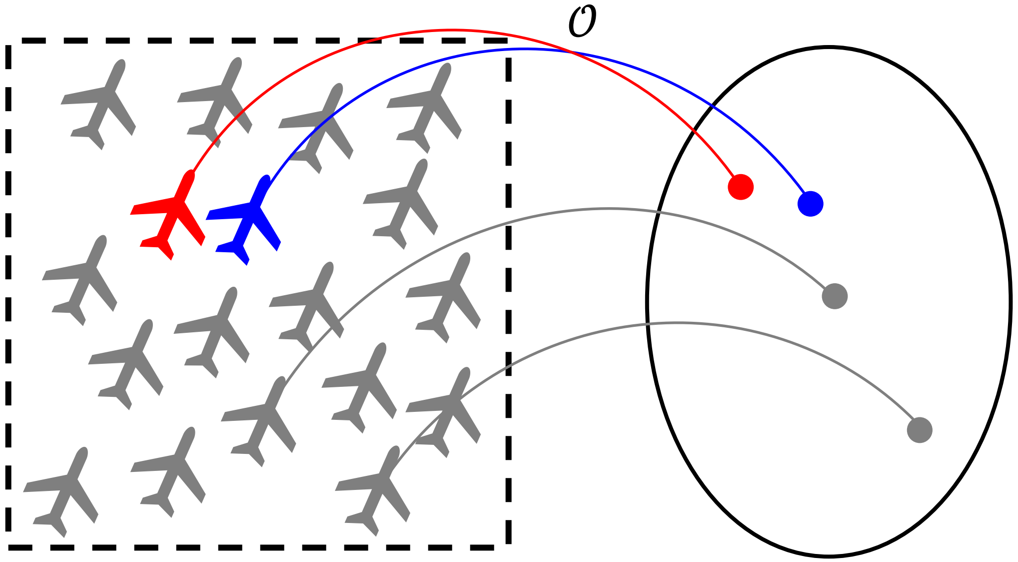


Jet angularities with $\beta \geq 1$:

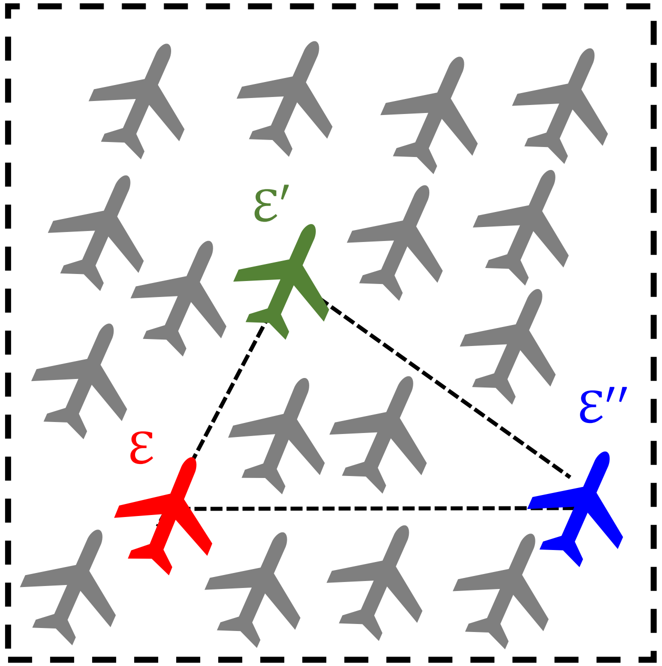
[C. Berger, T. Kucs, and G. Sterman, 0303051]

[A. Larkoski, J. Thaler, and W. Waalewijn, 1408.3122]

$$\lambda^{(\beta)} = \sum_{i=1}^{M} E_i\, \theta_i^{\beta}$$

$$\left|\lambda^{(\beta)}(\mathcal{E}) - \lambda^{(\beta)}(\mathcal{E}')\right| \leq \beta\ \mathrm{EMD}(\mathcal{E}, \mathcal{E}')$$

Exploring the Space of Jets

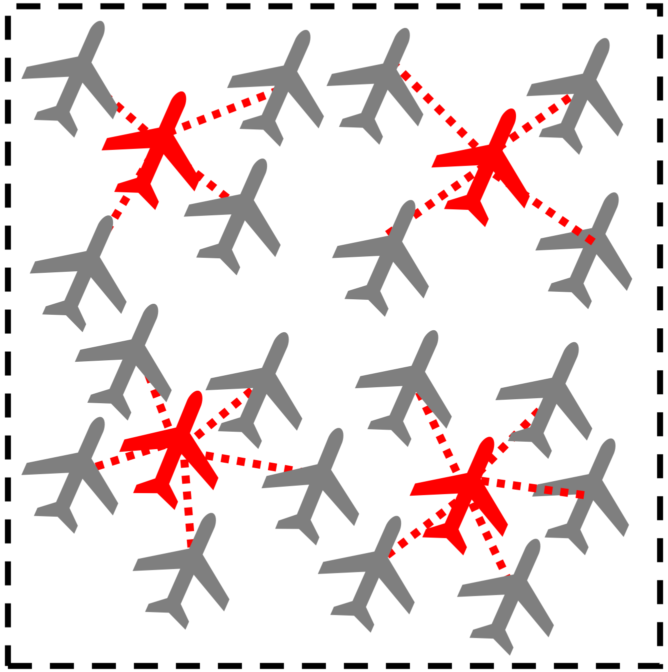


$$\text{EMD}(\mathcal{E}, \mathcal{E}') + \text{EMD}(\mathcal{E}', \mathcal{E}'') \geq \text{EMD}(\mathcal{E}, \mathcal{E}'')$$

Most Representative Jets

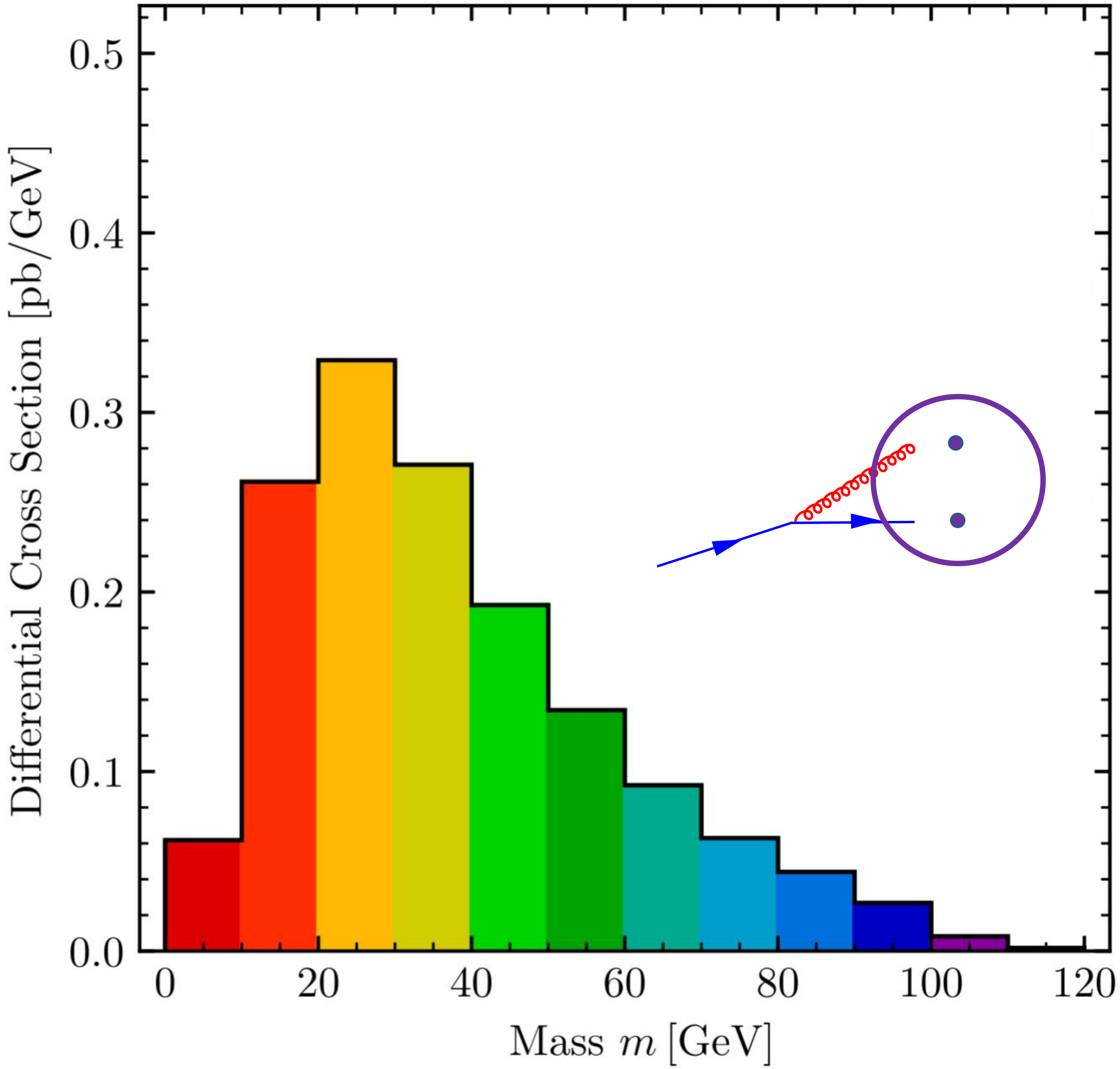
Jet Mass: $m = \left(\sum_{i=1}^{M} p_i^{\mu}\right)^2$

Measures how "wide" the jet is.



[Komiske, Mastandrea, **EMM**, Naik, Thaler, Phys. Rev. D, 1908.08542]

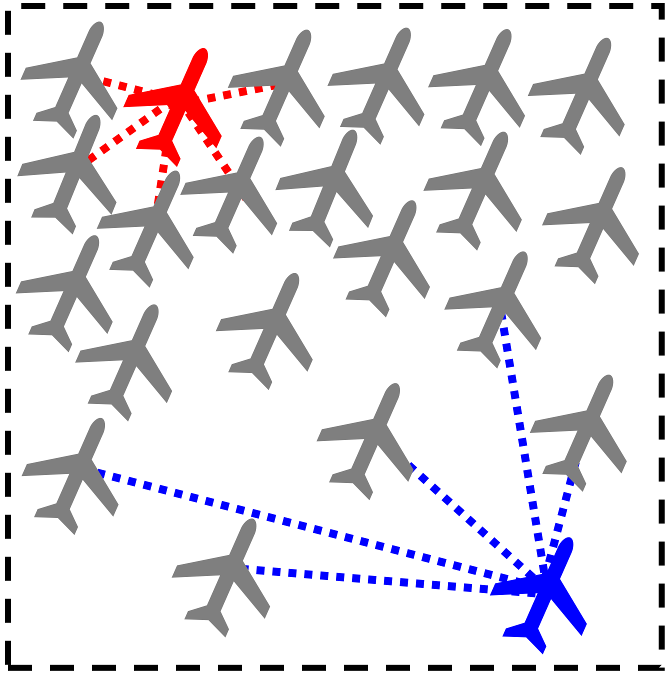
Jet Mass Histogram

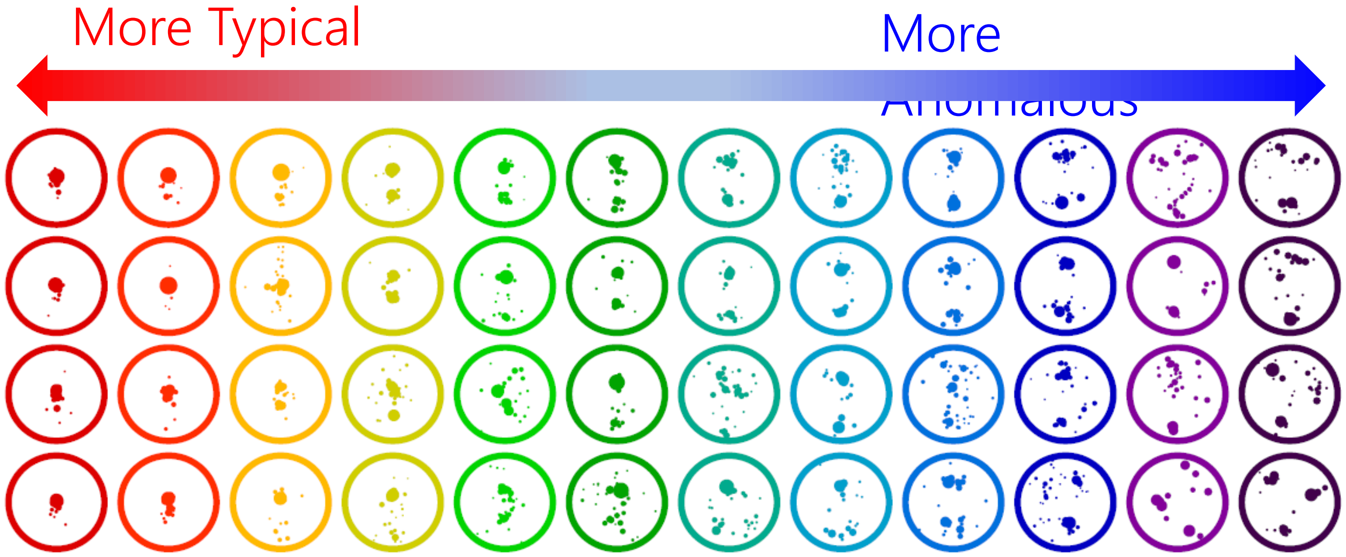


Towards Anomaly Detection

Mean EMD to Dataset:

$$\bar{Q}(\mathcal{E}) = \sum_{i=1}^{N} \mathrm{EMD}\,(\mathcal{E}, \mathcal{E}_i)$$



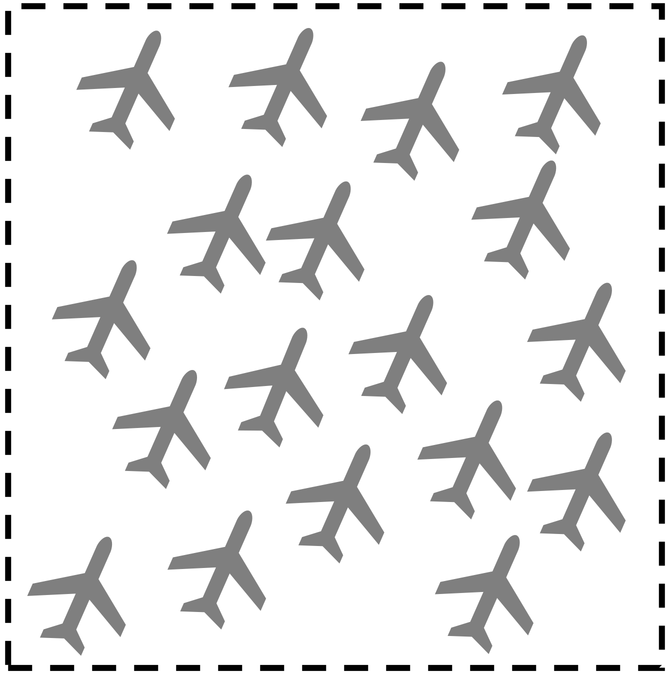


Complements recent developments in anomaly detection for collider physics.

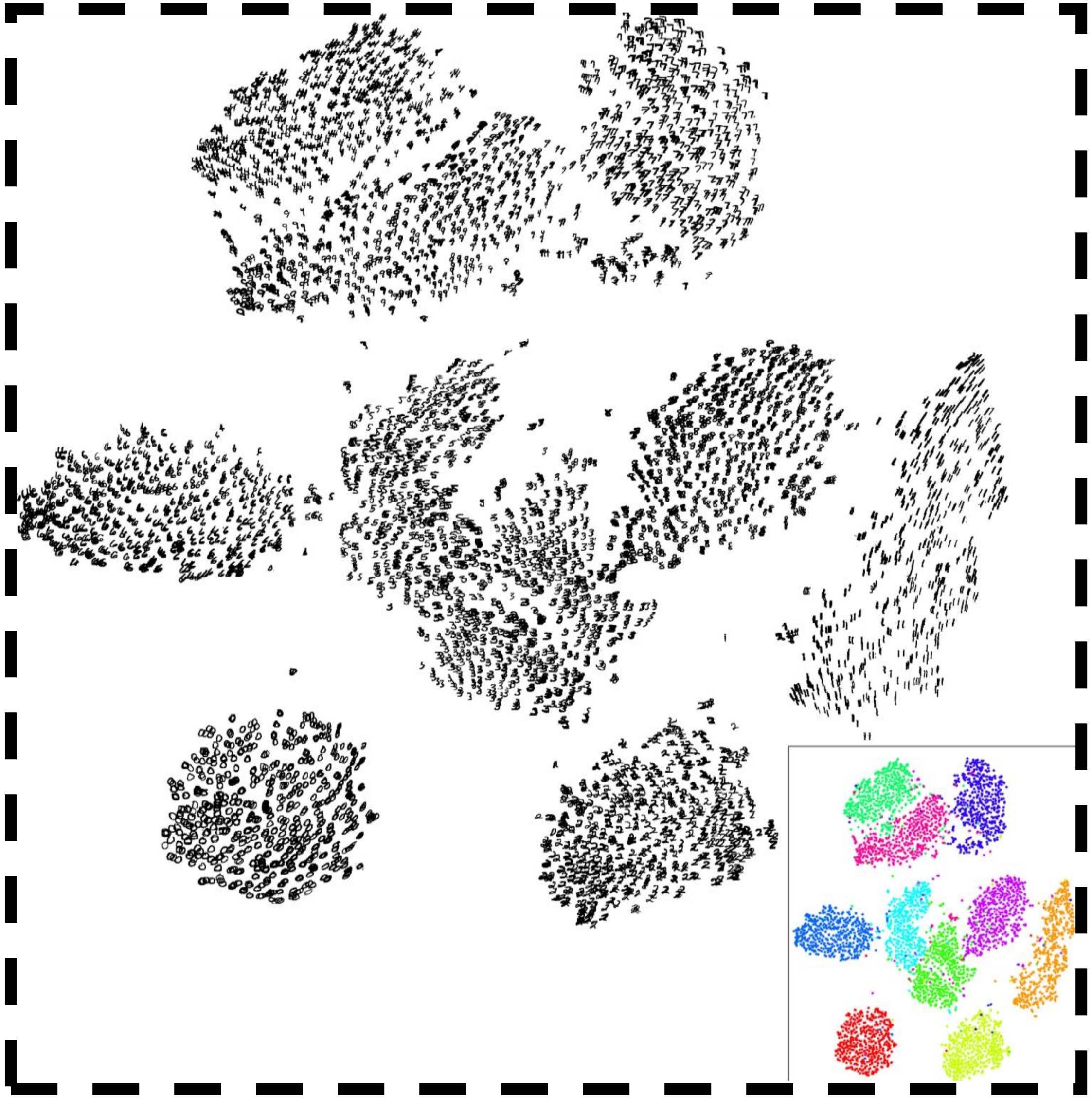
[Collins, Howe, Nachman, 1805.02664]
[Heimel, Kasieczka, Plehn, Thompson, 1808.08979]
[Farina, Nakai, Shih, 1808.08992]
[Cerri, Nguyen, Pierini, Spiropulu, Vlimant, 1811.10276]

Visualizing the Manifold

What does the space of jets look like?



[van der Maaten, Hinton, JMLR 2008]



t-SNE embedding

Visualizing the Manifold

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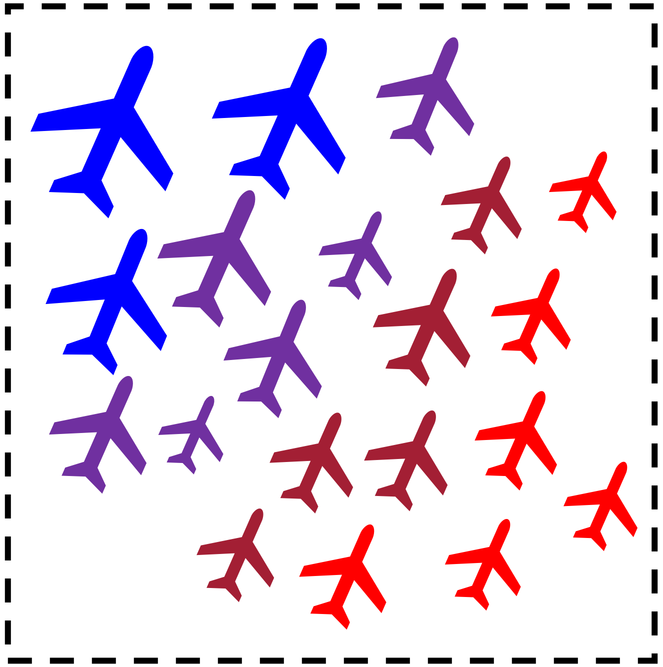

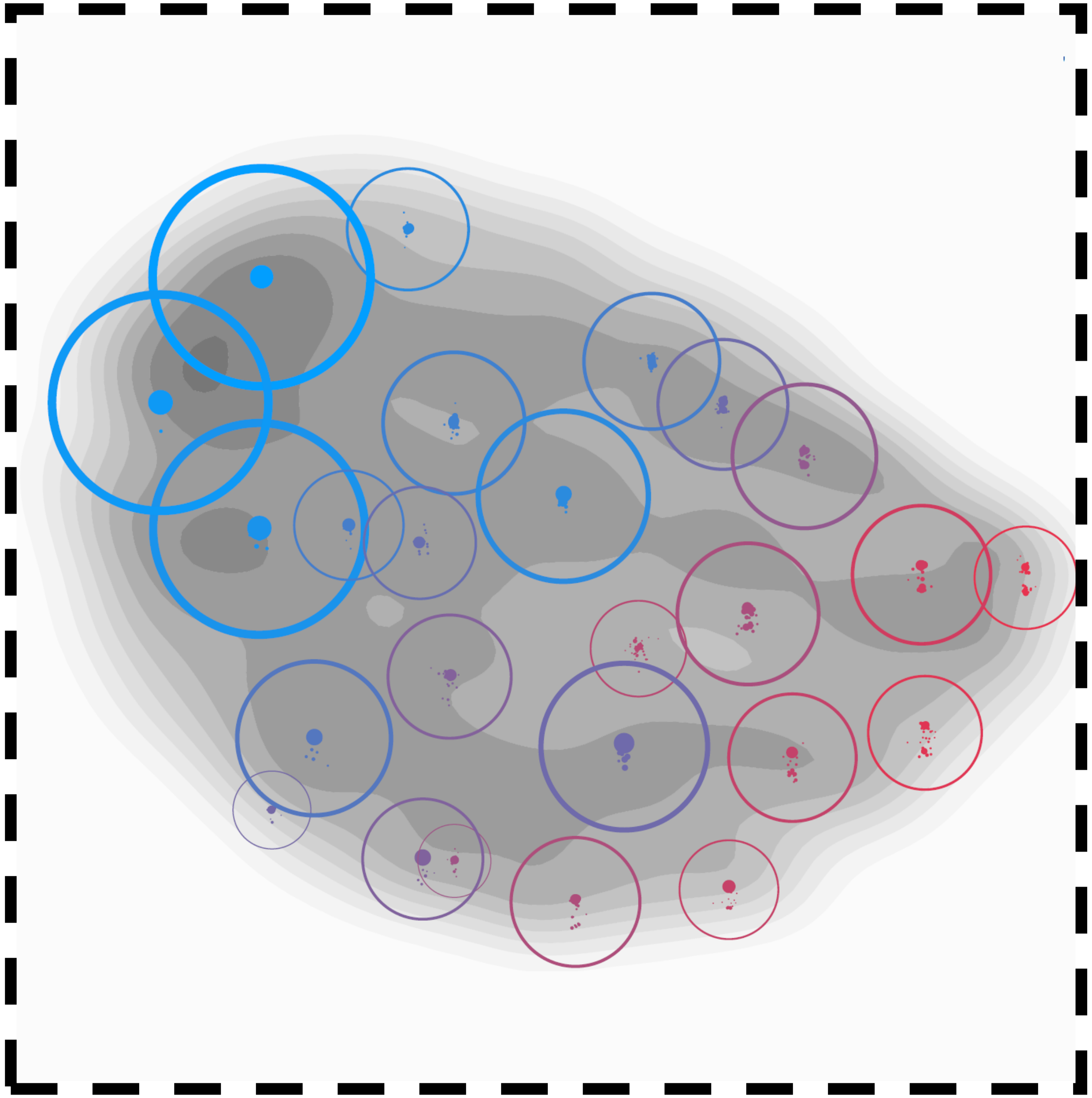

t-SNE embedding: 25-medoid jets shown

[van der Maaten, Hinton, JMLR 2008]

[Komiske, Mastandrea, **EMM**, Naik, Thaler, Phys. Rev. D, 1908.08542]

Visualizing the Manifold

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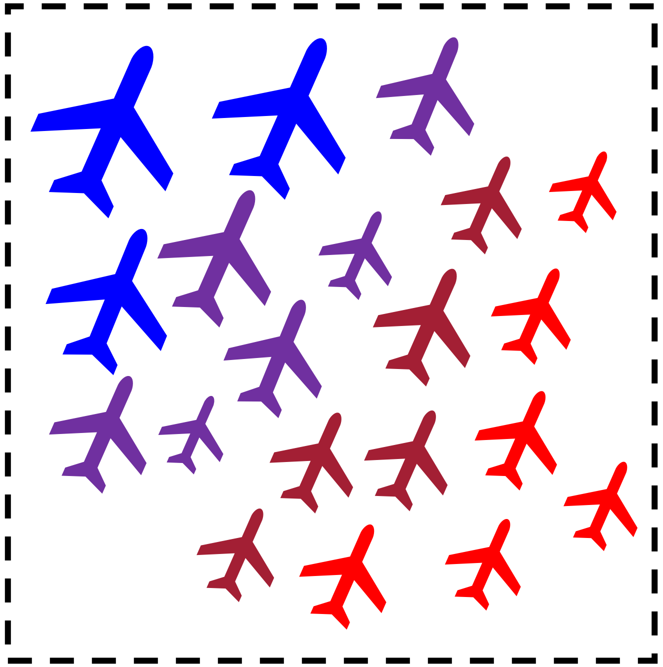

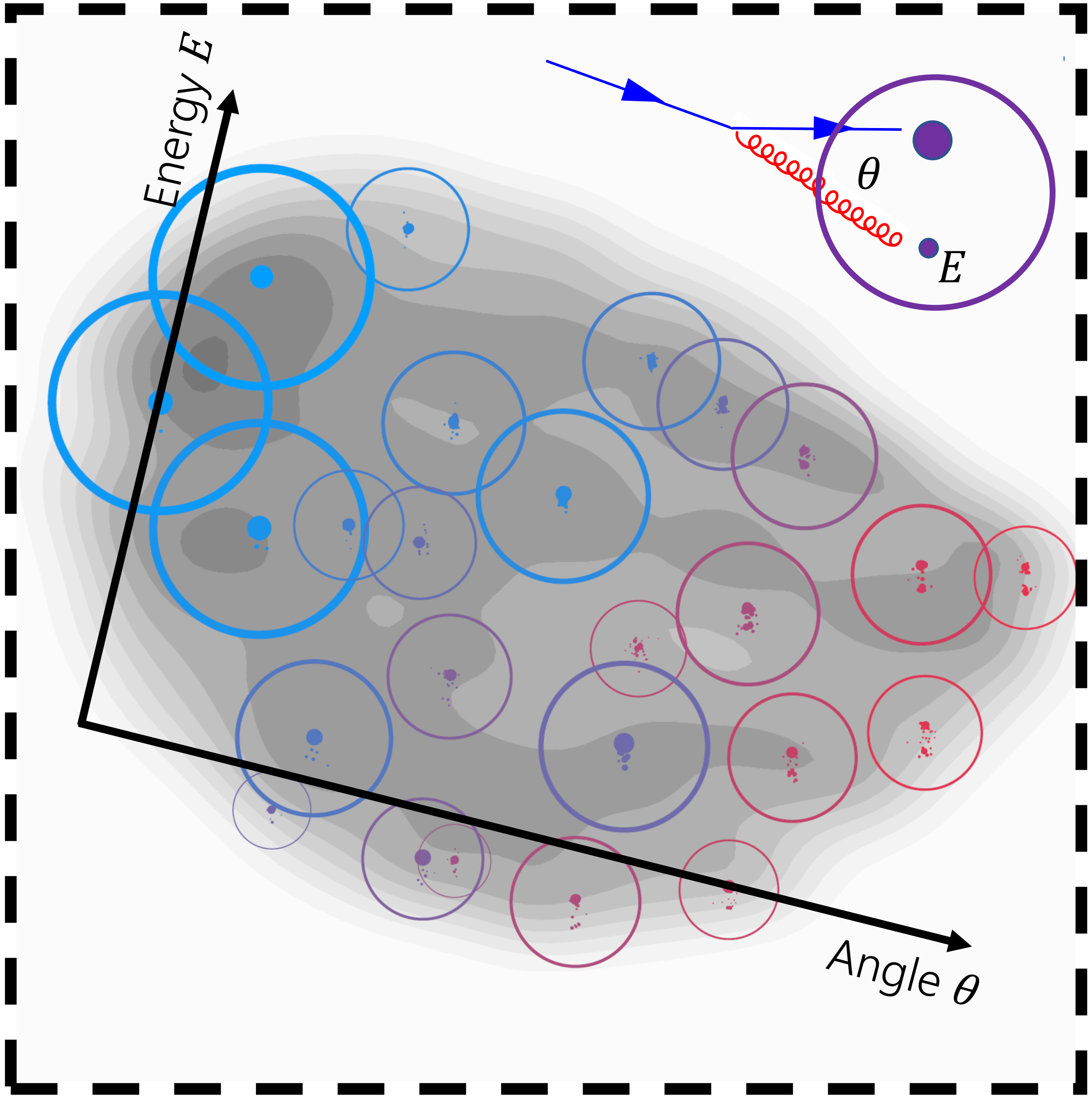

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